

Hadronic corrections to Bhabha scattering

Tord Riemann, DESY, Zeuthen

based on work with:

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Working Group on Radiative Corrections and Generators for Low
Energy Hadronic Cross Section and Luminosity

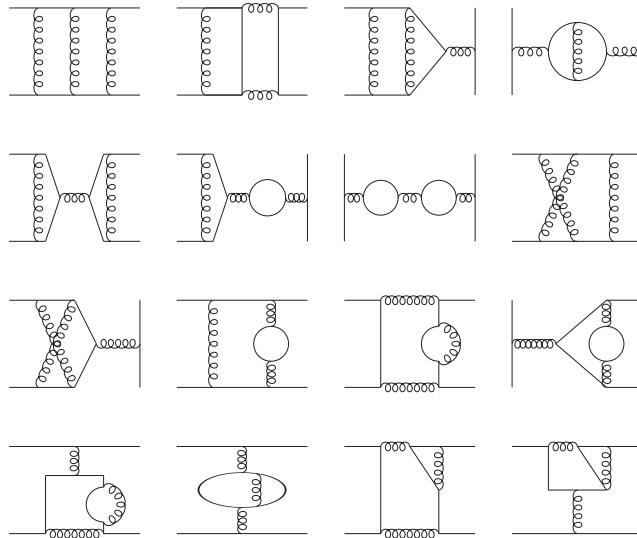
11 April 2008, Frascati

- **Introduction: Two-loop corrections to Bhabha Scattering**
- **Leptonic contributions with $m_e^2 \ll m_f^2 \ll s, t$**
ACGR: NPB 786 (2007) [arXiv:0704.2400]
- **Leptonic contributions with $m_e^2 \ll m_f^2, s, t$**
ACGR: APP B38 (2007) [arXiv:0710.5111] → see also talk by Roberto Bonziani
- **Hadronic contributions**
ACGR: PRL 100 (2008) [arXiv:0711.53847]
- **Summary**

Two Loop Bhabha Scattering

To calculate Bhabha scattering it is best to first compute $e^+e^- \rightarrow \mu^+\mu^-$, since it's closely related but has less diagrams.

There are 47 QED diagrams contributing to $e^+e^- \rightarrow \mu^+\mu^-$.



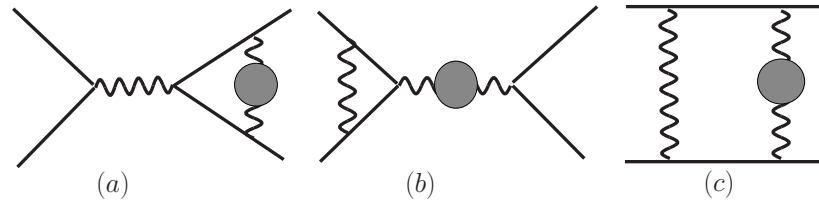
The Bhabha scattering amplitude can be obtained from $e^+e^- \rightarrow \mu^+\mu^-$ simply by summing it with the crossed amplitude (including fermi minus sign).

The diagrams with electrons and photons define an $n_f = 1$ problem.

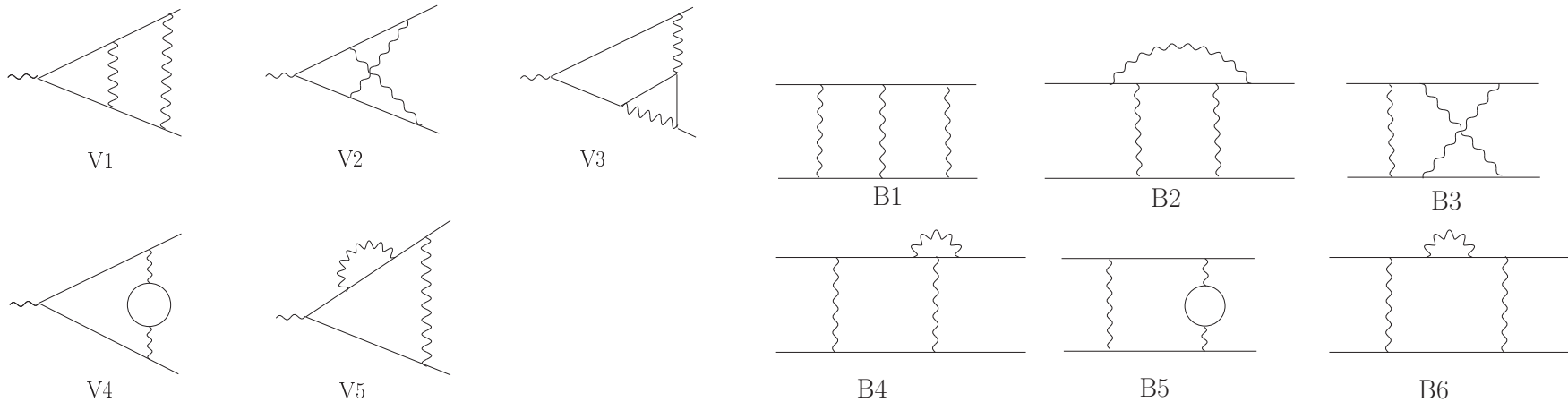
But there are additional ones with heavier fermions.

So we have to investigate an $n_f = 2$ problem

For self-energies starting with 1-loops, and for vertices and boxes starting with 2-loops:



The technical target: photonic and fermionic $N_f = 1, 2$ topologies

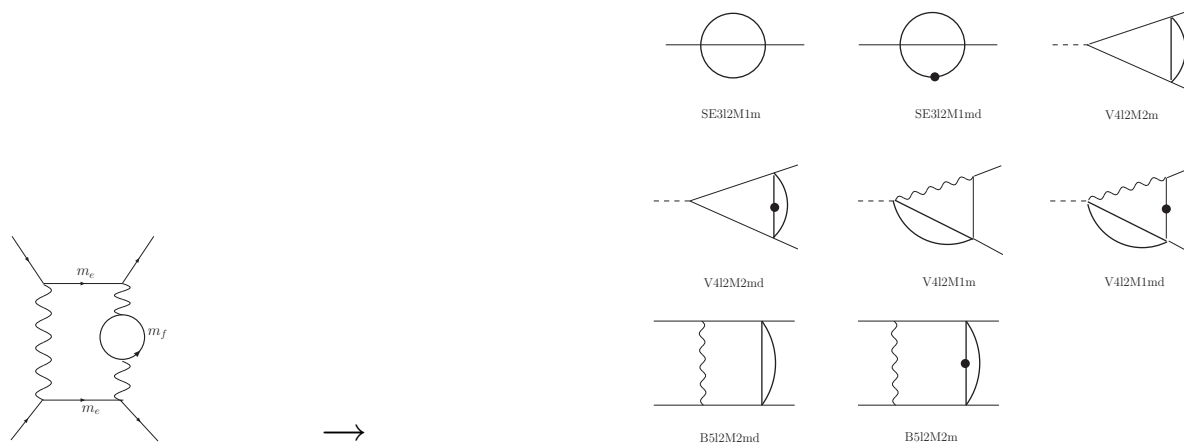


- The **unsolved** problem, even in the limit $m_e^2 \ll s, t$:
The **non-planar photonic 2-loop boxes B3**
- Finally the photonic corrections were derived from massless case by A. Penin ...
- ... and the $n_f = 1$ electron loops **V4, B5** by R. Bonciani et al., and later also by ACGR
- We think we know how to do the non-planar boxes, but it is not easy

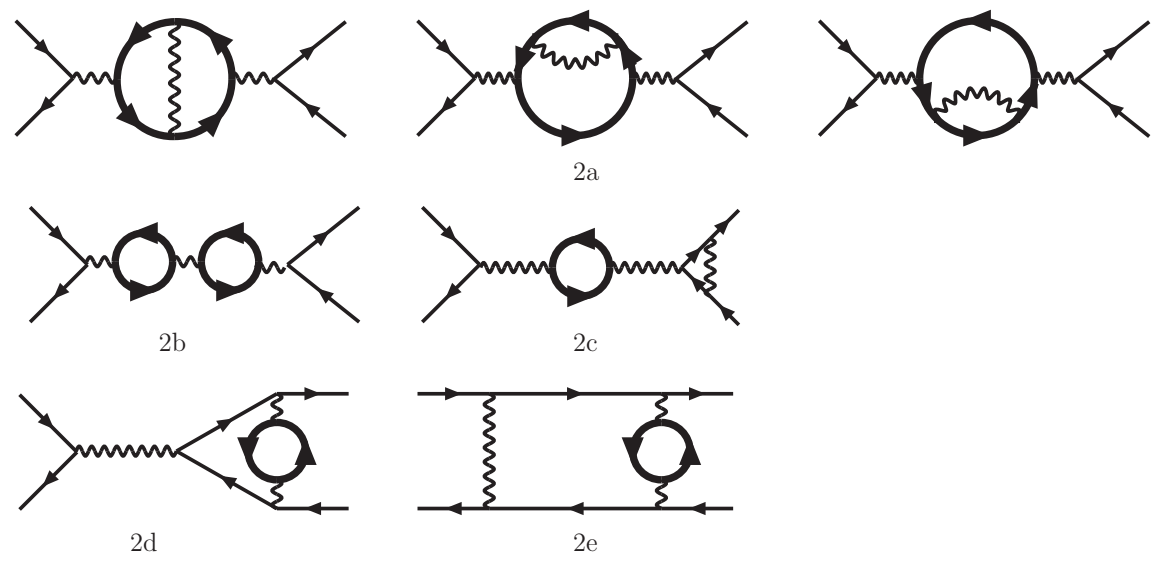
The $n_f = 2$ contributions have been determined in 2007

- **Self-energies** are not a two-masses-problem
- **2-vertices** are known (for $m_e^2 = m_f^2$ and $m_e^2 \ll m_f^2$): G. Burgers PLB 164 (1985), Kniehl, Krawczyk, Kühn, Stuart PLB 209 (1988)
- What is really new: the **2-boxes** with two different fermions involved

The 8 box-master integrals were identified in ACGR, PRD 71 (2005) [hep-ph/0412164]

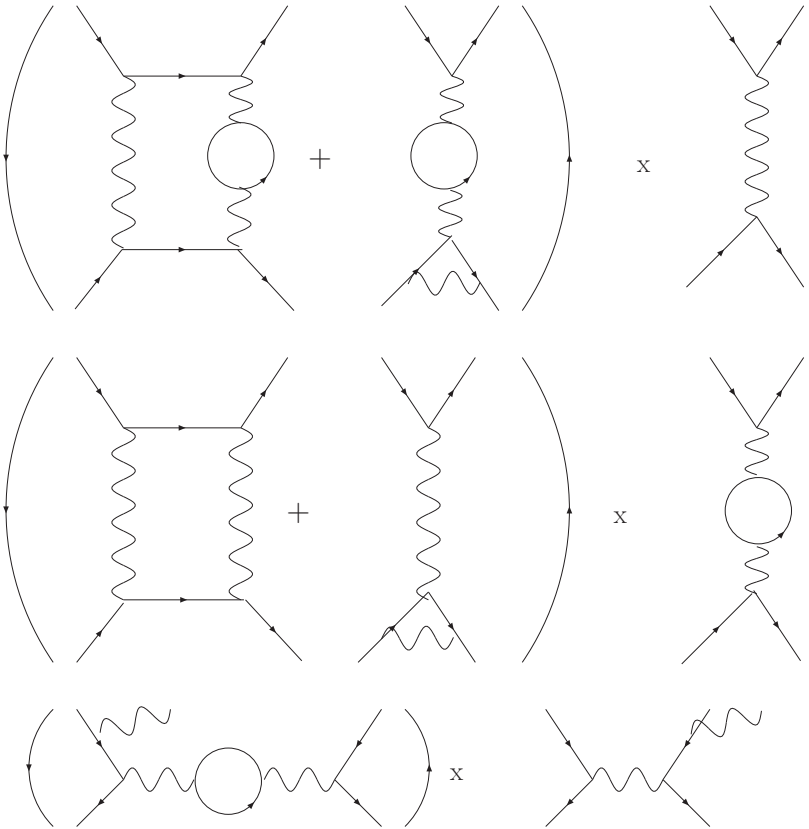


- $m_e^2 \ll m_f^2 \ll s, t$: Becher, Melnikov JHEP 6 (2007) and ACGR NPB 786 (2007)
- $m_e^2 \ll m_f^2, s, t$: ACGR 0710.5111 \rightarrow APP B38 (2007)
and Bonciani, Ferroglia, Penin 0710.4775 (2007)
- $m_e^2 \ll m_{hadrons}^2, s, t$: ACGR 0711.3847 \rightarrow PRL 100 (2008) **THIS TALK**



Classes of Bhabha-scattering 2-loop diagrams containing at least one fermion loop.

The 4 direct and 4 crossed fermionic 2-loop box diagrams have to be combined with other diagrams for an **IR-finite** contribution:



After combining the **2-loop** terms with the **loop-by-loop** terms and with **soft real** corrections:

$$\begin{aligned} \frac{d\sigma^{\text{NNLO}}}{d\Omega} + \frac{d\sigma_{\gamma}^{\text{NLO}}}{d\Omega} &= \frac{d\sigma^{\text{NNLO,e}}}{d\Omega} + \sum_{f \neq e} Q_f^2 \frac{d\sigma^{\text{NNLO},f^2}}{d\Omega} + \sum_{f \neq e} Q_f^4 \frac{d\sigma^{\text{NNLO},f^4}}{d\Omega} \\ &+ \sum_{f_1, f_2 \neq e} Q_{f_1}^2 Q_{f_2}^2 \frac{d\sigma^{\text{NNLO},2f}}{d\Omega}. \end{aligned}$$

The Box Corrections

The contribution of the renormalized two-loop box diagrams of class 2e is given by

$$\frac{d\sigma^{2e \times \text{tree}}}{d\Omega} = \frac{\alpha^2}{2s} \left[\frac{1}{s} A_1^{2e \times \text{tree}}(s, t) + \frac{1}{t} A_2^{2e \times \text{tree}}(s, t) \right]$$

Here the auxiliary functions can be conveniently expressed through **three independent form factors** $B_{i,f}^{(2)}(x, y)$, where $i = A, B, C$,

$$A_1^{2e \times \text{tree}}(s, t) = F_\epsilon^2 \sum_f Q_f^2 \text{Re} \left[B_{A,f}^{(2)}(s, t) + B_{B,f}^{(2)}(t, s) + B_{C,f}^{(2)}(u, t) - B_{B,f}^{(2)}(u, s) \right],$$

$$A_2^{2e \times \text{tree}}(s, t) = F_\epsilon^2 \sum_f Q_f^2 \text{Re} \left[B_{B,f}^{(2)}(s, t) + B_{A,f}^{(2)}(t, s) - B_{B,f}^{(2)}(u, t) + B_{C,f}^{(2)}(u, s) \right].$$

The normalization factor is

$$F_\epsilon = \left(\frac{m_e^2 \pi e^{\gamma_E}}{\mu^2} \right)^{-\epsilon}$$

How to evaluate the $N_f = 2$ diagrams?

We did it in 2 ways

- Decompose the 2-loop integrals to master integrals, solve them.

Here: In the limit $m_e^2 \ll m_f^2 \ll s, t, u$

This was done in hep-ph/0704240v2 \longrightarrow ACGR, NPB 786 (2007)

- Alternatively, rewrite the 2-loop integrals as dispersion integrals.

Decompose the loop integrals afterwards into master integrals

The master integrals are simpler, of one-loop type, but the numerical dispersion integration remains then.

Advantages of the dispersion integrals:

- get easily the range $m_e^2 \ll m_f^2, s, t, u$
- method applies also to hadronic insertions

Dispersion Integrals

$$\frac{g_{\mu\nu}}{q^2 + i\delta} \rightarrow \frac{g_{\mu\alpha}}{q^2 + i\delta} (q^2 g^{\alpha\beta} - q^\alpha q^\beta) \Pi_{\text{had}}(q^2) \frac{g_{\beta\nu}}{q^2 + i\delta},$$

the once-subtracted dispersion integral

$$\Pi_{\text{had}}(q^2) = -\frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dz}{z} \frac{\text{Im} \Pi_{\text{had}}(z)}{q^2 - z + i\delta}.$$

Finally, one relates $\text{Im} \Pi_{\text{had}}$ to the hadronic cross-section ratio R_{had} ,

$$\text{Im} \Pi_{\text{had}}(z) = -\frac{\alpha}{3} R_{\text{had}}(z) = -\frac{\alpha}{3} \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}(z)}{(4\pi\alpha^2)/(3z)},$$

For heavy fermion insertions, we have instead of $R_{\text{had}}(z)$:

$$R_f(z) = Q_f^2 C_f (1 + 2m_f^2/z) \sqrt{1 - 4m_f^2/z},$$

Replacing the $\Pi_{\text{had}}(q^2)$ in a vertex or in box diagram by the z -dispersion integral and exchanging the $\int d^4k$ with the $\int dz$ creates one-loop diagrams with a subsequent z -integration.

The kernel functions for the dispersion integrals

$$\Delta\alpha(x) = \Delta\alpha_{\text{had}}^{(5)}(x) + \Pi_e(x) + \sum_{f=\mu,\tau,t} \Pi_f(x)$$

$$\Delta\alpha_{\text{had}}^{(5)}(x) = \frac{\alpha}{\pi} \frac{x}{3} \int_{4m_\pi^2}^{\infty} dz \frac{R_{\text{had}}^{(5)}(z)}{z} \frac{1}{x - z + i\delta}$$

$$V_2(x) = V_{2e}(x) + V_{2\text{rest}}(x)$$

$$V_{2\text{rest}}(x) = \int_{4M^2}^{\infty} dz \frac{R(z)}{z} K_V(x + i\delta; z)$$

$$K_V(x; z) = \frac{1}{3} \left\{ -\frac{7}{8} - \frac{z}{2x} + \left(\frac{3}{4} + \frac{z}{2x} \right) \ln \left(-\frac{x}{z} \right) - \frac{1}{2} \left(1 + \frac{z}{x} \right)^2 \left[\zeta_2 - \mathbf{Li}_2 \left(1 + \frac{x}{z} \right) \right] \right\}$$

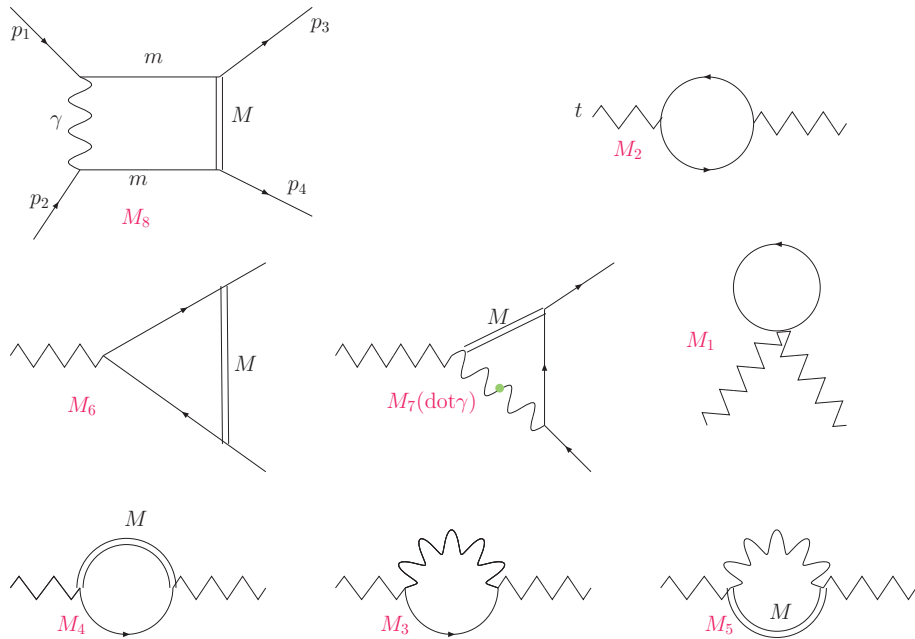
$$B_i(x, y) = \int_{4M^2}^{\infty} dz \frac{R(z)}{z} K_{\text{box},i}(x + i\delta, y + i\delta; z)$$

The $K_{\text{box},i}(x, y; z)$ are determined as linear combinations of one-loop integrals with mass $z = M^2$.

The dispersion master integrals for the $N_f = 2$ contributions

There are three box kernel functions, depending on m_e, m_f, s, t . with $m_e^2 \ll z = m_f^2, s, t$.
 They are IR-divergent.

The eight master integrals for the 2-loop boxes are:



The Box Corrections (repeated here from above)

The contribution of the renormalized two-loop box diagrams of class 2e is given by

$$\frac{d\sigma^{2e \times \text{tree}}}{d\Omega} = \frac{\alpha^2}{2s} \left[\frac{1}{s} A_1^{2e \times \text{tree}}(s, t) + \frac{1}{t} A_2^{2e \times \text{tree}}(s, t) \right]$$

Here the auxiliary functions can be conveniently expressed through three independent form factors $B_{i,f}^{(2)}(x, y)$, where $i = A, B, C$,

$$A_1^{2e \times \text{tree}}(s, t) = F_\epsilon^2 \sum_f Q_f^2 \text{Re} \left[B_{A,f}^{(2)}(s, t) + B_{B,f}^{(2)}(t, s) + B_{C,f}^{(2)}(u, t) - B_{B,f}^{(2)}(u, s) \right],$$

$$A_2^{2e \times \text{tree}}(s, t) = F_\epsilon^2 \sum_f Q_f^2 \text{Re} \left[B_{B,f}^{(2)}(s, t) + B_{A,f}^{(2)}(t, s) - B_{B,f}^{(2)}(u, t) + B_{C,f}^{(2)}(u, s) \right].$$

The normalization factor is

$$F_\epsilon = \left(\frac{m_e^2 \pi e^{\gamma_E}}{\mu^2} \right)^{-\epsilon}$$

Look e.g. at $B_{C,f}^{(2)}(t, s)$ for hadrons:

$$B_{C,had}^{(2)}(t, s) = \int_{4M_\pi^2}^{\infty} \frac{dz}{z} R_{had}(z) K_C(s, t, z)$$

And similarly for leptons:

$$4M_\pi^2 \longrightarrow 4m_l^2$$

$$R_{had}(z) \longrightarrow R_{lep}(z) \sim \sqrt{1 - \frac{4m_l^2}{z}} \left(1 + \frac{2m_l^2}{z}\right) + \epsilon R_{lep}^\epsilon(z)$$

Get:

$$B_{C,lep}^{(2)}(t, s) = \int_{4m_l^2}^{\infty} \frac{dz}{z} R_{lep}(z) K_C(s, t, z)$$

$$K_C(x, y; z) = F_\epsilon \sum_{i=1}^8 c_{Ci} M_i(s, t, z)$$

$$= \frac{1}{3m_e^2(y-z)} \left\{ 2 \frac{F_\epsilon}{\epsilon} x^2 L_x + 4\zeta_2 x^2 \left(\frac{z}{y} - 2\right) - 2(x^2 + y^2 + xy) L_x \right.$$

$$+ x^2 \left(\frac{z}{y} - 1\right) L_y + 2x^2 \left(\frac{z}{y} - 1\right) L_y^2 + 4x^2 L_x L_y + x^2 \left(\frac{z}{y} - 1\right) \ln\left(\frac{z}{m_e^2}\right)$$

$$- 2x^2 \left(\frac{z}{y} - \frac{1}{2}\right) \ln^2\left(\frac{z}{m_e^2}\right) + 4x^2 \left(\frac{z}{y} - 1\right) \ln\left(\frac{z}{m_e^2}\right) \ln\left(1 - \frac{z}{y}\right)$$

$$+ 2x^2 \ln\left(\frac{z}{m_e^2}\right) L_x - x^2 \left(\frac{z}{y} + \frac{y}{z} - 2\right) \ln\left(1 - \frac{z}{y}\right) - 4x^2 \ln\left(1 - \frac{z}{y}\right) L_x$$

$$\left. + 4x^2 \left(\frac{z}{y} - 1\right) \text{Li}_2\left(\frac{z}{y}\right) - 2x^2 \text{Li}_2\left(1 + \frac{z}{x}\right) \right\}.$$

The contributing masters are:

$$M_1 = N \int d^D k \frac{1}{k^2 - m^2}, \quad (1)$$

$$M_2 = N \int d^D k \frac{1}{(k^2 - m^2)[(k - p_1 - p_2)^2 - m^2]}, \quad (2)$$

$$M_3 = N \int d^D k \frac{1}{k^2(k - p_1 + p_3)^2}, \quad (3)$$

$$M_4 = N \int d^D k \frac{1}{(k^2 - m^2)[(k - p_3)^2 - z]}, \quad (4)$$

$$M_5 = N \int d^D k \frac{1}{(k^2 - z)(k - p_1 + p_3)^2}, \quad (5)$$

$$M_6 = N \int d^D k \frac{1}{(k^2 - z)[(k + p_3)^2 - m^2][(k + p_3 - p_1 - p_2)^2 - m^2]}, \quad (6)$$

$$M_7 = N \int d^D k \frac{1}{(k^2 - z)[(k + p_3)^2 - m^2](k + p_3 - p_1)^2}, \quad (7)$$

$$M_8 = N \int d^D k \frac{1}{(k^2 - z)[(k + p_3)^2 - m^2](k + p_3 - p_1)^2[(k + p_3 - p_1 - p_2)^2 - m^2]}, \quad (8)$$

where

$$F_\epsilon = N = m^{2\epsilon} \frac{e^\gamma \epsilon}{i\pi^{2-\epsilon}}. \quad (9)$$

and e.g. the box integral $M_8 = Bo1$ is:

$$\begin{aligned}
 Bo1 = & (4*ep*z^2 + 2*\text{Log}[-(m^2/t)] - 4*ep*\text{Log}[me]*\text{Log}[-(m^2/t)] - \\
 & 4*ep*\text{Log}[1 - m^2/t]*\text{Log}[-(m^2/t)] + 3*ep*\text{Log}[-(m^2/t)]^2 - \\
 & 2*\text{Log}[-(me^2/t)] + 4*ep*\text{Log}[me]*\text{Log}[-(me^2/t)] + \\
 & 4*ep*\text{Log}[1 - m^2/t]*\text{Log}[-(me^2/t)] - 2*ep*\text{Log}[-(m^2/t)]* \\
 & \text{Log}[-(me^2/t)] - ep*\text{Log}[-(me^2/t)]^2 + \text{Log}[-(m^2/s)]* \\
 & (4*ep*\text{Log}[me] + 4*ep*\text{Log}[1 - m^2/t] - 2*(1 + ep*\text{Log}[-(m^2/t)] + \\
 & ep*\text{Log}[-(me^2/t)])) + 2*ep*\text{PolyLog}[2, (m^2 + s)/s])/ \\
 & (2*ep*s*(m^2 - t))
 \end{aligned}$$

$d\sigma / d\Omega$ [nb] \sqrt{s} [GeV]	1	10	91	500
LO QED	46.6409	0.466409	0.00563228	0.000186564
LO Zfitter	46.643	0.468499	0.127292	0.0000854731
NNLO (e)	-0.230927	-0.00453987	-0.0000919387	$-4.28105 \cdot 10^{-6}$
NNLO ($e + \mu$) “	-0.256679	-0.00570942	-0.000122796	$-5.90469 \cdot 10^{-6}$
NNLO ($e + \mu + \tau$) “		-0.00586082	-0.000135449	$-6.7059 \cdot 10^{-6}$
NNLO ($e + \mu + \tau + t$) “				$-6.6927 \cdot 10^{-6}$
NNLO photonic	2.07476	0.0358755	0.000655126	0.0000284063
NNLO IR e	-0.19927	-0.00359349	-0.0000672264	$-2.95317 \cdot 10^{-6}$
NNLO IR μ (analytic)	-0.0314292	-0.00134635	-0.0000335037	$-1.66781 \cdot 10^{-6}$
NNLO IR μ (dispersion)	-0.0333538	-0.00134663	-0.0000335037	$-1.66781 \cdot 10^{-6}$
NNLO IR τ (analytic)		-0.00021027	-0.0000162977	$-1.00877 \cdot 10^{-6}$
NNLO IR τ (dispersion)		-0.000272634	-0.0000163119	$-1.00878 \cdot 10^{-6}$

Table 1: Numerical values for the NNLO corrections to the differential cross section respect to the solid angle. Results are expressed in nanobarns for a scattering angle $\theta = 90^\circ$. Empty entries are related to cases where the high-energy approximation cannot be applied.

Using R_{had}

This is a topic by itself, because R_{had} is basically unpublished.

N.N.1:

Fuer R(s) mit Fehlern, Kontinuum + Resonanzen haben wir nur unsere interne Arbeitsversion.

N.N.2:

This procedure is a follow up of complicated programs, which unfortunately do not exist in a really user-friendly form.

N.N.3:

I understand that for your problem it is probably too cumbersome (and time-consuming) to use the data.

N.N.4:

es hat etwas gedauert, bis ich in meinen alten Verzeichnissen auf einer 1994er Vax am MPI fuendig geworden bin.

So, finally, we might reproduce the old estimates given for the vertex dispersion relation in **Kniesl, Krawczyk, Kühn, Stuart (1988) → finally we have numerics, but with larger errors than necessary**

Final formula and results

We distinguish 3 different categories of 2-loop contributions:

- Running α
- the irreducible 2-loop vertices
- the 'rest': irreducible vertices and boxes plus 2-loop boxes

$$\begin{aligned}
 \frac{d\bar{\sigma}}{d\Omega} = & c \int_{4M_\pi^2}^{\infty} dz \frac{R_{\text{had}}(z)}{z} \frac{1}{t-z} F_1(z) & (10) \\
 + & c \int_{4M_\pi^2}^{\infty} \frac{dz}{z(s-z)} \left\{ R_{\text{had}}(z) \left[F_2(z) + F_3(z) \ln \left| 1 - \frac{z}{s} \right| \right] \right. \\
 - & R_{\text{h}}(s) \left[F_2(s) + F_3(s) \ln \left| 1 - \frac{z}{s} \right| \right] \left. \right\} \\
 + & c \frac{R_{\text{h}}(s)}{s} \left\{ F_2(s) \ln \left(\frac{s}{4M_\pi^2} - 1 \right) - 6\zeta_2 F_a(s) \right. \\
 + & \left. F_3(s) \left[2\zeta_2 + \frac{1}{2} \ln^2 \left(\frac{s}{4M_\pi^2} - 1 \right) + \mathbf{Li}_2 \left(1 - \frac{s}{4M_\pi^2} \right) \right] \right\},
 \end{aligned}$$

with $c = \alpha^4/(\pi^2 s)$ and $R_{\text{h}}(s) = \theta(s - 4M_\pi^2) R_{\text{had}}(s)$.

$$\begin{aligned}
F_1(z) = & \frac{1}{3} \left\{ 9 \bar{c}(s, t) \ln\left(\frac{s}{m_e^2}\right) + \left[-z^2 \left(\frac{1}{s} + \frac{2}{t} + 2 \frac{s}{t^2} \right) + z \left(4 + 4 \frac{s}{t} + 2 \frac{t}{s} \right) + \frac{1}{2} \frac{t^2}{s} + 6 \frac{s^2}{t} \right. \right. \\
& + 5s + 4t \left. \right] \ln\left(-\frac{t}{s}\right) + s \left(-\frac{z}{t} + \frac{3}{2} \right) \ln\left(1 + \frac{t}{s}\right) + \left[\frac{1}{2} \frac{z^2}{s} + 2z \left(1 + \frac{s}{t} \right) - \frac{11}{4} s - 2t \right] \ln^2\left(-\frac{t}{s}\right) \\
& - \left[\frac{1}{2} \frac{z^2}{t} - z \left(1 + \frac{s}{t} \right) + \frac{t^2}{s} + 2 \frac{s^2}{t} + \frac{9}{2} s + \frac{15}{4} t \right] \ln^2\left(1 + \frac{t}{s}\right) + \left[\frac{z^2}{t} - 2z \left(1 + \frac{s}{t} \right) + 2 \frac{s^2}{t} + 5s + \frac{5}{2} t \right] \\
& \times \ln\left(-\frac{t}{s}\right) \ln\left(1 + \frac{t}{s}\right) - 4 \left[\frac{t^2}{s} + 2 \frac{s^2}{t} + 3(s+t) \right] \left[1 + \text{Li}_2\left(-\frac{t}{s}\right) \right] - \left[\frac{t^2}{s} + 2 \frac{s^2}{t} + 3(s+t) \right] \ln\left(\frac{z}{s}\right) \ln\left(1 + \frac{t}{s}\right) \\
& - \left[2 \frac{z^2}{t} - 4z \left(1 + \frac{s}{t} \right) - 4 \frac{t^2}{s} - 2 \frac{s^2}{t} + s - \frac{11}{2} t \right] \zeta_2 + \left[z^2 \left(\frac{1}{s} + 2 \frac{s}{t^2} + \frac{2}{t} \right) - z \left(\frac{t}{s} + 2 \frac{s}{t} + 2 \right) \right] \ln\left(\frac{z}{s}\right) \\
& - \left[z^2 \left(\frac{1}{s} + \frac{1}{t} \right) + 2z \left(1 + \frac{s}{t} \right) + s + 2 \frac{s^2}{t} \right] \ln\left(\frac{z}{s}\right) \ln\left(1 + \frac{z}{s}\right) + \left[\frac{z^2}{s} + 4z \left(1 + \frac{s}{t} \right) - \frac{t^2}{s} - 4(s+t) \right] \\
& \times \ln\left(\frac{z}{s}\right) \ln\left(1 - \frac{z}{t}\right) - \left[z^2 \left(\frac{1}{s} + 2 \frac{s}{t^2} + \frac{2}{t} \right) - 2z \left(\frac{t}{s} + 2 \frac{s}{t} + 2 \right) + \frac{t^2}{s} + 2(s+t) \right] \ln\left(1 - \frac{z}{t}\right) \\
& + \left[\frac{z^2}{t} - 2z \left(1 + \frac{s}{t} \right) + 2 \frac{t^2}{s} + 8s + 4 \frac{s^2}{t} + 7t \right] \ln\left(1 - \frac{z}{t}\right) \ln\left(1 + \frac{t}{s}\right) + \left[\frac{z^2}{s} + 4z \left(1 + \frac{s}{t} \right) - \frac{t^2}{s} - 4(s+t) \right] \\
& \times \text{Li}_2\left(\frac{z}{t}\right) - \left[z^2 \left(\frac{1}{s} + \frac{1}{t} \right) + 2z \left(1 + \frac{s}{t} \right) + s + 2 \frac{s^2}{t} \right] \text{Li}_2\left(-\frac{z}{s}\right) - \left[\frac{z^2}{t} - 2z \left(1 + \frac{s}{t} \right) + \frac{t^2}{s} + 5s + 2 \frac{s^2}{t} + 4t \right] \\
& \times \text{Li}_2\left(1 + \frac{z}{u}\right) \left. \right\} + 4 \bar{c}(s, t) \ln\left(\frac{2\omega}{\sqrt{s}}\right) \left[\ln\left(\frac{s}{m_e^2}\right) + \ln\left(-\frac{t}{s}\right) - \ln\left(1 + \frac{t}{s}\right) - 1 \right],
\end{aligned}$$

and similarly for $F_2(z)$ and $F_3(z)$.

The $\int_{4M^2} dz F_i(z)$ gives from the lower integration bound the logarithmically enhanced terms $\ln(= M^2)^n$, e.g. from terms like $A(z) \ln(z/s)$ or from $B(z) \text{Li}_2\left(\frac{z}{s}\right)$.

Some numerical results

We will now discuss the numerical net effects arising from the $N_f = 2$ vertex plus box diagrams (i.e. excluding the pure running coupling effects):

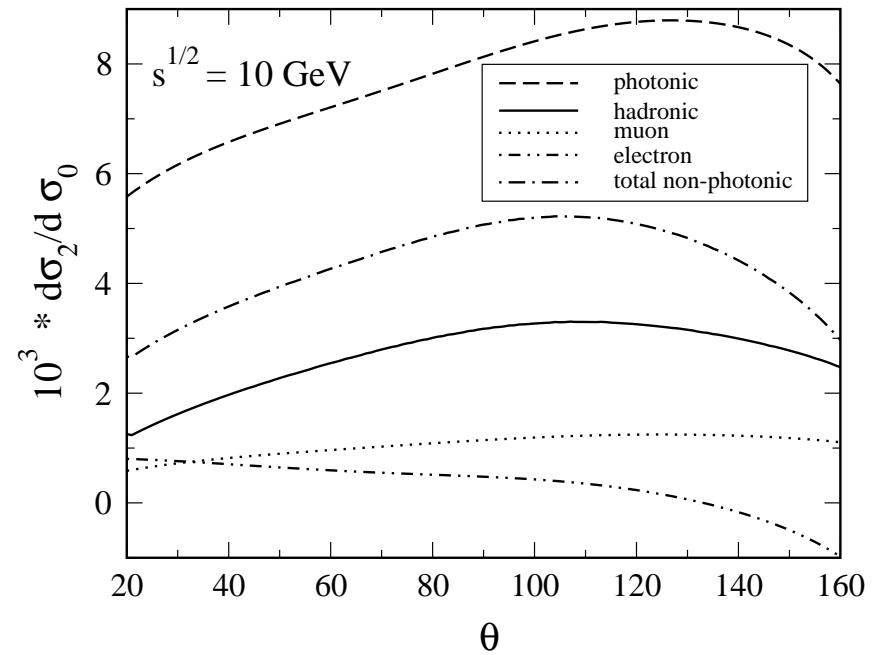
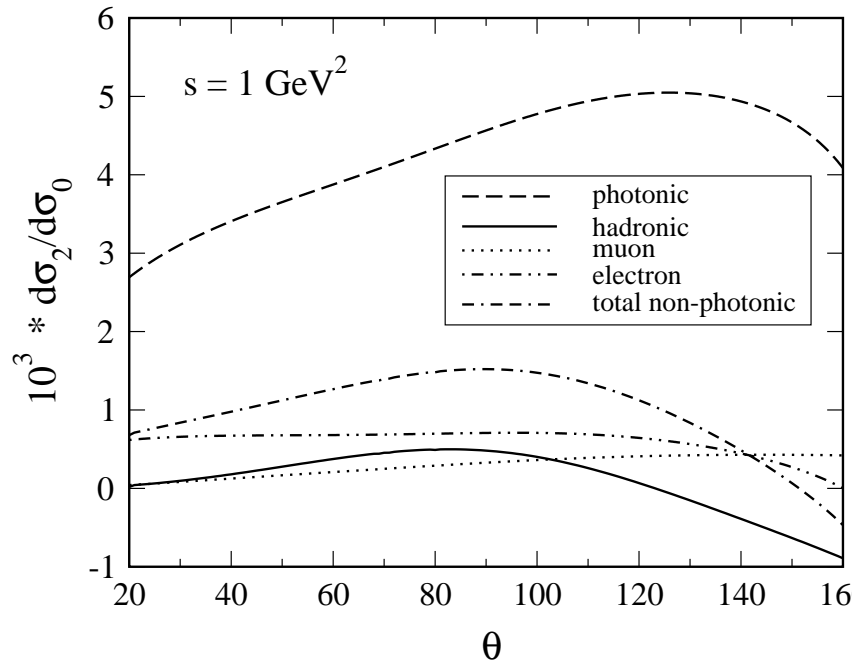
$$\frac{d\sigma_2}{d\Omega} = \frac{d\bar{\sigma}}{d\Omega} + \frac{d\sigma_v}{d\Omega},$$

with $d\bar{\sigma}/d\Omega$ from Eqn. (10). The expression for the irreducible vertex term $d\sigma_v/d\Omega$ derives directly from

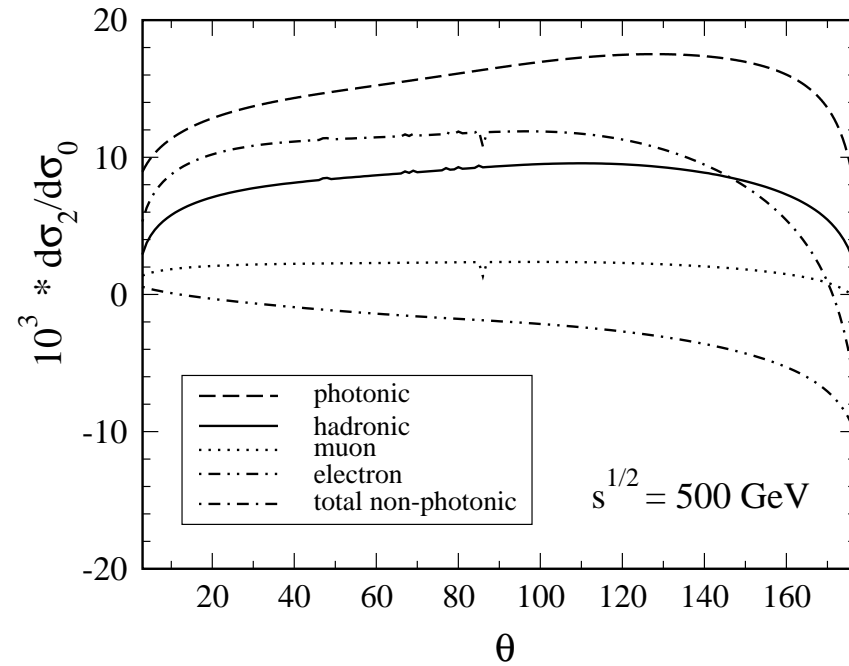
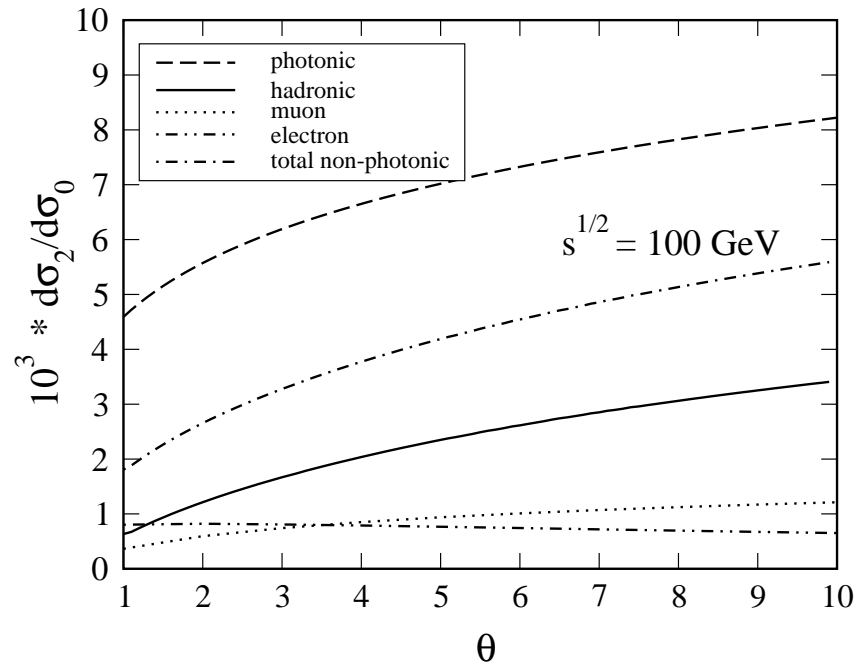
[Kniehl:1988id,webPage:2007x3]

. The $d\sigma_2/d\Omega$ is normalized to the pure photonic Bhabha Born cross section $d\sigma_0/d\Omega$:

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{s} \left(\frac{s}{t} + 1 + \frac{t}{s} \right)^2.$$



Two-loop $N_f = 2$ vertex and box corrections $d\sigma_2$ to Bhabha scattering in units of $10^{-3}d\sigma_0$ at meson factories, $\sqrt{s} = 1 \text{ GeV}$ (a) and $\sqrt{s} = 10 \text{ GeV}$ (b).



Two-loop $N_f = 2$ vertex and box corrections $d\sigma_2$ to Bhabha scattering in units of $10^{-3}d\sigma_0$ at ILC energies of $\sqrt{s} = 100$ GeV (GigaZ option) and $\sqrt{s} = 500$ GeV.

Summary

- We determine the $N_f = 2$ contributions to 2-loop Bhabha scattering, including the hadronic corrections
- They are small, but non-negligible at the scale 10^{-3} (\rightarrow **No LEP influencing**)
- Agreement for $m_e^2 \ll m_i^2 \ll s, t, u$ with:
"Two-loop QED corrections to Bhabha scattering"
Thomas Becher, Kirill Melnikov, arXiv:0704.3582 [hep-ph], JHEP
- Agreement for $m_e^2 \ll m_i^2, s, t, u$ with:
"Two-Loop Heavy-Flavor Contribution to Bhabha Scattering",
Roberto Bonciani, Andrea Ferroglia, Sacha Penin, arXiv:0710.4775v3 [hep-ph]
- To be evaluated yet:
 \rightarrow **1-loop diagrams with real photon emission**, interfering with real (Born) radiation, including 5-point functions
- Also: \rightarrow **Real pair production**
- Both items were studied already by Andrei Arbuzov, Kuraev, Shaitchatdenov (1998, small photon mass)