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Electroweak Parameters at HERA: Theoretical Aspects

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Abstract: The basic issues of electroweak parameters and their relation to deep inelastic scattering are presented. In particular we discuss the interdependence of the vector boson masses, the NC and CC coupling constants at the level of radiative corrections, and the implications for NC and CC cross sections at HERA.

1 Introduction

The fundamental process in neutral and charged current deep inelastic lepton-proton scattering is the 4-fermion scattering between the lepton and the quarks inside the nucleon. In lowest order this process is of pure electroweak origin, in the Standard Model mediated by the exchange of photons and weak Z^0, $W^\pm$ bosons.

The electroweak Standard Model contains, besides fermion masses, quark mixing angles, and the mass of the Higgs scalar, three free parameters in the gauge sector. For a comparison between theory and experiment three independent experimental input data are required for fixing the SU(2) and U(1) gauge coupling constants g_2, g_1, and the vacuum expectation value v of the Higgs field. It is, however, more practical to deal with parameters which have a more direct relation to experiments. For deep inelastic scattering a natural choice is given by the electromagnetic fine structure constant α and the masses of the Z and W bosons characterizing the photon, neutral current Z and charged current W exchange. The masses M_H, m_t of the Higgs boson and of the top quark enter the higher order calculations as additional free parameters unless a direct experimental determination of their values is available.

The existing calculations of electroweak radiative corrections for HERA processes (see [1] and the references therein) follow the lines of the on-shell scheme which treats the masses of the vector bosons M_Z and M_W in a symmetric way: the amplitudes for 4-fermion processes are expressed in terms of M_Z and M_W (besides α) as the basic quantities, together with m_t and M_H in the loop contributions. The Fermi constant G_μ, measured from the muon lifetime, can then be calculated as a constraint

$$G_\mu \;=\; G_\mu(\alpha, M_Z, M_W, M_H, m_t) \tag{1}$$

on the value of M_W after specifying the other quantities. The relation (1) comprises those radiative corrections to the muon lifetime which are beyond the traditional QED corrections to the Fermi model. It is thus one of the crucial tools for testing the electroweak theory at the quantum level. The great phenomenological relevance of (1) consists of the possibility to derive bounds on m_t, M_H, or to predict M_W for given values of m_t, M_H and confront it with experimental results on the W mass. Since M_W is the basic quantity for the deep-inelastic CC cross section, measurements in particular of σ_{CC} provide consistency checks of the vector boson mass interdependence in the Standard Model.

In this contribution we give a discussion of the basic theoretical relations between the electroweak parameters, in particular of the vector boson masses, the NC and CC coupling constants at the level of radiative corrections, and the implications for neutral and charged current cross sections at HERA. Special emphasis is devoted to the ratio $R_- = \sigma_{NC}(e^-)/\sigma_{CC}(e^-)$, and to the left-right asymmetry A_- from polarized electrons with respect to their dependence on the basic parameters of the Standard Model. The numerical results have been checked by various independent calculations yielding agreement at the level of a few per mille.

2 Parameters and renormalization schemes

For describing scattering processes between light fermions in lowest order we can neglect the exchange of Higgs bosons because of their small Yukawa couplings to the known fermions. The standard processes accessible by the experimental facilities are basically 4-fermion processes. These are mediated by the gauge bosons, in their coupling structure defined by the vertices for the fermions interacting with the vector bosons. The neutral and charged current vertices are given by

$$\frac{e}{2\sin\theta_W\cos\theta_W}\,\gamma_\mu\,(I_3^f - 2Q_f\,\sin^2\theta_W - I_3^f\gamma_5)\,, \qquad \frac{e}{2\sqrt{2}\sin\theta_W}\,\gamma_\mu(1-\gamma_5), \tag{2}$$

where Q_f and I_3^f denote the charge and the third isospin component of f. The mixing angle is related to the vector boson masses in general by

$$\sin^2\theta_W \;=\; 1 - \frac{M_W^2}{\rho_0\, M_Z^2} \tag{3}$$

where $\rho_0 \neq 1$ at the tree level in case of a more complicated Higgs system than with doublets only. When discussing radiative corrections we restrict ourselves mainly to the minimal model with $\rho_0 = 1$. The abbreviation s_W^2

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \tag{4}$$

will always be utilized for this particular meaning of $\sin^2\theta_W$ in terms of the boson mass ratio. Thus we are left with e, M_W, M_Z as the basic set of free parameters. In higher order, the parameters get contributions from self energy and vertex correction diagrams and it has to be specified how each measurable parameter is related to a defining experiment. The specification in terms of the parameters above is conventionally called the electroweak on-shell scheme [2]. Alternatively, one may use as basic parameters α, G_μ, M_Z [3], or α, G_μ, $\sin^2\theta_W$ with the mixing angle deduced from neutrino-electron scattering [4], or perform the loop calculations in the $\overline{MS}$ scheme [5]. The so-called $*$-scheme [6] is a different way of book-keeping in terms of effective couplings. After incorporating also the leading terms beyond the 1-loop order, differences between the various schemes (scheme dependence) appear only in higher order subleading terms and are numerically insignificant. In the W mass e.g. (keeping all the other parameters fixed) they amount at most to 20 MeV [7]. Here we proceed according to the on-shell scheme.

3 The vector boson masses

The masses M_W, M_Z of the vector bosons are correlated by means of the muon decay constant $G_\mu = 1.16639(2)\cdot 10^{-5}\,\mathrm{GeV}^{-2}$ and the electromagnetic fine structure constant $\alpha^{-1} = 137.0359895(61)$. In 1-loop order of the Standard Model, G_μ is given by the expression

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}s_W^2 M_W^2}\left[1+\Delta r(\alpha, M_W, M_Z, M_H, m_t)\right]\,. \tag{5}$$

The Z mass has been precisely measured at LEP to be [8]

$$M_Z = 91.175 \pm 0.021\,\mathrm{GeV}.$$

Already at 1-loop order, Δr is a relatively intricate expression, and the following decomposition is useful to identify large contributions of distinct origin parametrized by $\Delta\alpha$ and $\Delta\rho$:

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho + (\Delta r)_{\mathrm{rem}}\,. \tag{6}$$

$\Delta\alpha$ means the subtracted fermionic part of the photon vacuum polarization

$$\Delta\alpha = \Pi_f^\gamma(0) - \mathrm{Re}\,\Pi_f^\gamma(M_Z^2) \tag{7}$$

and contains the large logarithmic corrections from the light fermions:

$$\Delta\alpha = \sum_{lept}\frac{\alpha}{3\pi}\left(\log\frac{M_Z^2}{m_f^2} - \frac{5}{3}\right) + \Delta\alpha_{had}\,, \tag{8}$$

where

$$\Delta\alpha_{had} = -\frac{\alpha}{3\pi}\,M_Z^2\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds'\,\frac{R^\gamma(s')}{s'(s'-M_Z^2-i\varepsilon)}$$

with

$$R^\gamma(s) = \frac{\sigma(e^+e^-\to\gamma^*\to hadrons)}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)}$$

as an experimental quantity yielding [9]

$$\Delta\alpha = 0.0595\pm 0.0009\ \text{ for } M_Z = 91.175\,\mathrm{GeV}. \tag{9}$$

$\Delta\rho$ is the leading quadratic correction from a large top mass [10]

$$\Delta\rho = \frac{3\alpha}{16\pi s_W^2 c_W^2} \frac{m_t^2}{M_Z^2} . \tag{10}$$

All other terms are collected in the $(\Delta r)_{\text{rem}}$. It contains a term logarithmic in the top mass

$$(\Delta r)_{\text{rem}}^{top} = -\frac{\alpha}{4\pi s_W^2} \left(\frac{c_W^2}{s_W^2} - \frac{1}{3} \right) \log \frac{m_t}{M_Z} + \cdots \tag{11}$$

and the Higgs boson contribution which increases only logarithmically for large M_H at 1-loop according to the screening theorem [11]:

$$(\Delta r)_{\text{rem}}^{Higgs} \simeq \frac{\alpha}{16\pi s_W^2} \cdot \frac{11}{3} \left(\log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) . \tag{12}$$

The typical size of $(\Delta r)_{\text{rem}}$ is of the order ~ 0.01. For more details see e.g. [12].

The presence of large terms in Δr requires the consideration of higher than 1-loop effects. The modification of Eq. (5) in the following way

$$1 + \Delta r \;\rightarrow\; \frac{1}{(1 - \Delta\alpha) \cdot (1 + \frac{c_W^2}{s_W^2} \Delta\overline{\rho}) \;-\; (\Delta r)_{remainder}} \equiv \frac{1}{1 - \Delta r} \tag{13}$$

with

$$\Delta\overline{\rho} = 3\, \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}} \cdot \left[1 + \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}} (19 - 2\pi^2) \right] \;+\; \Delta\rho^{\alpha\alpha_s} \tag{14}$$

accomodates the following higher order terms (Δr in the denominator is an effective correction including higher orders):

- The resummation of $\Delta\alpha$: the replacement $1 + \Delta\alpha \;\rightarrow\; (1 - \Delta\alpha)^{-1}$ correctly takes into account all orders in the leading logarithmic corrections $(\Delta\alpha)^n$ according to the renormalization group [13].

- The resummation of the leading m_t^2 contribution [14] in terms of $\Delta\overline{\rho}$. Thereby, however, also irreducible higher order diagrams are required. $\Delta\overline{\rho}$ in Eq. (14) contains the irreducible 2-loop electroweak result [15] and the leading QCD contribution to the ρ-parameter

  $$\Delta\rho^{\alpha\alpha_s} = -\,\Delta\rho \cdot \frac{\alpha_s(m_t^2)}{\pi} \cdot \frac{2}{3} \left(\frac{\pi^2}{3} + 1 \right) . \tag{15}$$

  A comparison with the full $O(\alpha\alpha_s)$ calculation [16, 17] shows that this leading term takes into account the bulk of the strong interaction effects and yields a sufficiently good approximation with a deviation in M_W of less than 20 MeV.

- With the quantity $(\Delta r)_{\text{rem}}$ in the denominator non-leading higher order terms containing mass singularities of the type $\alpha^2 \log(M_Z/m_f)$ from light fermions are also incorporated [18].

At the 2-loop level, a heavy Higgs boson induces in $\Delta r_{\rm rem}$ a term quadratic in M_H [19], which is, however, numerically insignificant for $M_H < 1$ TeV [20].

The treatment of the higher order reducible terms in Eq. (13) can be further refined by performing in $(\Delta r)_{\rm rem}$ the following substitution

$$\frac{\alpha}{s_W^2} \to \frac{\sqrt{2}}{\pi} G_\mu M_W^2 (1-\Delta\alpha) \tag{16}$$

in the expansion parameter of the combination

$$\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) - \Delta\rho$$

after removing the UV singularity according to the $\overline{MS}$ scheme with $\mu = M_Z$. This is discussed in [21] and is equivalent to the method in the on-shell scheme described in the last reference of [2] as well as to the recipe given at the end of ref. [14]. Numerically this modification is of some importance in the M_W-M_Z correlation for very heavy top quarks above 250 GeV. As an example, for $m_t = 300$ GeV one obtains a change in M_W by about 40 MeV.

A general comment, however, is in order: The refined treatment of the non-leading reducible higher order terms can be considered as an improvement only in case that the 2-loop irreducible non-leading terms are essentially smaller in size. Irreducible contributions of the type $\alpha G_\mu m_t^2 \log(m_t/M_Z)$ are unknown, and one has to rely on the assumption that the suppression by $1/N_C$ relative to the 2-loop reducible term is not compensated by a large coefficient. For bosonic 2-loop terms reducible and irreducible contributions are a priori of the same size and one does not gain from resumming 1-loop terms. In order to be on the safe side, the differences caused by the summation of non-leading reducible terms should be considered as a theoretical uncertainty at the level of 1-loop calculations improved by higher order leading terms.

The quantity Δr on the r.h.s. of Eq. (13)

$$\Delta r \,=\, 1 - \frac{\pi\alpha}{\sqrt{2}G_\mu} \frac{1}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}$$

is experimentally determined by M_Z and the ratio M_W/M_Z. Theoretically, it can be computed from M_Z, G_μ, α after specifying the masses M_H, m_t. In Figure 1 we display the prediction for Δr as a function of m_t in various steps: the first order calculation with the geometrical resummation of the lowest order Δr, then including the electroweak higher order terms on the basis of Eq. (13), and finally including the QCD corrections related to m_t. Both electroweak and QCD higher order effects yield a positive shift to Δr and thus diminish the slope of the first order dependence on m_t for large top masses.

The implications of the higher order corrections on M_W are demonstrated in table 1. Here $M_Z = 91.175$GeV has been assumed and the parametrization of the light-quark contribution of Ref. [9] has been adopted. Nonperturbative QCD effects in gauge boson vacuum polarizations associated with the $t\bar{t}$ threshold, which can be estimated with help of dispersive methods [22], have been neglected in the third column of table 1. In the higher order electroweak results of table 1 the refinement as described above in Eq. (16) was taken into account.

Figure 1: Δr *in* $O(\alpha)$ *(dotted), in* $O(\alpha^2)$ *(full), and in* $O(\alpha^2 + \alpha\alpha_s)$ *(dashed).* $M_Z = 91.175$ *GeV,* $M_H = 300$ *GeV.*

m_t [GeV]	M_H [GeV]	$O(\alpha)$	$+O(\alpha^2)$	$+O(\alpha\alpha_s)$
90	60	79.959	79.957	79.927
90	300	79.861	79.860	79.831
90	1000	79.767	79.767	79.738
140	60	80.259	80.247	80.205
140	300	80.160	80.152	80.110
140	1000	80.063	80.060	80.017
200	60	80.723	80.673	80.600
200	300	80.620	80.580	80.507
200	1000	80.521	80.490	80.417

Table 1: Prediction of M_W (in GeV) for given m_t and M_H on the basis of Δr.

4 Neutral and charged current processes

4.1 Lowest order cross sections

In the quark-parton model the fundamental processes are considered to be the neutral current 4-fermion scattering processes between the electron and the quark constituents of the proton. Defining the kinematical variables for the electron-proton processes

$$e(l) + p(p) \to e'(l') + X(p_X), \quad e' = e, \nu_e$$

by

$$Q^2 \;=\; -(l-l')^2, \quad S = (l+p)^2, \quad x = \frac{Q^2}{2p\cdot(l-l')}, \quad y = \frac{Q^2}{xS}, \tag{17}$$

the differential cross section for the neutral current process in the parton picture is given by

$$\frac{d^2\sigma^{NC}}{dx\, dQ^2}(e_{L,R}) \;=\; \frac{2\pi\alpha^2}{xQ^4}\left[\left(1+(1-y)^2\right)F_2^{L,R} \;+\; \left(1-(1-y)^2\right)xF_3^{L,R}\right]. \tag{18}$$

The structure functions

$$\begin{aligned} F_2^{L,R} &= \sum_f [xq_f(x,Q^2)+x\bar{q}_f(x,Q^2)]\cdot A_f^{L,R}, \\ xF_3^{L,R} &= \sum_f [xq_f(x,Q^2)-x\bar{q}_f(x,Q^2)]\cdot B_f^{L,R} \end{aligned} \tag{19}$$

contain the quark (q_f) and anti-quark ($\bar{q}_f$) distribution functions as well as coupling constants and propagators corresponding to photon and Z boson exchange ($L=+, R=-$):

$$\begin{aligned} A_f^{L,R} &= Q_f^2 + 2Q_eQ_f(v_e\pm a_e)v_f\frac{Q^2}{Q^2+M_Z^2} + (v_e\pm a_e)^2(v_f^2+a_f^2)\left(\frac{Q^2}{Q^2+M_Z^2}\right)^2 \\ B_f^{L,R} &= -2Q_eQ_f(v_e\pm a_e)a_f\frac{Q^2}{Q^2+M_Z^2} + 2(v_e\pm a_e)v_fa_f\left(\frac{Q^2}{Q^2+M_Z^2}\right)^2. \end{aligned} \tag{20}$$

The parametrization of the lowest order cross section in terms of the vector and axial vector coupling constants $v_{e,f},\ a_{e,f}$ is quite general and applies to any model where a vector boson with v and a couplings is exchanged together with the photon. For a quantitative evaluation a well-known ambiguity in the Born level presentation shows up: the coupling constants can either be expressed by means of the expressions in (2), normalized to e, or in the following way with help of the Fermi constant making use of the lowest order version of relation (5):

$$\begin{aligned} a_f &= \left(\frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha}\right)^{1/2} I_3^f, \\ v_f &= \left(\frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha}\right)^{1/2} (I_3^f - 2\,Q_f\,s_W^2)\,. \end{aligned} \tag{21}$$

The numerical results obtained from these two parametrizations are different in general. Of course, after incorporating consistently the weak radiative corrections described next an unambigous result is obtained.

In a similar way one can write for the charged current (unpolarized) differential cross section

$$\frac{d^2\sigma^{CC}}{dxdQ^2} = \frac{\pi\alpha^2}{4s_W^4}\cdot\frac{1}{(Q^2+M_W^2)^2}\left[u+c+(1-y)^2(\bar{d}+\bar{s}+\bar{b})\right] \tag{22}$$

$$= \frac{G_\mu^2}{2\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2\left[u+c+(1-y)^2(\bar{d}+\bar{s}+\bar{b})\right]. \tag{23}$$

The second form has the advantage that it is not changed by large radiative corrections as shown in the next subsection.

4.2 Amplitudes and form factors

It is not our aim to repeat the details of the radiative corrections in deep inelastic scattering (see e.g. [1] and the references therein), in particular the QED corrections which are conventionally put on top of the amplitudes resp. cross sections dressed by the non-QED corrections. The non-QED (or weak) corrections are of importance since they affect the correlation between various observables and introduce the dependence on the unknown parameters of the Standard Model.

In a compact notation the weak corrections can be summarized in terms of effective matrix elements for the underlying parton subprocesses:

photon exchange amplitude :

The dressed photon amplitude can be written in the following way:

$$\mathcal{M}_\gamma = \frac{e^2}{1-\hat{\Pi}_f^\gamma(Q^2)} \cdot \frac{1}{Q^2} \cdot Q_e \gamma_\mu \otimes Q_f \gamma^\mu \tag{24}$$

with the fermionic vacuum polarization $\hat{\Pi}_f^\gamma$ subtracted at $Q^2 = 0$. Writing it in the denominator corresponds to the summation of the leading log terms from the light fermions in case of large Q^2. For the hadronic part the convenient parametrization of Ref. [9] is commonly used. The bosonic content of $\hat{\Pi}^\gamma$ has to be understood as expanded to $O(\alpha)$ and combined with the other diagrams in the form factors of the weak neutral current.

***Z* exchange amplitude :**

As is discussed in [23, 24] and references cited therein, the weak one-loop corrections to the neutral current amplitude $\mathcal{M}_Z$ for $eq \to eq$ can be expressed in terms of four weak form factors $(\rho_{eq}, \kappa_e, \kappa_q, \kappa_{eq})$, in the following way making use of dressed vector ($\bar{v}$) and axial-vector ($\bar{a}$) couplings (with $s = xS$):

$$\begin{aligned}\mathcal{M}_Z(s,Q^2) \sim \frac{1}{Q^2+M_Z^2}\left[\frac{G_\mu}{\sqrt{2}}M_Z^2\right] [&\bar{a}_e\bar{a}_q\gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5 + \bar{v}_e\bar{a}_q\gamma_\mu \otimes \gamma_\mu\gamma_5 \\ &+\bar{a}_e\bar{v}_q\gamma_\mu\gamma_5 \otimes \gamma^\mu + \bar{v}_{eq}\gamma_\mu \otimes \gamma^\mu],\end{aligned} \tag{25}$$

$$\bar{a}_e\bar{a}_q = \rho_{eq}(s,Q^2)\, I_3^e\, I_3^q, \tag{26}$$

$$\begin{aligned}\bar{v}_e\bar{a}_q &= \rho_{eq}(s,Q^2)\, I_3^e\, I_3^q\, \left[1-4|Q_e|s_W^2\,\kappa_e(s,Q^2)\right], \\ \bar{a}_e\bar{v}_q &= \rho_{eq}(s,Q^2)\, I_3^e\, I_3^q\, \left[1-4|Q_q|s_W^2\,\kappa_q(s,Q^2)\right],\end{aligned} \tag{27}$$

$$\bar{v}_{eq} = \bar{a}_e\bar{v}_q + \bar{v}_e\bar{a}_q - \bar{a}_e\bar{a}_q\left[1-16|Q_eQ_q|s_W^4\,\kappa_{eq}(s,Q^2)\right], \tag{28}$$

with s_W^2 from (4). In the Born approximation, $\rho = \kappa = 1$, and $\bar{v}_{eq} = \bar{v}_e\bar{v}_q$. The coupling $\bar{v}_{eq}$ has no parallel in the Born approximation. The above parametrization, however, is very close to a Born-like expression.

An alternative way of writing the corrections introduced above retains the axial-vector couplings in the form $\bar{a}_f = I_3^f$ and changes instead the overall normalization of the NC coupling strength by the factor $\rho_{eq}(s, Q^2)$:

$$G_\mu M_Z^2 \to \rho_{eq}(s, Q^2)\, G_\mu M_Z^2. \tag{29}$$

The other form factors can be absorbed into various effective weak mixing angles which depend on the fermion species and the kinematical variables:

$$s_W^2 \to \begin{cases} \kappa_e(s, Q^2) s_W^2 = s_e^2(s, Q^2) \\ \kappa_q(s, Q^2) s_W^2 = s_q^2(s, Q^2) \\ \sqrt{\kappa_{eq}(s, Q^2)}\, s_W^2 = s_{eq}^2(s, Q^2). \end{cases} \tag{30}$$

As may be seen from Figures 2 and 3, the dependencies of the form factors on the kinematics is numerically comparable to those on e.g. the t-quark mass which are constant in the leading order.

Figure 2: *The NC form factor* $\rho_{eu(\bar{u})}$ *for* $m_t = 130\, GeV$. *The horizontal dotted line corresponds to* $1 + \Delta\rho$. $x = 0.3$.

.

Figure 3a: *The effective mixing angle* s_e^2 *without heavy box diagrams for* $m_t = 130\ GeV$. *The horizontal full line denotes* s_W^2, *the dotted line corresponds to* $s_W^2 + c_W^2\,\Delta\rho$.

Figure 3b: *The effective mixing angle* s_e^2 *in* $eu(\bar{u}) \to eu(\bar{u})$ *including heavy box diagrams. Same signatures as in Figure 3a.* $x = 0.3$.

The universal parts of the form factors contain the quantity $\Delta\overline{\rho}$ from Eq. (14) as the leading m_t-contribution. Taking account of the higher order terms in powers of m_t^2 including the QCD correction, the following expressions are obtained [24, 25]

$$\rho_{eq} = \frac{1+\Delta\rho_{eq}^{\rm rem}}{1-\Delta\bar{\rho}}, \tag{31}$$

$$\kappa_f = (1+\Delta\kappa_f^{\rm rem})(1+\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\bar{\rho}), \tag{32}$$

$$\kappa_{eq} = (1+\Delta\kappa_{eq}^{\rm rem})(1+\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\bar{\rho})^2. \tag{33}$$

The quantities with the index 'rem' are the 1-loop form factors where the corresponding m_t^2 part in $O(\alpha)$ has been subtracted.

It should be noticed that the top- and Higgs dependence enters only through the universal parts of the form factors. The x-dependence is due to the box diagrams with the exchange of $2\,W$ and $2\,Z$. In Figures 2 and 3 we display the form factor ρ_{eq} and the corresponding effective mixing angle s_e^2 for the channel $q=u$ and $\bar{u}$. Without the heavy box diagrams both ρ and κ are the same for quarks and antiquarks.

W **exchange amplitude:**

The charged current amplitude including higher order contributions can be written in the following way:

$$\mathcal{M}_W(s,Q^2) = \frac{G_\mu}{4\sqrt{2}}\,\rho_{eq}^W(s,Q^2)\,\frac{M_W^2}{Q^2+M_W^2}\,\gamma_\mu\,[1-\gamma_5]\otimes\gamma^\mu\,[1-\gamma_5]\,. \tag{34}$$

Differently from the neutral current, only a single form factor ρ^W for each channel is required to accomodate the higher order contributions. In this representation, the weak radiative corrections are very small with very little dependence on m_t and M_H. The leading t-quark contributions are already absorbed when the matrix element is normalized with G_μ instead of α/s_W^2. Hence, we have

$$\rho_{eq}^W \equiv (\rho_{eq}^W)^{\rm rem}. \tag{35}$$

As an example, the form factor ρ_{eu}^W (without QED corrections) is explicitly shown in table 2. Its dependence on the electroweak parameters is only marginal, typically 0.1%.

5 Observables

5.1 Cross sections

From the amplitudes discussed in section 4.2 one obtains as measureable quantities the NC and CC differential and integrated cross sections, the ratio $R_\pm = \sigma^{NC}(e^\pm)/\sigma^{CC}(e^\pm)$, and a series of asymmetries, differentially as well as integrated.

Q^2 [GeV^2]	$m_t = 90$ GeV	$m_t = 200$ GeV
524	0.9924 0.9925	0.9919 0.9920
1002	0.9948 0.9948	0.9942 0.9943
10350	1.0072 1.0071	1.0064 1.0063

Table 2: The charged current form factor ρ_{eu}^W. Upper values: $M_H = 100\,\mathrm{GeV}$, lower values: $M_H = 1\,\mathrm{TeV}$. $x = 0.3$.

In the cross section formulae, the weak corrections are contained in the improved couplings. Details may be found in [26] for the NC process and in [27] for the CC process. The NC cross section including weak loops and QED corrections has the following structure which is realized in the package `TERAD91`, `DISEP` branch [28]:

$$\frac{d^2\sigma^{NC}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4} \sum_B \sum_b \sum_{q,\bar{q}} c_b K(B)[V(B)R_b^V(B) \pm A(B)R_b^A(B)]. \tag{36}$$

The different sums extend over: quarks q and anti-quarks $\bar{q}$; photon and Z exchange, $B = \{(\gamma\gamma), (\gamma Z), (ZZ)\}$. For leptonic [quarkonic, interference] radiation, $c_b = Q_e^2$ [Q_q^2, $Q_e Q_q$]. In case of the Born cross section, $c_b = 1$, and no summation over b is required.

The QED corrections are collected in the $R_b^{V,A}(B)$ together with the parton distributions (see [1]). The expressions for the weak corrections, contained in the generalized improved couplings V, A and in the cross section normalization K, are given by

$$\begin{aligned} V(\gamma, Z) &= \bar{v}_{eq}, \\ V(Z, Z) &= (\bar{v}_e^2 + \bar{a}_e^2)\bar{a}_q^2 + \bar{a}_e^2\bar{v}_q^2 + \bar{v}_{eq}^2, \\ A(\gamma, Z) &= \bar{a}_e\bar{a}_q, \\ A(Z, Z) &= 2\bar{a}_e\bar{a}_q(\bar{v}_e\bar{v}_q + \bar{v}_{eq}). \end{aligned} \tag{37}$$

The dressed couplings in these expressions are those of Eq. (26-28). Further,

$$\begin{aligned} K(\gamma, Z) &= 2|Q_e Q_q| F_A(Q^2)\chi(Q^2), \\ K(Z, Z) &= \left[\chi(Q^2)\right]^2. \end{aligned} \tag{38}$$

The factor $F_A(Q^2) = [1 - \Delta\alpha(Q^2)]^{-1}$ describes the running of α in the QED part, and χ is the Z boson propagator with a suitable normalization factor:

$$\chi(Q^2) = \frac{G_\mu M_Z^2}{8\pi\alpha\sqrt{2}} \cdot \frac{Q^2}{Q^2 + M_Z^2}. \tag{39}$$

According to our previous discussion, the NC and CC cross sections in the Standard Model are determined by the input parameters G_μ, α, M_Z once m_t and M_H are specified. Table 3 contains the integrated unpolarized cross sections σ^{NC} and σ^{CC} as functions of the top quark and Higgs masses. Quark distributions are from [29], set 1. Here, QCD

m_t [GeV]	M_H [GeV]	σ_a^{NC} [nb]	σ_a^{CC} [nb]	σ_b^{NC} [nb]	σ_b^{CC} [nb]
90	60	0.211	0.0257	0.0999	0.0230
90	1000	0.211	0.0256	0.0998	0.0229
120	60	0.211	0.0257	0.0999	0.0231
120	1000	0.211	0.0256	0.0998	0.0230
150	60	0.211	0.0258	0.1000	0.0232
150	1000	0.211	0.0257	0.0999	0.0231
200	60	0.211	0.0260	0.1000	0.0233
200	1000	0.211	0.0259	0.1000	0.0232
250	60	0.212	0.0262	0.1000	0.0235
250	1000	0.212	0.0261	0.1000	0.0234

Table 3: Unpolarized NC and CC cross sections from the basic input parameters. $M_Z = 91.175$ GeV. $x > 0.1$. a: $Q^2 > 500\,\mathrm{GeV}^2$, b: $Q^2 > 1000\,\mathrm{GeV}^2$. No QED corrections.

corrections were included in Δr only, but not in the weak form factors entering the cross section formulae.

The NC cross section is dominated by the photon exchange amplitude unless Q^2 is restricted to $Q^2 > 1000\,\mathrm{GeV}^2$. Hence, the prominent part of the NC cross section is determined by the running electromagnetic fine structure constant whereas the more interesting electroweak effects associated with the NC form factors only show up at very

Figure 4: $d\sigma^{NC}(\rho)/d\sigma^{NC}(\rho = 1)$ *for* $x = 0.5$

high Q^2. The influence of the ρ form factors in the NC amplitude on the differential cross section is demonstrated in Figure 4.

From table 3 one can read off that only σ^{CC} has a variation with the electroweak input at the level of 1% or more. Since the CC differential cross section including electroweak corrections has a very simple form close to that in (23)

$$\frac{d^2\sigma^{CC}}{dx\,dQ^2} = \frac{G_\mu^2}{2\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \left[\sum_{f=u,c} q_f\,(\rho_{ef}^W)^2 + (1-y)^2 \sum_{f=d,s,b} \bar{q}_f\,(\rho_{e\bar{f}}^W)^2\right] \tag{40}$$

and the ρ^W have practically no dependence on the electroweak input, σ^{CC} is determined (almost) exclusively through M_W, also in higher order. This is different when the representation (22) is chosen for the CC cross section, where α, M_Z, M_W are considered as independent input quantities without the G_μ-constraint (1). In this case, the electroweak corrections to the lowest order form (22) are large and depend sizeably on m_t (and also on M_H):

$$\frac{d^2\sigma^{CC}}{dx\,dQ^2} = \frac{\pi\alpha^2}{4s_W^4}\cdot\left(\frac{1}{1-\Delta\tilde{r}}\right)^2\cdot\frac{1}{(Q^2+M_W^2)^2}\left[\sum_{f=u,c} q_f(\tilde{\rho}_{ef}^W)^2 + (1-y)^2 \sum_{f=d,s,b} \bar{q}_f(\tilde{\rho}_{e\bar{f}}^W)^2\right]. \tag{41}$$

$\tilde{\rho}^W$ are the CC form factors in (34), but expressed in terms of M_W instead of G_μ. $\Delta\tilde{r}$ is given by

$$\Delta\tilde{r} = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \frac{c_W^4}{s_W^4}\frac{(\Delta\rho)^2}{1-\Delta\alpha} + \cdots\text{(higher orders)} + (\Delta r)_{\text{rem}} \tag{42}$$

with $\Delta\alpha$ from (7) and $\Delta\rho$ in the form (10) supplemented by the irreducible 2-loop pieces in (14). The difference to Δr in (13) consists in expressing $\Delta\tilde{r}$ exclusively in terms of $\alpha, M_Z, M_W, m_t, M_H$ avoiding G_μ at all. [1] In order to study the dependence of (41) on M_W, m_t and to impose the G_μ-constraint in an independent second step, it is important for consistency that all quantities on the r.h.s. of (41) are free of G_μ and expressed in terms of M_W. Superimposing the condition (1) explicitly given by

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2(1-\frac{M_W^2}{M_Z^2})}\cdot\frac{1}{1-\Delta\tilde{r}(\alpha, M_Z, M_W, m_t, M_H)} \tag{43}$$

is equivalent to utilizing (40) for the calculation of the CC cross section yielding the same results in a more immediate way. In order to avoid unnecessary confusion we recommend the use of Eq. (40). In cases where the separate discussion of (41) and (43) is wanted, attention has to be paid to the different functional dependence of the electroweak corrections on the parameters.

It should be noted that the structure of the CC cross section as given in Eq. (40) is quite general and comprises extensions of the minimal model by new physics affecting only the radiative corrections. The form factors ρ^W become model dependent, but do not get any large corrections from modifications of the static ρ parameter. Such changes in ρ would influence the NC effective couplings, at the same level as a heavy top quark in the minimal model, but due to the suppression of the Z exchange part in the NC cross section do not give rise to visible effects. We thus can conclude that the determination of M_W from σ^{CC} or from σ^{CC} normalized to σ^{NC} on the basis of the Standard Model formulae remains valid also in a large class of extended models.

[1] Summing up all orders in $\Delta\rho$ [30] and imposing the G_μ-constraint reproduces the result in (13).

5.2 The ratio $R_- = \sigma^{NC}(e^-)/\sigma^{CC}(e^-)$

The practical use of the absolute cross sections for consistency checks of the electroweak theory or for getting information on the electroweak parameters is limited by systematic uncertainties. An earlier study [31] reports an error of $\delta M_W = 800$ MeV on the W mass measurement from the absolute CC cross section. This was obtained on the basis of $\mathcal{L} = 250$ pb^{-1} with $\delta\mathcal{L} = 1\%$.

Systematic errors, in particular from the overall normalization of the cross sections, can be reduced by making use of cross section ratios. We define the quantity

$$R_- = \frac{\sigma^{NC}(e^- p \to e^- X)}{\sigma^{CC}(e^- p \to \nu_e X)} \tag{44}$$

as the ratio of electron-proton cross sections (for earlier studies of this quantity in the Born approximation see [32, 33]). We first consider the integrated cross sections, and come back to the differential distributions in further course. From a purely theoretical point of view, both integrated and differential quantities are equivalent, determined essentially by M_W as the relevant electroweak parameter in the W propagator. The effective coupling strength at $Q^2 \neq 0$, given by $\rho_{eq}^W G_\mu$, differs very little from the CC coupling at $Q^2 = 0$.

Figure 5: *Unpolarized R_- versus M_W for $M_H = 60\ GeV$ ($\circ$) and $M_H = 1\ TeV$ ($\Box$). The marked points correspond to the values $m_t = 90, 120, 150, 200, 250, 300\ GeV$. $x > 0.1$, $Q^2 > 500\ GeV^2$.*

According to our previous discussion in section 5.1, it is predominantly σ^{CC} yielding the main dependence of R_- on M_W. Therefore, by its normalization to σ^{NC}, one gains through the suppression of systematic uncertainties. On the other hand, σ^{NC} is also dependent on M_W via the form factors, although very weakly. But this weak effect is sufficient to compensate part of the M_W-dependence of σ^{CC} in R_-. As a consequence, the theoretical sensitivity of R_- with respect to M_W is slightly diminished compared to that of σ^{CC} itself. From an experimental point of view, however, the net effect advocates the ratio R_- (see the contribution of Brisson et al. [34]).

On the basis of the expressions (36), (40) in section 5.1 it is straight-forward to obtain the interdependence of R_- and M_W. This correlation is displayed in Figure 5. For a fixed value of M_H, the only parameter varying along the curve is m_t. Different Higgs masses yield different curves, which, however, are not separable. The dependence of R_- on M_W therefore is practically unique. This is the concrete substantiation of the qualitative argumentation given above in section 5.1. A variation of m_t from 90 to 200 GeV induces a shift in R_- by $\Delta R_-/R_- = 0.011$, corresponding to $\Delta M_W = 675$ MeV. Inversely, a measurement of $R_- \pm \delta R_-$ with a global error δR_- can be converted into a range of M_W. $\delta R_-/R_- = 1\%$ corresponds to $\delta M_W = 630$ MeV. Also shown in Figure 5 is the present result on M_W from combined LEP and $p\bar{p}$ collider W and Z mass measurements [35] yielding $M_W = 80.14 \pm 0.27$ GeV.

We want to point out that this information on M_W is obtained from the W propagator and hence can be interpreted as a 'direct' W mass measurement. Of course, the coupling strength itself given by G_μ, contains indirect information on M_W in terms of the relation (43) resp. (5) and (13). But even the most precise measurement of G_μ from the muon lifetime is not a measurement of M_W since the relation between both quantities is model dependent. In the Standard Model this relation depends on m_t, and at least one additional independent experimental input (besides α and M_Z) is required to obtain a bound on M_W. HERA can provide an input in terms of R_-. In figure 6 we demonstrate the extraction of M_W (expressed as s_W^2) from G_μ by use of the relation (43) (full curve). The dotted band corresponds to the range obtained from a fixed value of R_- with $\delta R_-/R_- = 1\%$. Thereby R_- is calculated in the representation of σ^{CC} given in (41). Also the NC cross section has to be used in this representation. The central value of R_- is fixed such that for $m_t = 90$ GeV the corresponding value of s_W^2 coincides with the one obtained from G_μ. For simplicity only the terms associated with $\Delta\alpha$ and $\Delta\rho$ were taken into account, neglecting $(\Delta r)_{\rm rem}$ in (42) and also in (43) ('improved Born approximation'). The figure hence should not be used for quantitative purposes but is a demonstration of the basic features.

The observations obtained from Figure 6 are as follows:

- There is no bound on M_W (or s_W^2) from the integral $R_-(M_Z, M_W, m_t)$ or from $G_\mu(M_Z, M_W, m_t)$ separately unless m_t is restricted from somewhere else.
- Combining R_- with G_μ yields a bound on M_W, determined by the intersection of the curves. It results from the different slopes of the curves due to the Q^2-dependent W propagator in R_-. This bound is the same as the one which would be obtained from $R_-(M_Z, G_\mu, m_t)$ based on the representation (40).

It is important that the radiative corrections in the NC part of R_- are also taken into account. They modify the top-dependence only slightly, but due to the flat intersection small changes in the slope induce rather big shifts in M_W.

Figure 6: s_W^2 *from* G_μ *(full curve) and from* $R_- \pm \delta R_-$ *with* $\delta R_-/R_- = 0.01$ *(dotted curves). Same cuts as Figure 5.*

So far we have discussed R_- in terms of integrated cross sections. The quantity R_- can also be defined differentially with the kinematical variables in the differential cross sections $d\sigma^{NC,CC}/dxdy$ defined via the current jets,

$$R_-(x,y) = \frac{d\sigma^{NC}(e^-p \to e^-X)}{d\sigma^{CC}(e^-p \to \nu_e X)} \tag{45}$$

in bins in (x,y) given by the resolution in these variables. In a systematic study by Bernardi et al. [36] the bining was derived for which the smearing corrections can be delt with at a scale of $\pm 20\%$ corrections. In the subsequent analysis this bining is applied demanding further a minimum value of 500 GeV^2 for Q^2 and 7 GeV for $p_\perp$. Smearing effects require to use $x \leq 0.7$, $0.03 \leq y \leq 0.8$ [36, 37]. In order to fairly reduce the uncertainty due to the sea quark contribution we demand $x \geq 0.1$.

In the following the statistical precision of the M_W measurement from $R_-(x,y)$ is derived assuming an integrated luminosity of $\mathcal{L} = 200\ pb^{-1}$ and using the parton distributions [29] (set 1). The statistical weights for the bins are calculated from $d\sigma^{NC,CC}$ for $m_t = 140$ GeV and $M_H = 300$ GeV. The differential neutral and charged current cross sections are computed with the code `TERAD91` [28] according to the expressions in Eq. (36) and (40). The statistical analysis is carried out using the χ^2–method applying the code `STATAN` [38]. Fitting M_W in $R_-(x,y)$ (with α, G_μ, M_Z fixed) to the data sample in the bins, one obtains

$$\delta M_W^{stat} = \pm 910 \mathrm{MeV}\,. \tag{46}$$

Although, the investigation of the high Q^2 behaviour of charged current observables at HERA will likely reveal the propagator effect, it may not contribute to an independent precision measurement of M_W.

We also want to address the influence of the QED and QCD corrections on the quantity $R_-(x,y)$ in (45). Here, we consider the corrections integrating over the phase space of the radiated photon and gluon, respectively. The $O(\alpha)$ QED corrections are calulated in the leading log approximation [39] with the code `HELIOS` [40] and are shown in Figure 7 using the parton distributions [29] (set 1). The correction $\delta_{QED}(R_-)$ is only weakly dependent on the choice of the set of parton distributions for $x \geq 0.1$ in the kinematical range used in the analysis. The largest corrections occur at lower x, however, for $x \geq 0.1$ the correction to R_- is smaller than 2%.

Figure 7: *LLA QED corrections to $R_-(x,y)$ using jet measurements*

For the calculation of the $\mathcal{O}(\alpha_s)$ corrections we use the DIS scheme [41], i.e. keeping $xq_f(x,Q^2) + x\bar{q}_f(x,Q^2)$ invariant to all orders. This appears to be the most natural scheme for deep inelastic scattering. The corrections were derived in [42]. For $x \geq 0.1$ the corrections to R_- are shown in Figure 9 and turn out to be smaller than 0.5 % for all y. Both corrections are small and are only weakly dependent on the choice of electroweak parameters.

Figure 7: $O(\alpha_s)$ *QCD corrections to* $R_-(x,y)$ *in the DIS scheme*

The ratio R_- is also influenced by the choice of parton distributions. For illustration, the ratio $\delta_{R_-}^{Par} = R_-(MT)/R_-(HMRS)$ with the parametrizations MT [43] and HMRS [44] varies from +0.3% to –3% between $x = 0.1$ and $x = 0.7$. Thereby the parton distributions have been extrapolated in Q^2 to the kinematical range of HERA. This shows that, in spite of the stability of different fitting procedures, the absolute size of the parton distributions has to be determined by a direct measurement in the kinematical range under investigation. As shown in [45] parton distributions for some flavours may be measured also directly for luminosities of the order of $\mathcal{L} = 400\ pb^{-1}$. Thus, one may hope that these aspects can be controlled applying different methods (see also the discussions in [34, 46]).

5.3 Asymmetries

Besides the unpolarized cross sections a useful set of measureable quantities is given by the various asymmetries from the NC processes involving left- and right-handed electrons and/or positrons:

- the polarization asymmetry (left-right asymmetry)

$$A_\pm = \frac{d\sigma(e_L^\pm) - d\sigma(e_R^\pm)}{d\sigma(e_L^\pm) + d\sigma(e_R^\pm)} \tag{47}$$

- the mixed charge-polarization asymmetry

$$B_{\pm} = \frac{d\sigma(e_L^{\pm}) - d\sigma(e_R^{\mp})}{d\sigma(e_L^{\pm}) + d\sigma(e_R^{\mp})} \tag{48}$$

- the charge asymmetry (left-handed, right-handed, or unpolarized)

$$C = \frac{d\sigma(e^-) - d\sigma(e^+)}{d\sigma(e^-) + d\sigma(e^+)}. \tag{49}$$

All these asymmetries vanish for purely electromagnetic interactions in lowest order. They are therefore sensitive to the structure of the weak interaction, however, in general influenced by QED radiative corrections. For earlier studies on the asymmetries see [33, 47]. For the polarization asymmetry the QED corrections are very small. Moreover, the asymmetries are practically insensitive to the uncertainties in the hadronic input. The dependence of A_- on the parameters of the Standard Model is displayed in Figure 9 (input: α, M_Z, G_μ). Also shown is the effect of the higher than 1-loop order terms resulting from m_t.

Figure 9: *Polarization asymmetry A_- for $P_L = -0.8$. $M_H = 45, 300, 1000$ GeV. The curve deviating for large m_t corresponds to $O(\alpha)$. $x = 0.2$, $y = 0.5$*

The mixed asymmetry B_+ is nearly independent of the parameters of the Standard Model and to extensions at the level of radiative corrections. On the other hand, it has some sensitivity to extra Z bosons of an extended gauge group [48].

6 Virtual new physics?

Extensions of the minimal model affecting only the vector boson self energies can be parametrized in terms of universal contributions to the form factors for the NC and CC amplitudes to be added to the ρ_i, κ_i in (29),(30) and to ρ_{eq}^W in (34). Making use of the expansion for the self-energies from new physics $[q^2 = -Q^2]$

$$\Sigma^{ij}(q^2) = \Sigma^{ij}(0) + q^2 \Pi^{ij}(q^2) \tag{50}$$

one obtains the following general expressions:

$$\begin{aligned}
\Delta\rho_Z(q^2) &= \Delta\rho(0) + \Pi^{ZZ}(M_Z^2) - \frac{q^2\Pi^{ZZ}(q^2) - M_Z^2\Pi^{ZZ}(M_Z^2)}{q^2 - M_Z^2}, \\
\Delta\rho_W(q^2) &= \Pi^{WW}(M_W^2) - \frac{q^2\Pi^{WW}(q^2) - M_W^2\Pi^{WW}(M_W^2)}{q^2 - M_Z^2}, \\
\Delta\kappa(q^2) &= \Delta\kappa(M_Z^2) - \frac{c_W}{s_W}\left[\Pi^{\gamma Z}(q^2) - \Pi^{\gamma Z}(M_Z^2)\right].
\end{aligned} \tag{51}$$

The static quantity $\Delta\rho(0)$ is a modification of the ρ-parameter, and $\Delta\kappa(M_Z^2)$ enters the universal part of the effective mixing angle at the Z resonance. Both have already been significantly constrained by the precision data from LEP. $\Delta\rho_W$ is not accessible at LEP. Detailed discussions of the q^2-dependent terms in (49) require the specification of the kind of new physics. Only for very simple situations which may occur in the presence of heavy objects in the loops, general statements are possible:

For $\Pi^{ij} = constant$, all extra contributions vanish except $\Delta\rho(0)$ and $\Delta\kappa(M_Z^2)$, which are already very much constraint by LEP.

The next simplest case would be

$$\Pi^{ij} \simeq \frac{\alpha}{\pi}\, c^{ij}\, \frac{q^2}{\Lambda^2} \tag{52}$$

where Λ is the scale of a new physics object and c^{ij} the coupling to the vector bosons i, j relative to the electromagnetic coupling. In order to estimate the sensitivity of the ratio R_- to the possible extra contribution

$$\Delta\rho_W = -\frac{\alpha}{\pi}\, c^{WW}\, \frac{q^2}{\Lambda^2}$$

we varied Λ and c^{WW} with the result that $\Lambda \simeq 300$ GeV and $c^{WW} = 40$ would be required for producing a visible effect of about 1% in R_-. This means a rather low lying new scale with an unnatural large coupling strength to the charged vector bosons only (a strong contribution to Z can again be excluded from the LEP data). We thus can conclude that the appearance of observable new physics in the radiative corrections is very unlikely.

References

[1] G. Kramer, H. Spiesberger, “Radiative corrections to *ep* scattering, a survey”, these proceedings;
H. Spiesberger et al., report of the working group on “Radiative Corrections”, these proceedings

[2] A. Sirlin, Phys. Rev. D 22 (1980) 971;
W. J. Marciano, A. Sirlin, Phys. Rev. D 22 (1980) 2695;
J. Fleischer, F. Jegerlehner, Phys. Rev. D 23 (1981) 2001;
K.I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, T. Muta, Suppl. Prog. Theor. Phys. 73 (1982) 1;
Z. Hioki, Phys. Rev. Lett. 65 (1990) 683, E:1692; Z. Phys. C 49 (1991) 287;
D. Bardin, P. Christova, O. Fedorenko, Nucl. Phys. B 197 (1982) 1;
A. Akhundov, D. Bardin, T. Riemann, Nucl. Phys. B 276 (1986) 1;
M. Böhm, W. Hollik, H. Spiesberger, Fortschr. Phys. 34 (1986) 687;
M. Consoli, W. Hollik, F. Jegerlehner, in: *Z Physics at LEP 1*, eds. G. Altarelli, R. Kleiss, C. Verzegnassi, CERN 89-08 (1989), Vol. 1, p. 7;
W. Hollik, Fortschr. Phys. 38 (1990) 165

[3] G. Passarino, R. Pittau, Phys. Lett. 228 B (1989) 89

[4] M. Veltman, Phys. Lett. 91 B (1980) 95;
M. Green, M. Veltman, Nucl. Phys. B 169 (1980) 137, E: Nucl. Phys. B 175 (1980) 547;
F. Antonelli, M. Consoli, G. Corbo, Phys. Lett. 91 B (1980) 90;
F. Antonelli, M. Consoli, G. Corbo, O. Pellegrino, Nucl. Phys. B 183 (1981) 195;
D. Bardin, O. Fedorenko, N. Shumeiko, J. Phys. G 7 (1981) 195;
G. Passarino, M. Veltman, Phys. Lett. 237 B (1990) 537

[5] W.J. Marciano, A. Sirlin, Phys. Rev. Lett. 46 (1981) 163;
A. Sirlin, Phys. Lett. 232 B (1989) 123;
G. Degrassi, S. Fanchiotti, A. Sirlin, Nucl. Phys. B 351 (1991) 49

[6] D.C. Kennedy, B.W. Lynn, Nucl. Phys. B 322 (1989) 1;
M. Kuroda, G. Moultaka, D. Schildknecht, Nucl. Phys. B 350 (1991) 25

[7] W. Hollik, H.J. Timme, Z. Phys. C 32 (1986) 125;
F. Jegerlehner, Z. Phys. C 32 (1986) 425

[8] J.R. Carter, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva 1991 (to appear)

[9] F. Jegerlehner, preprint Paul-Scherrer-Institut, PSI-PR-91-16 (1991), in: *Progress in Particle and Nuclear Physics*, ed. A. Fässler, Pergamon Press, Oxford, U.K. 1991, updated from:
H. Burkhardt, F. Jegerlehner, G. Penso, C. Verzegnassi,
Z. Phys. C 43 (1989) 497

[10] D.A. Ross, M. Veltman, Nucl. Phys. B 95 (1975) 135 ;
M. Veltman, Nucl. Phys. B 123 (1977) 89

[11] M. Veltman, Acta Phys. Polon. 8 (1977) 475; Phys. Lett. 70B (1977) 253

[12] G. Burgers, F. Jegerlehner, B. Kniehl, J. Kühn, in *Z Physics at LEP 1*, eds. G. Altarelli, R. Kleiss, C. Verzegnassi, CERN 89-08 (1989), Vol. 1 , p. 55

[13] W.J. Marciano, Phys. Rev. D 20 (1979) 274

[14] M. Consoli, W. Hollik, F. Jegerlehner, Phys. Lett. B 227 (1989) 167.

[15] J.J. van der Bij and F. Hoogeveen, Nucl. Phys. B 283 (1987) 477

[16] A. Djouadi and C. Verzegnassi, Phys. Lett. 195B (1987) 265;
A. Djouadi, Nuovo Cim. 100A (1988) 357

[17] D. Yu. Bardin, A.V. Chizhov, Dubna preprint E2-89-525 (1989);
B.A. Kniehl, Nucl. Phys. B 347 (1990) 86

[18] A. Sirlin, Phys. Rev. D 29 (1984) 89

[19] J. van der Bij and M. Veltman, Nucl. Phys. B 231 (1984) 205 ;
J.J. van der Bij, Nucl. Phys. B 248 (1984) 141

[20] F. Halzen, B.A. Kniehl, Nucl. Phys. B 353 (1991) 567

[21] S. Fanchiotti, A. Sirlin, in *M.A.B. Bég Memorial Volume*, eds. A. Ali and P. Hoodbhoy (World Scientific, Singapore, 1991) p. 58

[22] B.A. Kniehl, J.H. Kühn, R.G. Stuart, Phys. Lett. 214 B (1988) 621 ;
B.A. Kniehl, A. Sirlin, DESY 91-103 and MAD/PH/670, Nucl. Phys. **B** (in print)

[23] D.Yu. Bardin, M.S. Bilenky, G.V. Mitselmakher, T. Riemann, M. Sachwitz, Z. Phys. C 44 (1989) 493

[24] D. Bardin, W. Hollik, T. Riemann, Z. Phys. C 49 (1991) 485

[25] A. Akhundov, D. Bardin, M. Bilenky, P. Christova, S. Riemann, T. Riemann, M. Sachwitz, H. Vogt, package `DIZET`, version 4.04 (21 Aug 1991); version 2 was described in: D. Bardin et al., Comput. Phys. Commun. **59** (1990) 303; a description of version 4.04 is in preparation. See also references cited in the code.

[26] D. Bardin, C. Burdik, P. Christova, T. Riemann, Z. Phys. C 42 (1989) 679

[27] D. Bardin, C. Burdik, P. Christova, T. Riemann, Z. Phys. C 44 (1989) 149

[28] A. Akhundov, D. Bardin, L. Kalinovskaya, T. Riemann, package `TERAD91`, these proceedings, Vol. 3.

[29] D.W. Duke, J.F. Owens,Phys. Rev. D 30 (1984) 49

[30] see the last two references of [2]

[31] G. Cozzika, D. Haidt, G. Ingelmann, in: *Proceedings of the HERA Workshop*, Hamburg 1987, ed. R.D. Peccei, DESY 1988, Vol. 2, p. 713

[32] J.F. Wheater, Nucl. Phys. B 233 (1984) 365

[33] J. Blümlein, M. Klein, T. Riemann, in: *Proceedings of the HERA Workshop*, Hamburg 1987, ed. R.D. Peccei, DESY 1988, Vol. 2, p. 687

[34] V. Brisson et al., “The measurement of electroweak parameters at HERA”, these proceedings

[35] H. Plothow-Besch, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva 1991 (to appear)

[36] G. Bernardi, W. Hildesheim et al., these proceedings

[37] J. Blümlein, M. Klein, in preparation

[38] J. Blümlein, `FORTRAN` code `STATAN`, unpublished

[39] J. Blümlein, Phys. Lett. 271 B (1991) 267

[40] J. Blümlein, `FORTRAN` code `HELIOS`, these proceedings, Vol. 3

[41] G. Altarelli, R.K. Ellis, G. Martinelli, Nucl. Phys. B 157 (1979) 461

[42] B. Humpert, W.L. van Neerven, Nucl. Phys. B 184 (1981) 225;
J.G. Körner, E. Mirkes, G.A. Schuler, Int. J. Mod. Phys. A 4, (1989) 1781

[43] J. Morfin, Wu–Ki Tung, Z. Phys. C 52 (1991) 13

[44] P.N. Harriman, A.D. Martin, W.J. Stirling, R.G. Roberts, Phys. Rev. D 42 (1990) 798

[45] G. Ingelman, R. Rückl, Z. Phys. C 44 (1989) 291

[46] J. Blümlein, M. Klein, T. Naumann, T. Riemann, in: *Proceedings of the HERA Workshop*, Hamburg 1987, ed. R.D. Peccei, DESY 1988, Vol. 1, p. 67;
J. Blümlein, M. Klein, "Structure Functions and QCD Tests", these proceedings

[47] H. Spiesberger, in: *Proceedings of the HERA Workshop*, Hamburg 1987, ed. R.D. Peccei, DESY 1988, Vol. 2, p. 605;
C. Kiesling, *ibidem*, p. 653;
O. Gry, G.D. Heath, E. Paul, *ibidem*, p. 719

[48] F. Cornet, R. Rückl, in: *Proceedings of the HERA Workshop*, Hamburg 1987, ed. R.D. Peccei, DESY 1988, Vol. 2, p. 771