Scheme invariant evolution of non-singlet DIS structure functions at N3LO

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| February 01, 2022

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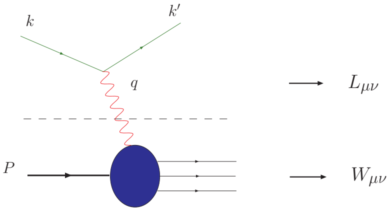
Outline

Introduction: Theory of deep inelastic scattering



- Kinematic invariants:

$$Q^2 = -q^2, \qquad x = \frac{Q^2}{2P.q}$$

- The cross section factorizes into leptonic and hadronic tensor:

$$\frac{d^2\sigma}{dQ^2dx} \sim L_{\mu\nu} W^{\mu\nu}$$

- The hadronic tensor can be expressed through structure functions:

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi) \langle P, | \left[J_\mu^{em}(\xi), J_\nu^{em}(\xi)\right] |P\rangle$$

$$= \frac{1}{2x}\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right) F_L(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x,Q^2)$$

$$+ i\epsilon_{\mu\nu\rho\sigma}\frac{q^\rho S^\sigma}{q\cdot P} g_1(x,Q^2) + i\epsilon_{\mu\nu\rho\sigma}\frac{q^\rho\left(q\cdot PS^\sigma - q\cdot SP^\sigma\right)}{(q\cdot P)^2} g_2(x,Q^2)$$

- F_L, F_2, g_1 and g_2 contain contributions from both, charm and bottom quarks.

Introduction

Why are Heavy Flavor Contributions important?

- They form a significant contribution to all structure functions particularly at small x and high Q^2.
- Concise 3-loop corrections are needed to determine $\alpha_s(M_Z)$, m_c and perhaps m_b.
- The accuracy of measurements at the LHC reaches a level of precision requiring 3-loop VFNS matching.

NNLO: [S. Alekhin, J. Blümlein, S. Moch and R. Placakyte (Phys. Rev. D96 (2017))]

$\alpha_s(M_Z^2) = 0.1147 \pm 0.0008$

$m_c(m_c) = 1.252 \pm 0.018(exp) \; {}^{+0.03}_{-0.02} \; (scale), \; {}^{+0.00}_{-0.07} \; (thy)\text{GeV}$ ($\overline{\text{MS}}$-scheme)

- Dedicated high luminosity DIS experiment are necessary to measure $\alpha_s(M_Z^2)$ at highest precision.
- One possibility: EIC measurement of F_2^p and F_2^d.
- Consider scheme invariant evolution to N^3LO.

Scheme invariant evolution

Non-singlet and singlet evolution

- partonic twist-2 evolution
  - Necessity of the extraction of quite a lot of parton distributions needed.
  - In the singlet case: also of the gluon (not coupling to electro-weak gauge bosons).
  - These functions are universal, but scheme-dependent.
- Scheme invariant non-singlet evolution
  - Prepare the observable purely experimentally.
  - One input observable at Q_0^2 has to be measured.
  - $\Lambda_{\rm QCD}$ is the only parameter to be determined in the correlated fit w.r.t. the measured input.
- Scheme invariant singlet evolution
  - Two input distributions need to be measured (F_2, F_L), $F_2, \partial F_2/\partial \ln(Q^2)$, $[g_1, \partial g_1/\partial \ln(Q^2)]$.
  - F_L measurement normally difficult at high precision.
  - No choice–free experimental representation possible, due to the flavor structure.
  - This actually implies reference to a third measured distribution at Q_0^2.

Decide for $F_2^{\rm NS}$, which requires a huge luminosity.

Physical key measurements require high experimental and theoretical efforts, but lead to key results.
Their derivation is our only mission.

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The Formalism: Scheme invariant NS evolution

The flavor decomposition

$$v^{\pm}_{k^2-1} = \sum_{l=1}^{k} (q_l \pm \bar{q}_l) - k(q_k \pm \bar{q}_k),$$

with q_i the quark distributions and

$$\begin{aligned} v_1^{\pm} &= 0 \\ v_3^{\pm} &= (u \pm \bar{u}) - (d \pm \bar{d}) \\ v_8^{\pm} &= (u \pm \bar{u}) + (d \pm \bar{d}) - 2(s \pm \bar{s}), \end{aligned}$$

one has

$$\begin{aligned} q_i + \bar{q}_i &= \frac{1}{N_F}\Sigma - \frac{1}{i} v^{+}_{i^2-1} + \sum_{l=i+1}^{N_F} \frac{1}{l(l-1)} v^{+}_{l^2-1}, \\ \Sigma &= \sum_{l=1}^{N_F} (q_l + \bar{q}_l). \end{aligned}$$

The NS structure functions

The nucleon structure functions for pure photon exchange in the case of three light flavors (u, d, s) are then given at leading order (LO) by

$$F_2^p = x\left[\frac{2}{9}\Sigma + \frac{1}{6}v_3^+ + \frac{1}{18}v_8^+\right]$$
$$F_2^d = \frac{1}{2}\left[F_2^p + F_2^n\right] = x\left[\frac{2}{9}\Sigma + \frac{1}{18}v_8^+\right]$$

and

$$F_2^{\mathrm{NS}} = F_2^p - F_2^d = \frac{1}{6}xC_q^{\mathrm{NS},+} \otimes v_3^+$$
$$xg_1^{\mathrm{NS}} = xg_1^p - xg_1^d = \frac{1}{6}x\Delta C_q^{\mathrm{NS},+} \otimes \Delta v_3^+,$$

with $\otimes$ the Mellin convolution,

$$\mathrm{M}[f(x)](N) = \int_0^1 dx x^{N-1} f(x), \qquad \mathrm{M}[(A \otimes B)(x)](N) = \mathrm{M}[A(x))](N) \cdot \mathrm{M}[B(x))](N).$$

Analogous relations hold for the polarized structure function g_1.

The non-singlet structure functions can be measured by purely experimental projections.

The NS structure functions

The evolution operator:

$$\frac{d}{d\ln(Q^2)}\ln\left[F^{\rm NS}(Q^2)\right] = \frac{d}{d\ln(Q^2)}\ln\left[C^{\rm NS}(Q^2)\right] + \frac{d}{d\ln(Q^2)}\ln\left[q^{\rm NS}(Q^2)\right].$$

Its solution is given by

$$F^{\rm NS}(Q^2) = E_{\rm NS}(Q^2,Q_0^2)\cdot F^{\rm NS}(Q_0^2).$$

The Wilson coefficient is given by

$$C(Q^2) = 1+\sum_{k=1}^{\infty} a_s^k(Q^2)C_k, \qquad C_k = c_k + h_k\left(L_c, L_b\right), \quad a_s = \frac{\alpha_s}{4\pi}$$

Here c_k denote the expansion coefficients of the massless Wilson coefficients and h_k of the massive Wilson coefficient, with

$$L_c = \ln\left(\frac{Q^2}{m_c^2}\right), \qquad L_b = \ln\left(\frac{Q^2}{m_b^2}\right)$$

and $m_{c,b}$ are the on–shell charm and bottom quark masses.

The NS structure functions

$$h_1 = 0 \quad (1)$$
$$h_2 = \hat{h}_2(L_c) + \hat{h}_2(L_b) \quad (2)$$
$$h_3 = \hat{h}_3(L_c) + \hat{h}_3(L_b) + \hat{\hat{h}}_3(L_c, L_b), \quad (3)$$

where $\hat{h}_i$ denote the single mass and $\hat{\hat{h}}_3$ the double mass contributions. One may rewrite the differential operator

$$\frac{d}{d\ln(Q^2)} = \frac{da_s(Q^2)}{d\ln(Q^2)} \cdot \frac{d}{da_s(Q^2)}, \qquad \frac{da_s}{d\ln(Q^2)} = -\sum_{k=0}^{\infty} \beta_k a_s^{k+2}. \quad (4)$$

$$\frac{d}{da_s} \ln\left[q^{\mathrm{NS}}(Q^2)\right] = -\frac{1}{2} \frac{\sum_{k=0}^{\infty} a_s^{k+1} P_k^{\mathrm{NS}}}{\sum_{k=0}^{\infty} \beta_k a_s^{k+2}}, \quad (5)$$

where β_k are expansion coefficients of the QCD–β function and $P_{k,qq}^{\mathrm{NS}} \equiv P_k^{\mathrm{NS}}$ are the splitting functions.

The NS structure functions: scaling violations

Evolution operator:

$$
\begin{aligned}
E_{\mathrm{NS}}(Q^2, Q_0^2) \quad = \quad & \left(\frac{a}{a_0}\right)^{-\frac{P_0}{2\beta_0}} \Bigg\{ 1 + \frac{a - a_0}{2\beta_0^2} \Bigg\{ \left[1 + a^2 C_2(Q^2) - a_0^2 C_2(Q_0^2) \right] \left(2\beta_0^2 C_1 - \beta_0 P_1 + \beta_1 P_0 \right) \\
& - \frac{(a^2 - a_0^2)}{4\beta_0^3} \left(2\beta_0^2 C_1 - \beta_0 P_1 + \beta_1 P_0 \right) \left[2\beta_0^3 C_1^2 + \beta_0^2 P_2 - \beta_0 \beta_1 P_1 + \left(\beta_1^2 - \beta_0 \beta_2 \right) P_0 \right] \\
& + \frac{(a^2 + a a_0 + a_0^2)}{3\beta_0^2} \Big[2\beta_0^4 C_1^3 - \beta_0^3 P_3 + \beta_0^2 \beta_1 P_2 + \left(\beta_0^2 \beta_2 - \beta_0 \beta_1^2 \right) P_1 \\
& + \left(\beta_0^2 \beta_3 - 2\beta_0 \beta_1 \beta_2 + \beta_1^3 \right) P_0 \Big] + \frac{a - a_0}{4\beta_0^2} \left(2\beta_0^2 C_1 - \beta_0 P_1 + \beta_1 P_0 \right)^2 \\
& + \frac{(a - a_0)^2}{24\beta_0^4} \left(2\beta_0^2 C_1 - \beta_0 P_1 + \beta_1 P_0 \right)^3 - \frac{a + a_0}{2\beta_0} \Big[2\beta_0^3 C_1^2 + \beta_0^2 P_2 - \beta_0 \beta_1 P_1 \\
& + P_0 \left(\beta_1^2 - \beta_0 \beta_2 \right) \Big] \Bigg\} + a^2 C_2(Q^2) - a_0^2 C_2(Q_0^2) - C_1 \left[a^3 C_2(Q^2) - a_0^3 C_2(Q_0^2) \right] \\
& + a^3 C_3(Q^2) - a_0^3 C_3(Q_0^2) \Bigg\}
\end{aligned}
$$

The NS structure functions: scaling violations

Heavy flavor single mass corrections:

$$
\begin{aligned}
\hat{h}_2^{(Q)} &= -\frac{\beta_{0,Q}}{4} P_{qq,(0)} \ln^2\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\hat{P}^{\rm NS}_{qq,(1)} \ln\left(\frac{Q^2}{m^2}\right) + a_{qq}^{(2),\rm NS} + \frac{\beta_{0,Q}}{4}\zeta_2 P_{qq,(0)} + \hat{C}_q^{(2),\rm NS} \\
\hat{h}_3^{(Q)} &= -\frac{1}{6} P_{qq,(0)}\beta_{0,Q}\left(\beta_0 + 2\beta_{0,Q}\right)\ln^3\left(\frac{Q^2}{m^2}\right) + \frac{1}{4}\Big[-2P^{\rm NS}_{qq,(1)}\beta_{0,Q} + 2\hat{P}^{\rm NS}_{qq,(1)}\left(\beta_0+\beta_{0,Q}\right) \\
& -\beta_{1,Q}P_{qq,(0)}\Big]\ln^2\left(\frac{Q^2}{m^2}\right) - \frac{1}{2}\Big[-\hat{P}^{\rm NS}_{qq,(2)} - \left(4a_{qq,Q}^{(2),\rm NS} + \zeta_2\beta_{0,Q}P_{qq,(0)}\right)\left(\beta_0+\beta_{0,Q}\right) \\
& -P_{qq,(0)}\beta_{1,Q}^{(1)}\Big]\ln\left(\frac{Q^2}{m^2}\right) + 4\bar{a}_{qq,Q}^{(2),\rm NS}\left(\beta_0+\beta_{0,Q}\right) + P_{qq,(0)}\beta_{1,Q}^{(2)} + \frac{1}{6}P_{qq,(0)}\beta_0\beta_{0,Q}\zeta_3 \\
& +\frac{1}{4}P^{\rm NS}_{qq,(1)}\beta_{0,Q}\zeta_2 - 2\delta m_1^{(1)}\beta_{0,Q}P_{qq,(0)} - \delta m_1^{(0)}\hat{P}^{\rm NS}_{qq,(1)} + 2\delta m_1^{(-1)}a_{qq,Q}^{(2),\rm NS} + a_{qq,Q}^{(3),\rm NS} \\
& +\left[-\frac{\beta_{0,Q}}{4}P_{qq,(0)}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\hat{P}^{\rm NS}_{qq,(1)}\ln\left(\frac{Q^2}{m^2}\right) + a_{qq}^{(2),\rm NS} + \frac{\beta_{0,Q}}{4}\zeta_2 P_{qq,(0)}\right]C_q^{(1),\rm NS} \\
& +\hat{C}_q^{(3),\rm NS}.
\end{aligned}
$$

The NS structure functions: scaling violations

Heavy flavor two mass corrections:

$$\hat{\hat{h}}_3^{\rm NS} = P_{qq,(0)}\beta_{0,Q}^2\left[\frac{2}{3}\left(L_c^3+L_b^3\right)+\frac{1}{2}\left(L_c^2L_b+L_cL_b^2\right)\right]-\beta_{0,Q}\hat{P}_{qq,(1)}^{\rm NS}\left(L_c^2+L_b^2\right)-\left[4a_{qq,Q}^{(2),\rm NS}\beta_{0,Q}\right.$$
$$\left.-\frac{1}{2}\beta_{0,Q}^2P_{qq,(0)}\zeta_2\right](L_c+L_b)+8\bar{a}_{qq,Q}^{(2),\rm NS}\beta_{0,Q}+\tilde{a}_{qq,Q}^{(3),\rm NS}(m_c,m_b,Q^2).$$

$$\hat{f}(x,N_F)=f(x,N_F+1)-f(x,N_F).$$

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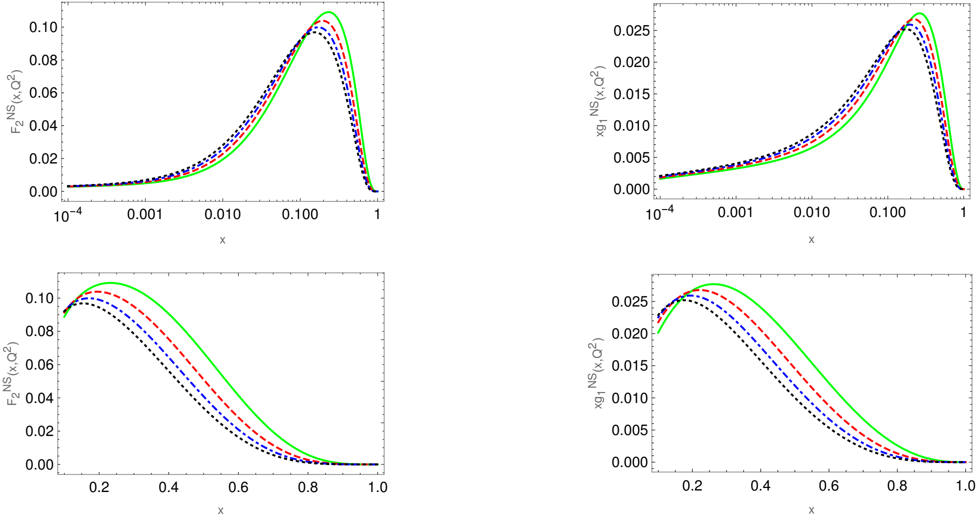
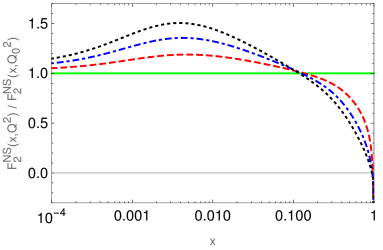
The NS structure functions: scaling violations

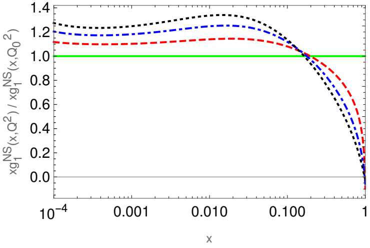


Full lines: $Q^2 = 10$ GeV^2, dashed lines $Q^2 = 10^2$ GeV^2 dash-dotted lines $Q^2 = 10^3$ GeV^2 dotted lines $Q^2 = 10^4$ GeV^2.

The NS structure functions: scaling violations

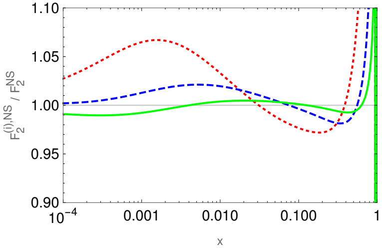
Relative size of the scaling violations, $Q_0^2 = 10\ \text{GeV}^2$.

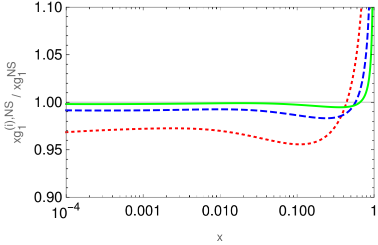




The NS structure functions: scaling violations

Relative effect of the higher order corrections, normalized to the N^{3}LO corrections at $Q^2 = 100$ GeV^2

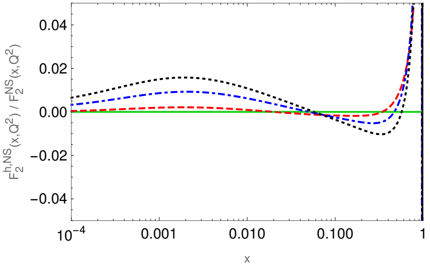


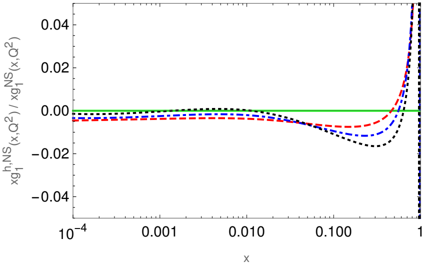


Dotted lines: LO, dashed lines: NLO, full lines NNLO.

The NS structure functions: scaling violations

The relative effects of the heavy flavor corrections.





dashed lines: $Q^2 = 10^2$ GeV^2, dash-dotted lines: $Q^2 = 10^3$ GeV^2, dotted lines: $Q^2 = 10^4$ GeV^2.

The NS structure functions: scaling violations

The 4-loop anomalous dimensions

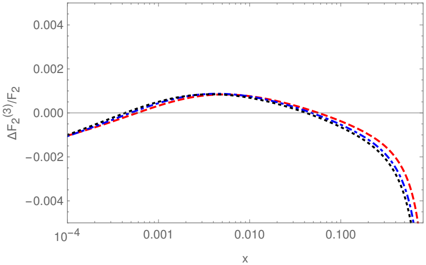
N	$\delta\gamma^{+,\mathrm{NS}}$	N	$\delta\gamma^{-,\mathrm{NS}}$
2	0.208822541	1	0.0
4	0.123728742	3	0.147102092
6	0.087155544	5	0.101634935
8	0.064949195	7	0.074593595
10	0.049680399	9	0.056598595
12	0.038394815	11	0.043633919
14	0.029638565	13	0.033767853
16	0.022602035	15	0.025956941

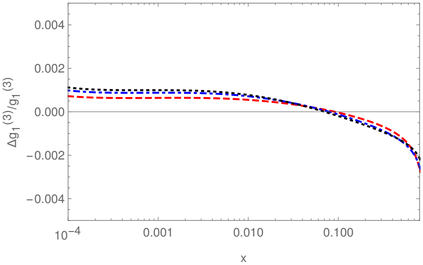
Table: The relative error comparing the exact moments [Baikov & Chetyrkin, Nucl. Phys. Proc. Suppl. 160 (2006) 76–79 ... S. Moch, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, JHEP 1710 (2017) 041 [arXiv:1707.08315]] of the four–loop anomalous dimensions, $\gamma^{(3),\pm,\mathrm{NS}}$, with the Padé approximation $P^{\pm\mathrm{NS}}_{qq,(3)} \approx (P^{\pm\mathrm{NS}}_{qq,(2)})^2 / P^{\pm\mathrm{NS}}_{qq,(1)}$.

From the 2nd moment, which agrees within 21%. The accuracy improves to 2.2% for the known even moments at N = 16 and to 2.6% for the odd moments at N = 15.

The NS structure functions: scaling violations

The effect of a $\pm 100\%$ error on the 4–loop splitting function.





dashed lines: $Q^2 = 10^2$ GeV^2, dash-dotted lines: $Q^2 = 10^3$ GeV^2, dotted lines: $Q^2 = 10^4$ GeV^2.

In our 2006 analysis this effect implied $\delta\Lambda_{\rm QCD} = \pm 2$ MeV at an experimental uncertainty of 26 MeV. The moments even allow to significantly improve about this.

Conclusions

- A precision determination of the strong coupling constant $\alpha_s(M_Z)$ requires a high luminosity measurement of a sufficiently simple inclusive observable.
- The measurement must be carried out under stringent systematic control.
- Such a measurement would have been possible in the past: air-core toroid (proposal, 1988). HERA data cannot be used, since DIS of deuterons have not been measured. Under sufficient preparation, it can be carried out at the EIC using proton and deuteron targets. LHeC may also perform such a measurement, provided also deuteron data will be available and the statistics for the non–singlet measurement is high enough.
- The theoretical analysis method is then scheme invariant evolution in the flavor non-singlet case for the structure function $F_2(x, Q^2)$.
- Both the light and heavy flavor corrections for this quantity are known on the level of the twist 2 approximation for $Q^2 \geq 10$ GeV^2, $W^2 \geq 15$ GeV^2, to measure $\alpha_s(M_Z)$ at an accuracy well below the 1% level.
- This is one way to decide what is the correct value of $\alpha_s(M_Z^2)$.