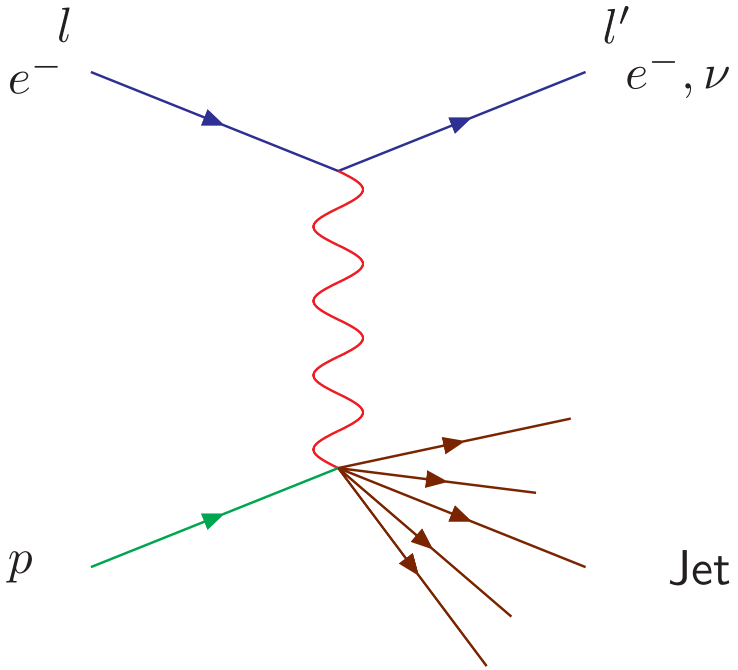
QCD Evolution of Unpolarized Parton Distributions

Johannes Blümlein
DESY

- Introduction
- Theory of Scale Evolution
- QCD Analysis of Unpolarized Structure Functions
- A few Remarks on Polarized Structure Functions
- Moments of Parton Densities
- $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$
- Outlook

DEEPLY INELASTIC SCATTERING

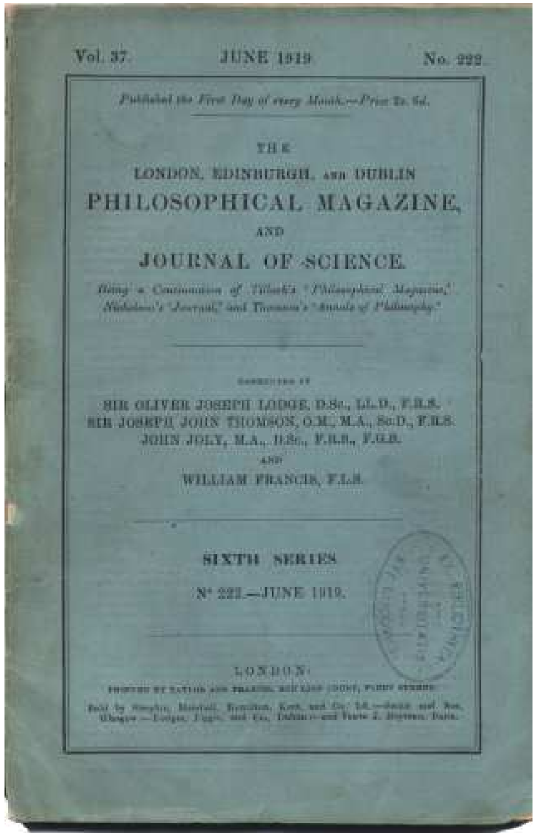


space – like process : $q^2 = (l - l')^2 = -Q^2 < 0 \quad W^2 = (p+q)^2 \geq M_p^2$

$$x = \frac{Q^2}{2p.q}, \qquad y = \frac{p.q}{p.l} \qquad 0 \leq x, y \leq 1$$

Prehistory and History

Discovery of the Proton (1919)



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SIR JOSEPH JOHN THOMSON, O.M., M.A., Sc.D., F.R.S.
JOHN JOLY, M.A., D.Sc., F.R.S., F.G.S.
AND
WILLIAM FRANCIS, F.L.S.

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LONDON:

"We must conclude that the nitrogen atom is disintegrated under the intense forces developed in a close collision with a swift alpha particle, and that the hydrogen atom which is liberated formed a constituent part of the nitrogen nucleus."

-Ernest Rutherford

particle zoo: e^-, p

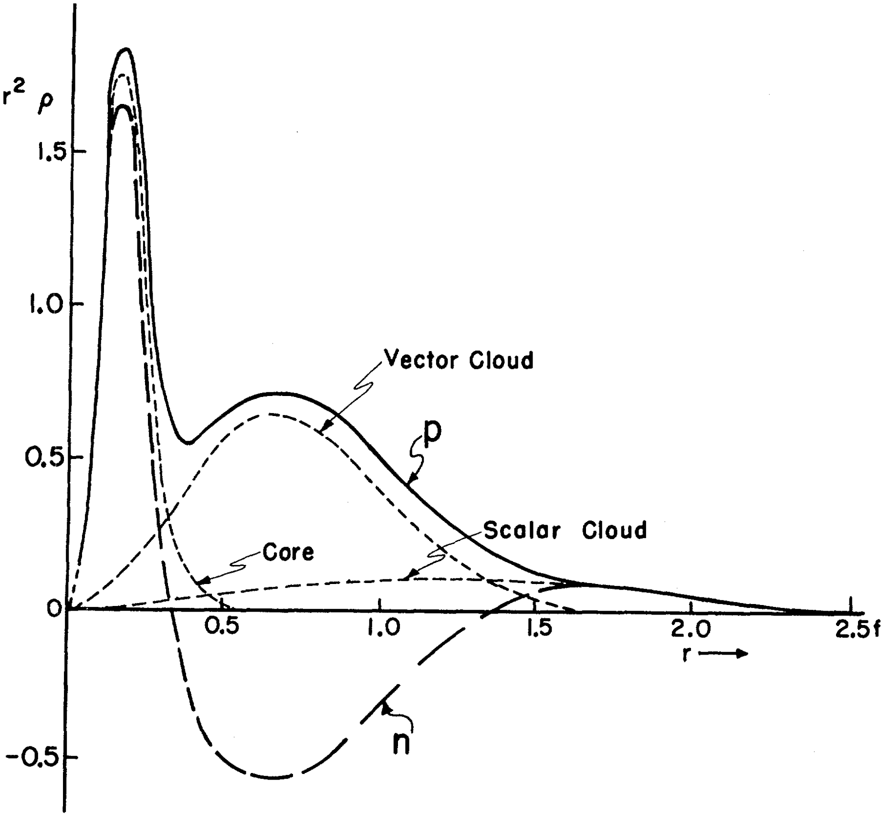
Nucleons at rising spatial resolution

$Q^2 \approx 0.5 M_N^2$: Hofstadter's Experiments 1950-1960

R. Hofstadter

(1915-1990)

Olson, Schopper, Wilson (1961)



The SLAC-MIT Experiments

Discovery of Scaling

SLAC

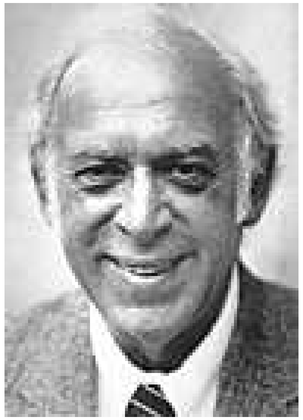
SLAC-MIT detector

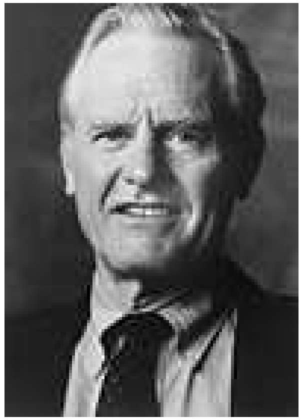
W. Panofsky (1919-2007)

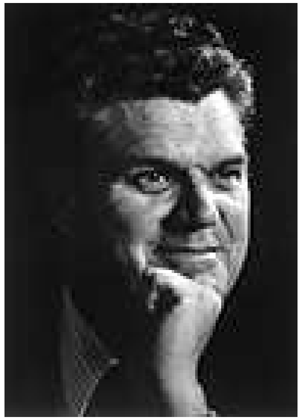
The SLAC-MIT Experiments

An American Success Story:

Discovery of Scaling







$Q^2 \approx 3M_N^2$ J. Friedman *1930 H. Kendall (1926-1999) R. Taylor *1929 (1968/69)

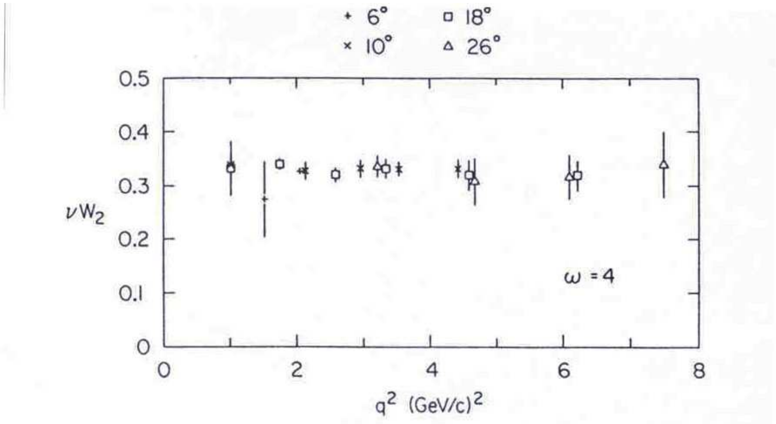


FIG. 13. An early observation of scaling: νW_2 for the proton as a function of q^2 for $W > 2$ GeV, at $\omega = 4$.

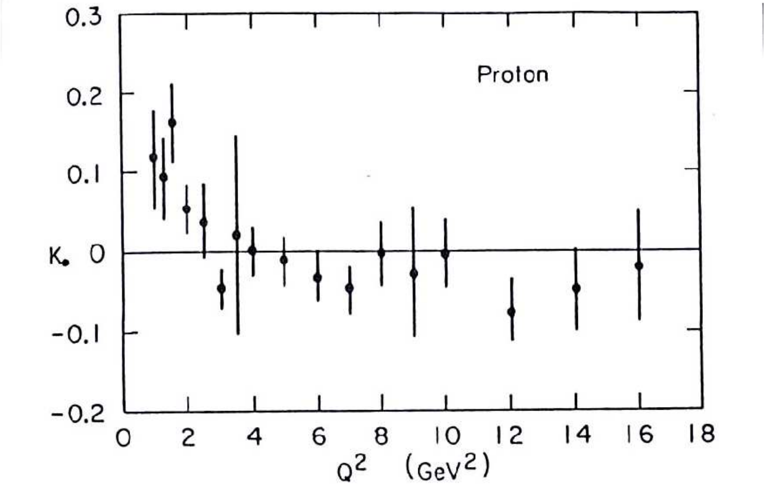


FIG. 18. The Callan-Gross relation: K_0 vs q^2, where K_0 is defined in the text. These results established the spin of the partons as 1/2.

precise measurements in a new kinematic region confirm a theoretical prediction

J. Bjorken
*1934

scaling:

$$\lim_{Q^2,\nu\to\infty,x=\text{fixed}} F_i(\nu,Q^2) = F_i(x)$$

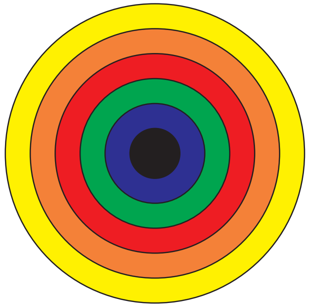
R. Feynman
(1918-1988)

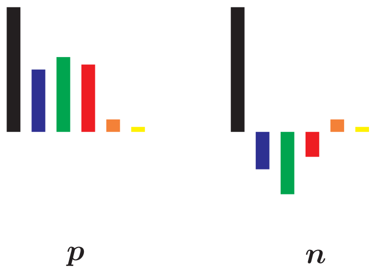
and find the constituents of hadrons,
the partons.

$$W_i(x,Q^2) = \sum_i dx_i \int_0^1 e_i^2 f(x_i)\delta\left(\frac{q.p_i}{M^2} - \frac{Q^2}{M^2}\right)$$

$\Longrightarrow$ **The measurement of F_L was instrumental to rule out vector-meson dominance models etc.**

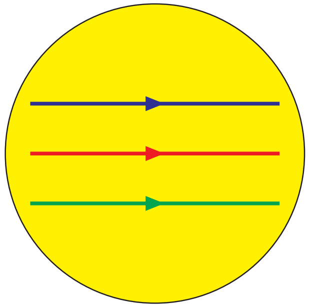
$Q^2 \sim 0.5 \cdot M_p^2$





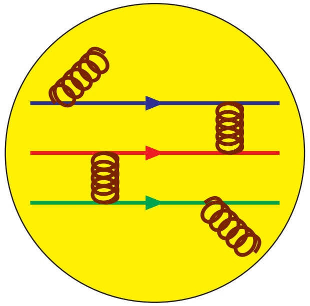
Charge Distribution

$Q^2 \sim 3 \cdot M_p^2$



Scaling

$Q^2 \sim 10...500 \cdot M_p^2$



Violation of Scaling

Today: $1 < Q^2 < 50.000$ **GeV**^{2} $\equiv 1/10.000 R_p$

DIS Structure Functions @ Twist 2

$$F_j(x,Q^2) = \hat{f}_i(x,\mu^2) \otimes \sigma_j^i\left(\alpha_s, \frac{Q^2}{\mu^2}, x\right)$$

$$\uparrow \mathrm{bare\ pdf} \qquad \uparrow \mathrm{sub-system\ cross-sect.}$$

$$= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i\left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\mathrm{finite\ \ pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k\left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\mathrm{finite\ \ Wilson\ \ coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Evolution Equations

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-2\gamma_\psi(g)\right]F_i(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\gamma_\kappa^N(g)-2\gamma_\psi(g)\right]f_k(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-\gamma_\kappa^N(g)\right]C_j^k(N)=0$$

CALLAN–SYMANZIK equations for mass factorization

$\equiv$ ALTARELLI–PARISI evolution equations

x-space :

$$\frac{d}{d\log(\mu^2)}\left(\begin{array}{c} q^+(x,Q^2) \\ G(x,Q^2) \end{array}\right)=\frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s)\otimes\left(\begin{array}{c} q^+(x,Q^2) \\ G(x,Q^2) \end{array}\right)$$

$$\boldsymbol{P}(x,\alpha_s)=\boldsymbol{P}^{(0)}(x)+\frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x)+\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{P}^{(2)}(x)+\ldots$$

Evolution Equations

$$\frac{da_s(\mu^2)}{d\ln\mu^2} = -\sum_{k=0}^{\infty}\beta_k a_s^{k+2}(\mu^2), \quad a_s(\mu^2) = \frac{\alpha_s(\mu^2)}{4\pi}$$

$$a_s(\mu^2) = \frac{a_s(\mu_0^2)}{1 + a_s(\mu_0^2)\beta_0 \ln(\mu^2/\mu_0^2)}$$

$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_R N_F > 0 \Longrightarrow$ asymptotic freedom

Solution in Mellin space :

$$\frac{df_{NS}(\mu^2)}{d\ln\mu^2} = a_s(\mu^2)P_{NS}^{(0)}(N)f_{NS}(N) + O(a_s^2)$$

$$f_{NS}(\mu^2,N) = f_{NS}(\mu_0^2,N)\left(\frac{a_s(\mu^2)}{a_s(\mu_0^2)}\right)^{-P_{NS}^{(0)}(N)/\beta_0}[1+O(a_s)]$$

$$F_{NS}(Q^2,N) = C_{NS}(Q^2/\mu^2,N)\cdot f_{NS}(\mu^2,N), \quad \mu^2 = \text{factorization scale}$$

Evolution Equations

LO splitting functions :

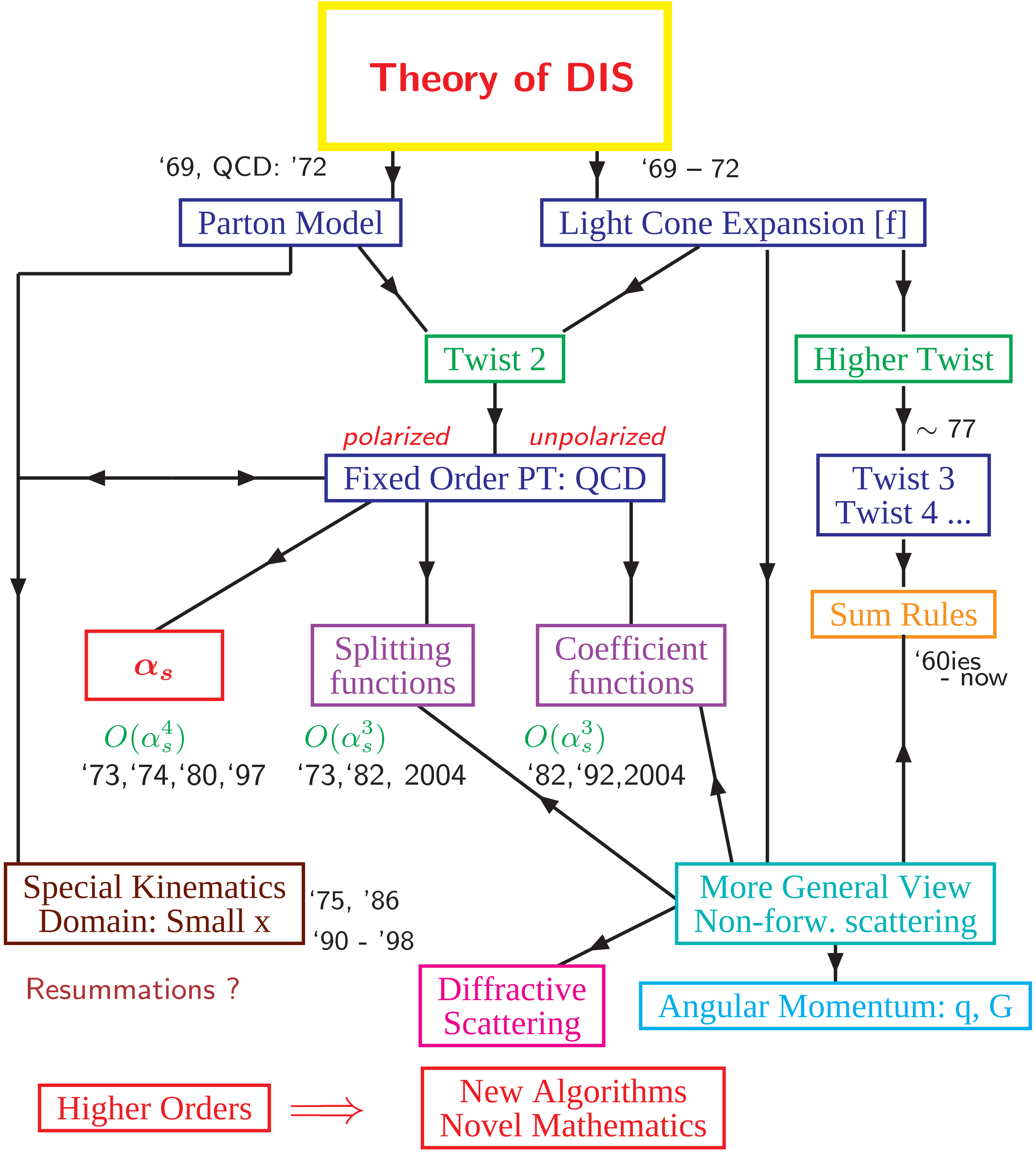
$$P_{qq}^{(0)}(x) = P_{NS}^{(0)}(x) \quad = \quad 2C_F \left(\frac{1+x^2}{1-x}\right)_+$$

$$P_{NS}^{(0)}(N) \quad = \quad -2C_F \left[2S_1(N-1) - \frac{(N-1)(3N+2)}{2N(N+1)}\right]$$

$$P_{qq}^{(0)}(N) \quad = \quad \int_0^1 dx x^{N-1} P_{qq}^{(0)}(x)$$

$$\int_0^1 dx \, [f(x)]_+ g(x) \quad = \quad \int_0^1 dx \, [g(x) - g(1)] \, f(x)$$

- No distrribution valued components in $P_{NS}(N)$.
- Harmonic sums appear.
- More involved, but similar expressions also for Wilson coefficients and all HO corrections.



Theory of DIS

'69, QCD: '72

'69 – 72

Parton Model

Light Cone Expansion [f]

Twist 2

Higher Twist

∼ 77

polarized

unpolarized

Fixed Order PT: QCD

Twist 3
Twist 4 ...

Sum Rules

'60ies
\- now

α_s

Splitting functions

Coefficient functions

$O(\alpha_s^4)$
'73,'74,'80,'97

$O(\alpha_s^3)$
'73,'82, 2004

$O(\alpha_s^3)$
'82,'92,2004

Special Kinematics
Domain: Small x

'75, '86

'90 - '98

More General View
Non-forw. scattering

Resummations ?

Diffractive Scattering

Angular Momentum: q, G

Higher Orders $\Longrightarrow$ New Algorithms
Novel Mathematics

1

Status of Highest Order Calculations

- Running α_s: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$

  Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;

  $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;

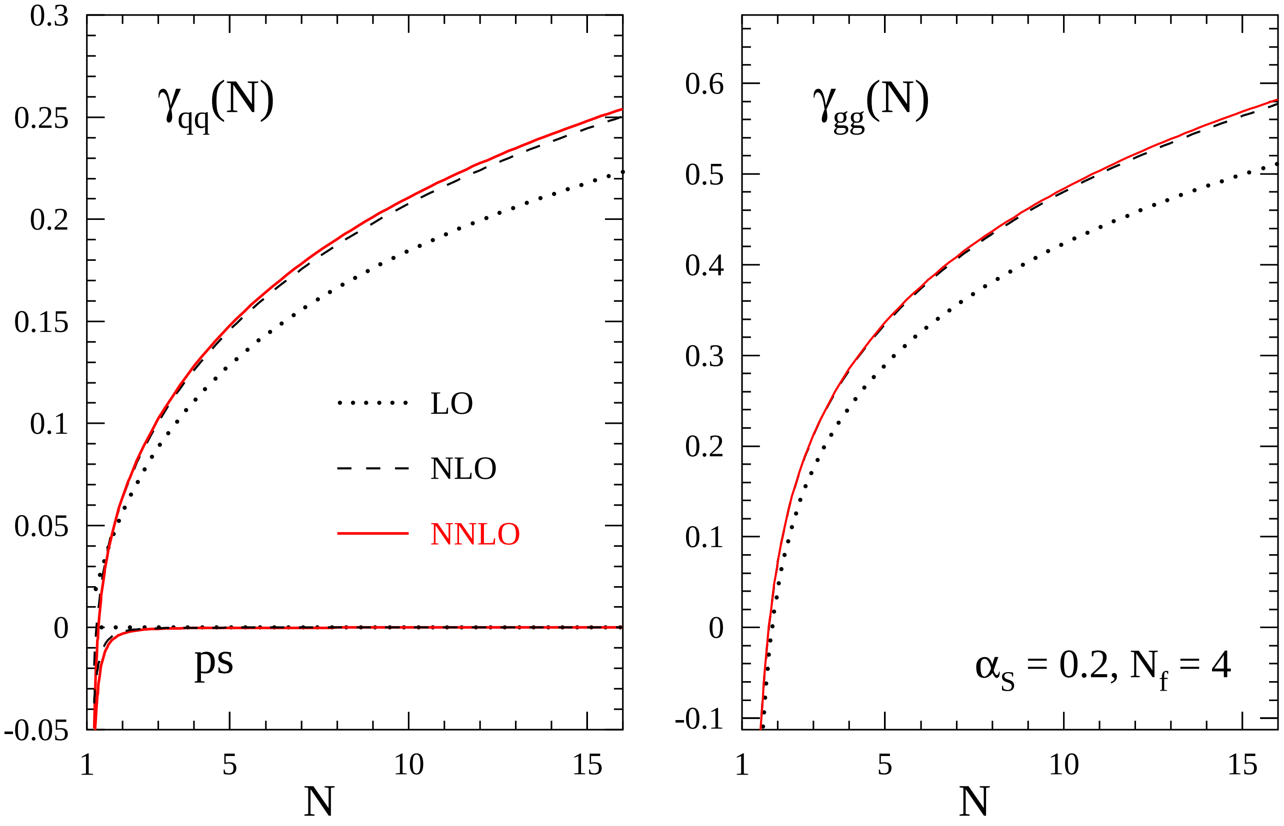
  Kumano, Miyama; Vogelsang; 1997; Gracey 2006
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993

  Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$ Watson 1982
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L: $O(\alpha_s^3)$

  Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,

  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_2: $O(\alpha_s^2 \varepsilon)$: all operators (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: First contributions to the moments of the operator matrix elements, Bierenbaum, Blümlein, Klein, 2008

Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004

Complex Analysis of these Functions

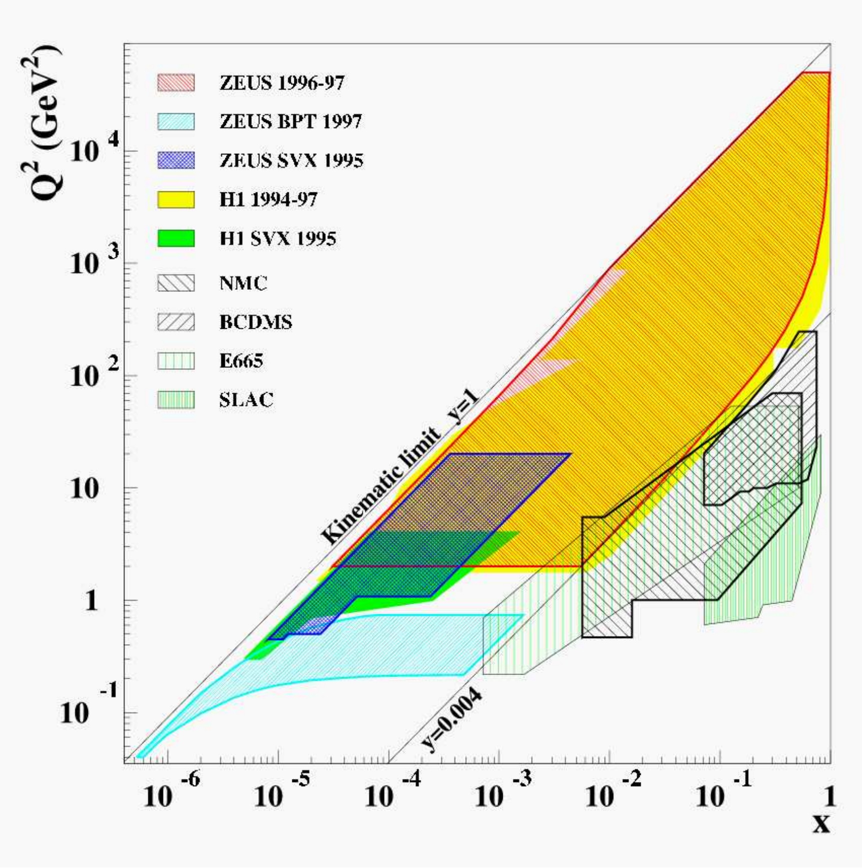
- Construct exact analytic continuations to complex N
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z+1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all Q^2
- Include the heavy flavor Wilson coefficients in Mellin Space
- Perform a single fast, numerical Mellin inversion
(at high precision)

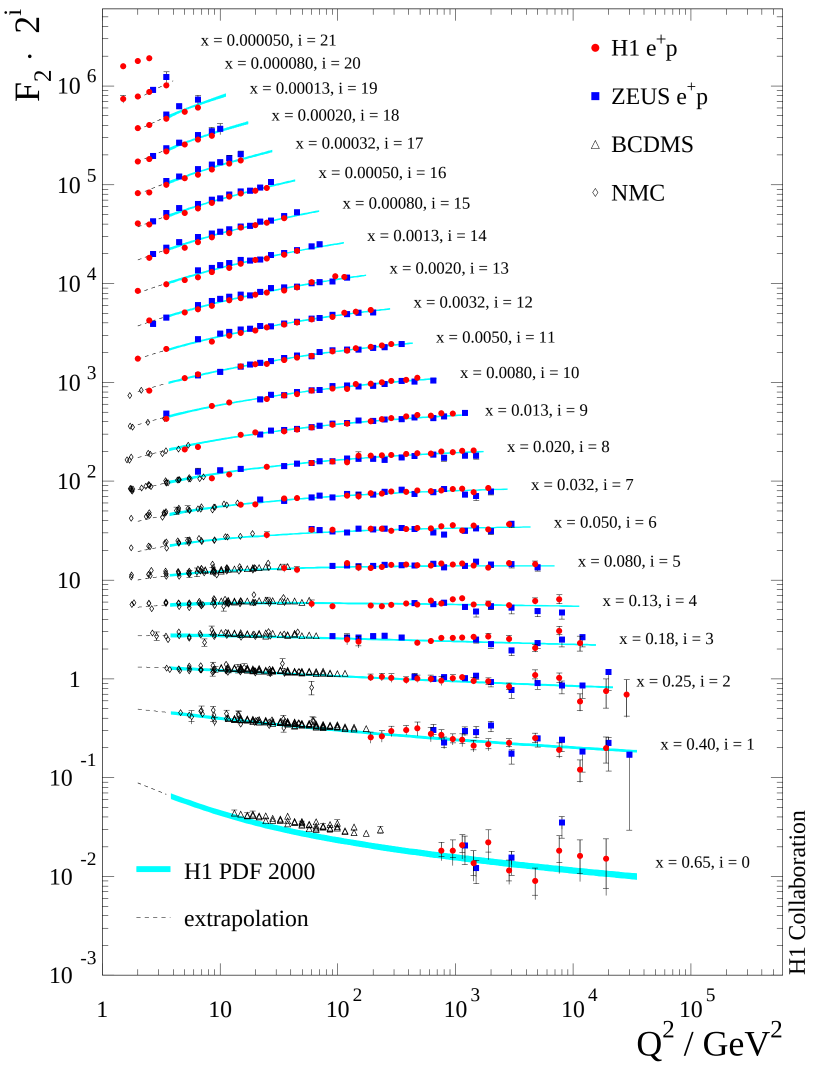
$\Rightarrow$ **Fastest and most Precise Way of Analysis**

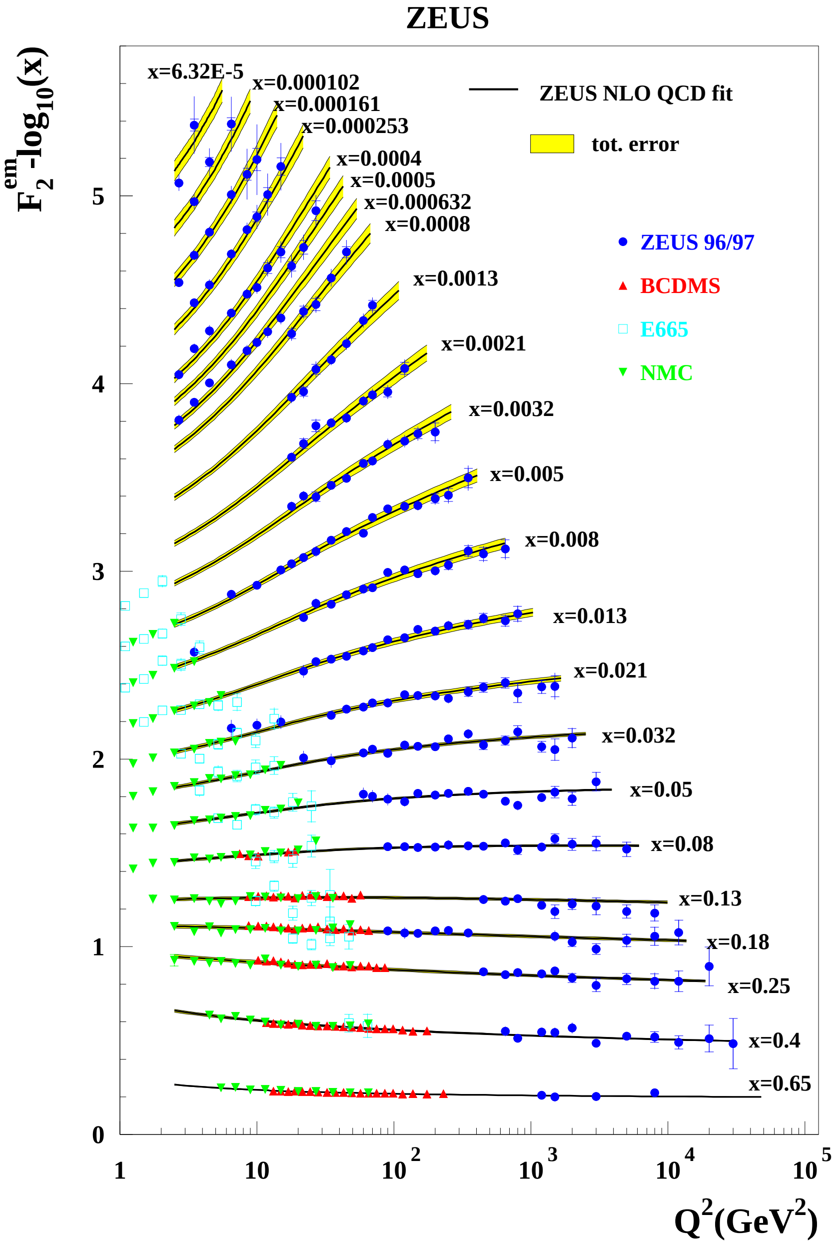
2. QCD Analysis of Unpolarized Structure Functions

DIS range
Nucleon structure:

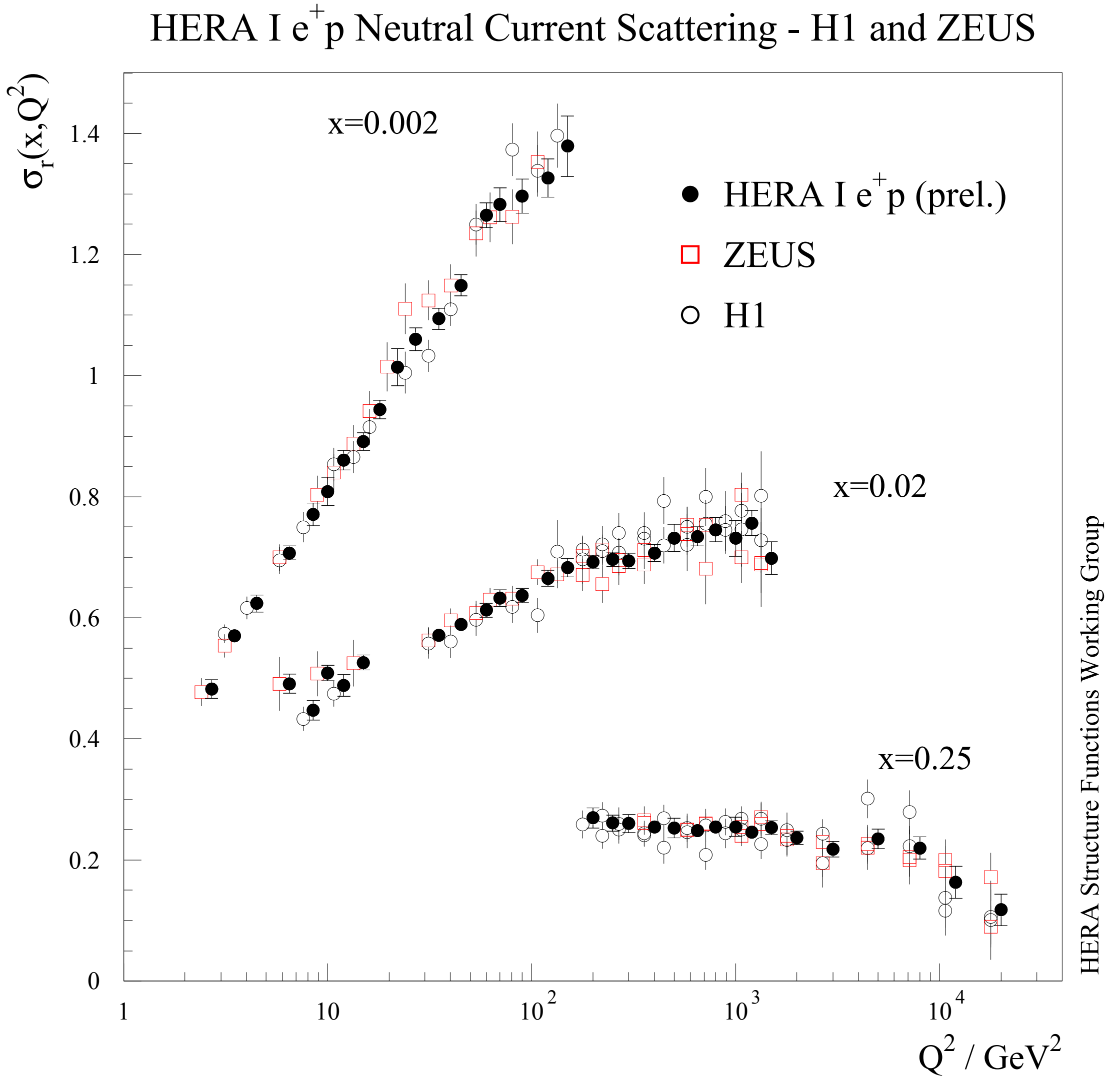
$10^{-5} < x < 0.9,$
$1 < Q^2 < 50.000 \mathrm{GeV}^2$




$F_2 \cdot 2^i$
Q^2 / GeV^2
x = 0.000050, i = 21
x = 0.000080, i = 20
x = 0.00013, i = 19
x = 0.00020, i = 18
x = 0.00032, i = 17
x = 0.00050, i = 16
x = 0.00080, i = 15
x = 0.0013, i = 14
x = 0.0020, i = 13
x = 0.0032, i = 12
x = 0.0050, i = 11
x = 0.0080, i = 10
x = 0.013, i = 9
x = 0.020, i = 8
x = 0.032, i = 7
x = 0.050, i = 6
x = 0.080, i = 5
x = 0.13, i = 4
x = 0.18, i = 3
x = 0.25, i = 2
x = 0.40, i = 1
x = 0.65, i = 0
● H1 e^+p
■ ZEUS e^+p
△ BCDMS
◊ NMC
H1 PDF 2000
extrapolation
H1 Collaboration
10^{-3}
10^{-2}
10^{-1}
1
10
10^2
10^3
10^4
10^5
10^6
1
10
10^2
10^3
10^4
10^5

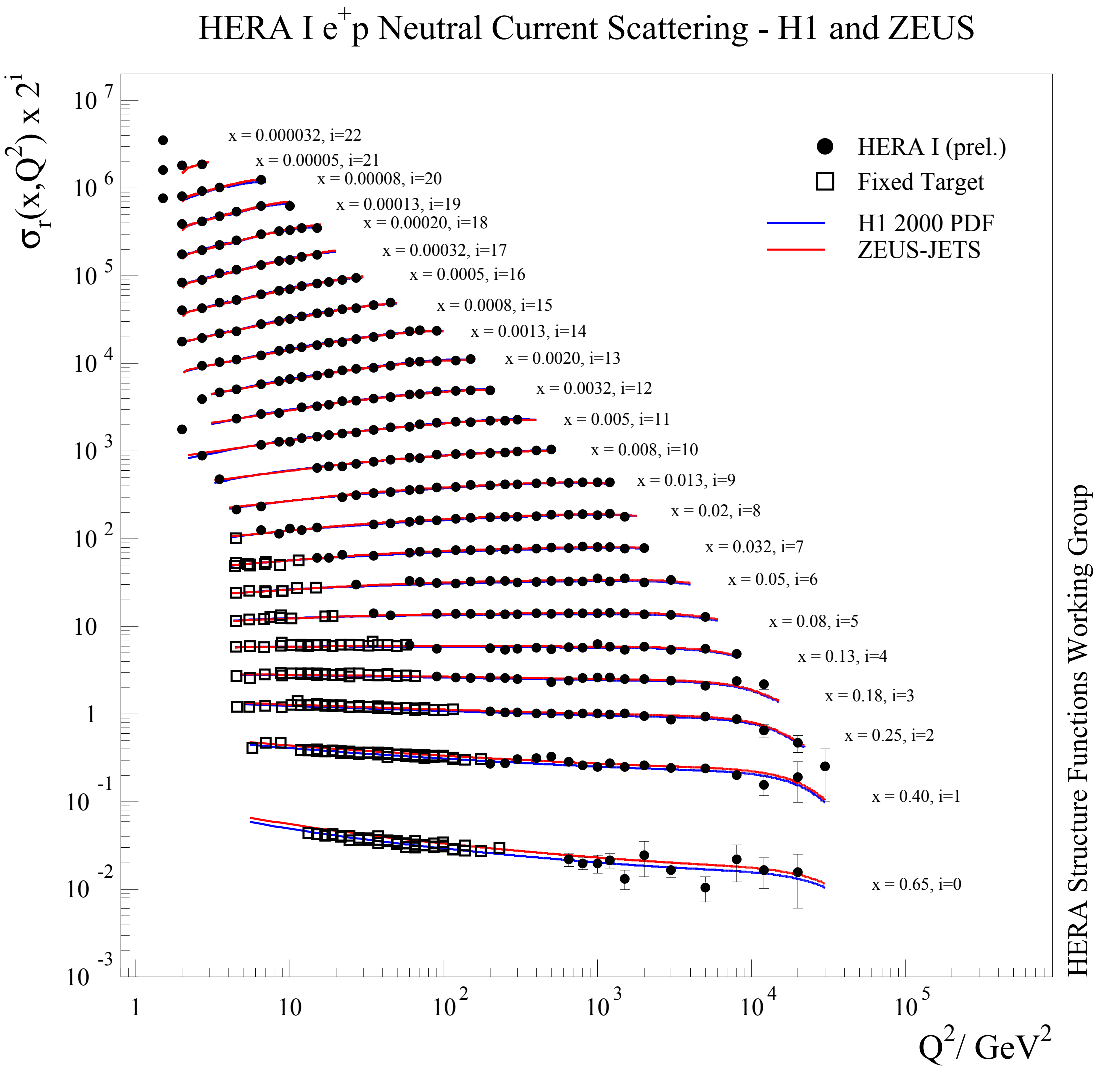

ZEUS
F_2^{em}-$\log_{10}(x)$
Q^2(GeV^2)
ZEUS NLO QCD fit
tot. error
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
0
1
2
3
4
5
1
10
10^2
10^3
10^4
10^5

New ZEUS + H1 averaged $F_2(x, Q^2)$



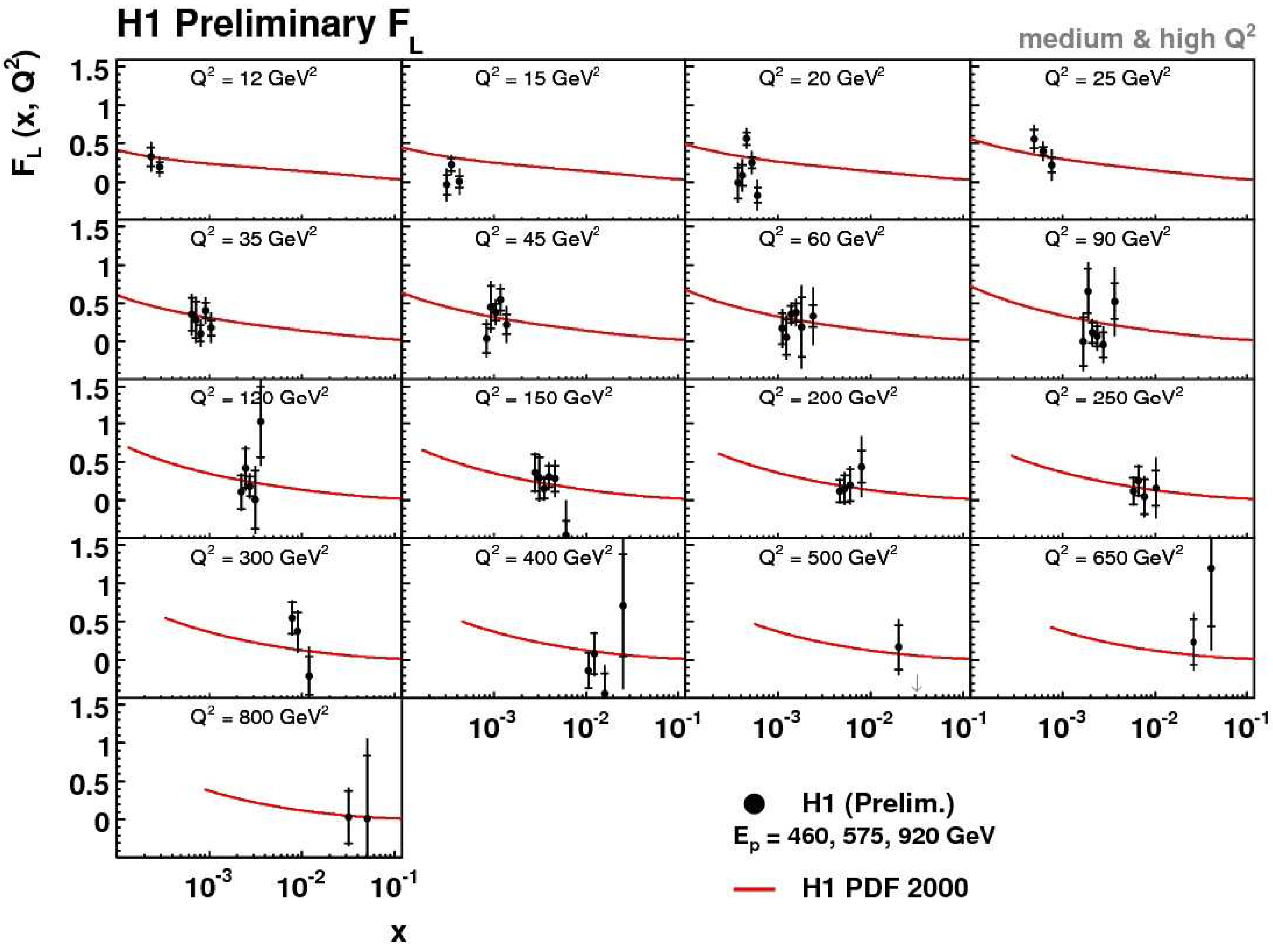
DIS08 Joël Feltesse

New ZEUS + H1 averaged $F_2(x, Q^2)$

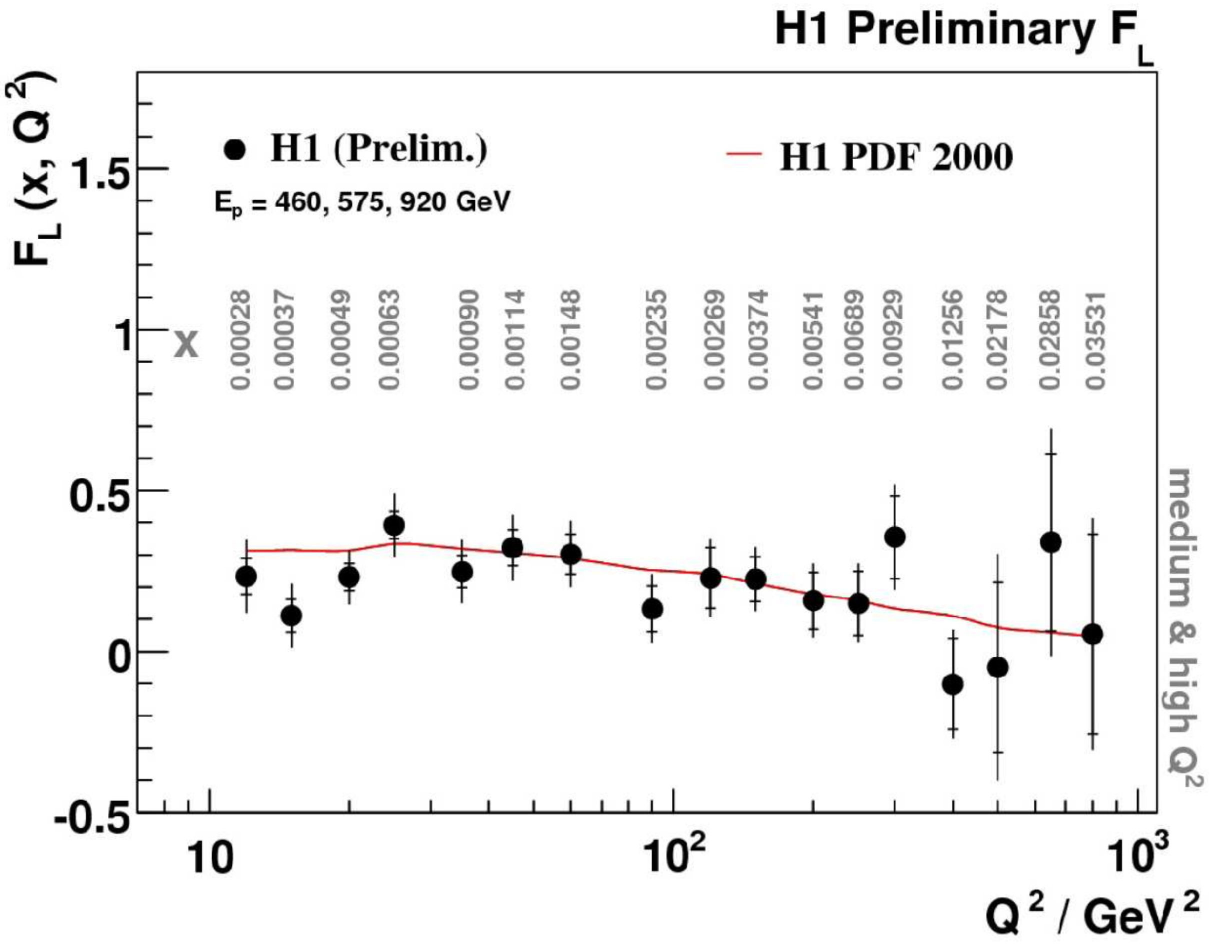


DIS08 Joël Feltesse

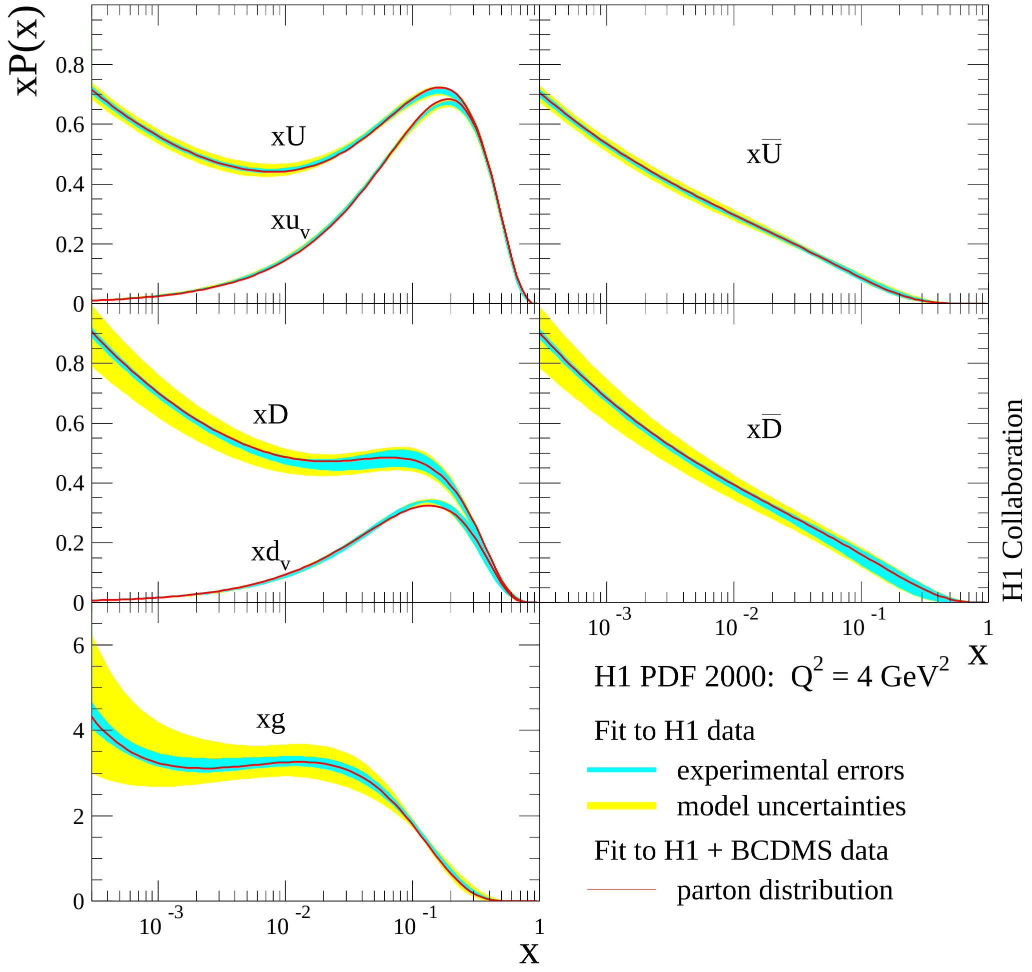
Direct $F_L(x, Q^2)$ Measurement at HERA



Direct $F_L(x, Q^2)$ Measurement at HERA

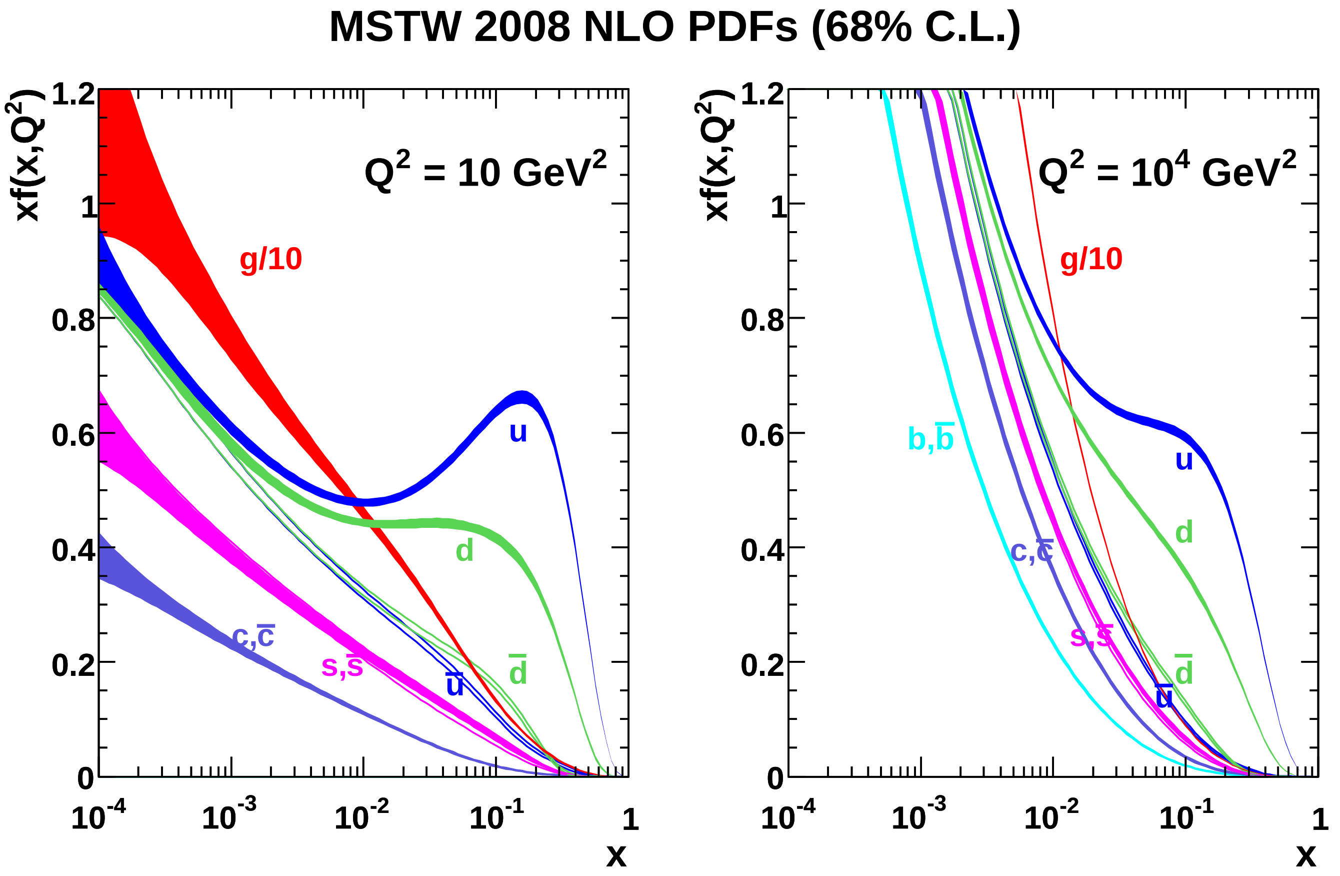


Parton Distributions: Overview

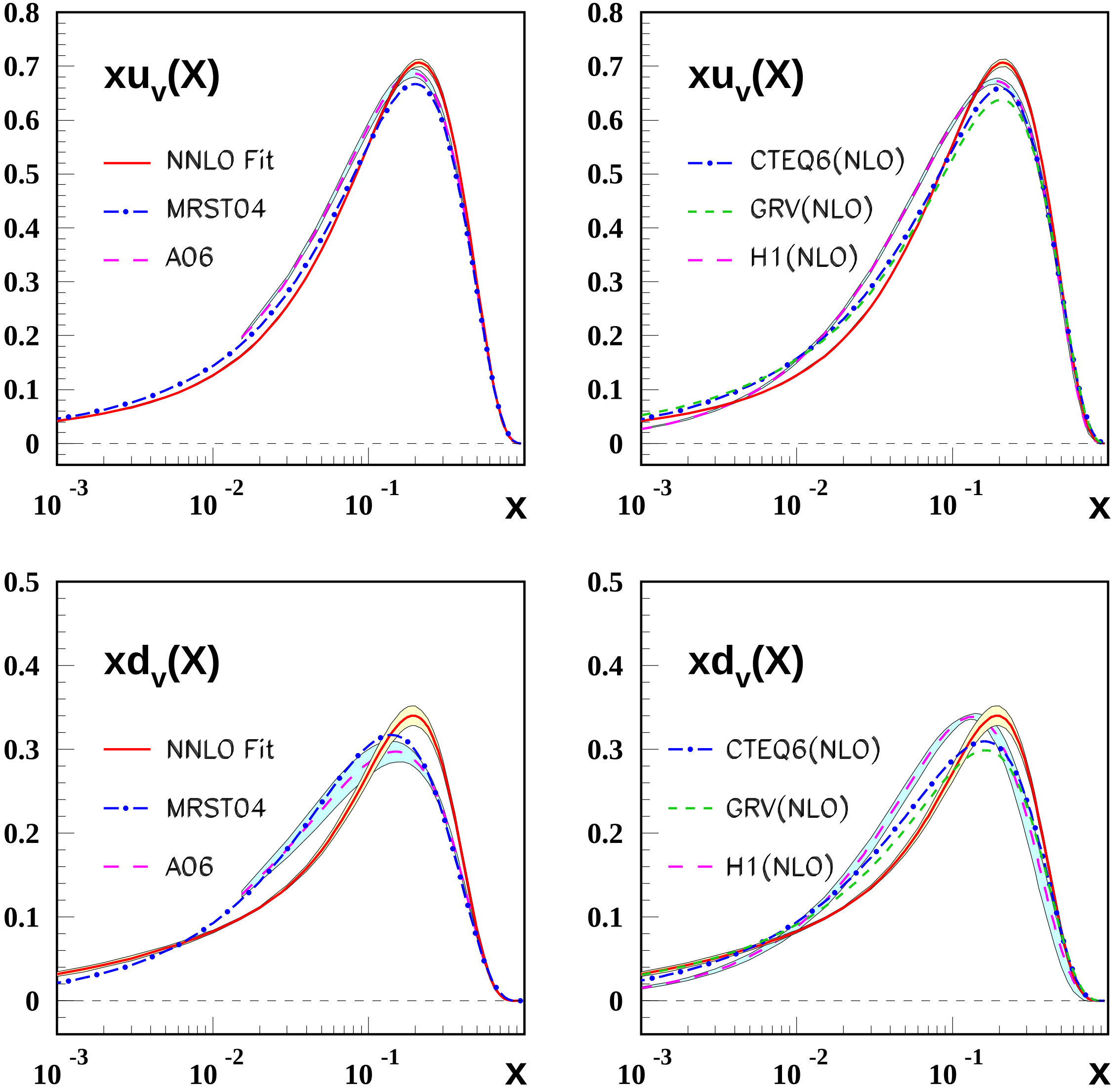


H1

Parton Distributions: Overview



World Data Analysis: Valence Distributions



World data:
NS-analysis

$W^2 > 12.5$ GeV^2, $Q^2 > 4$ GeV^2

N^3LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ ± 2 MeV in Λ_{QCD}

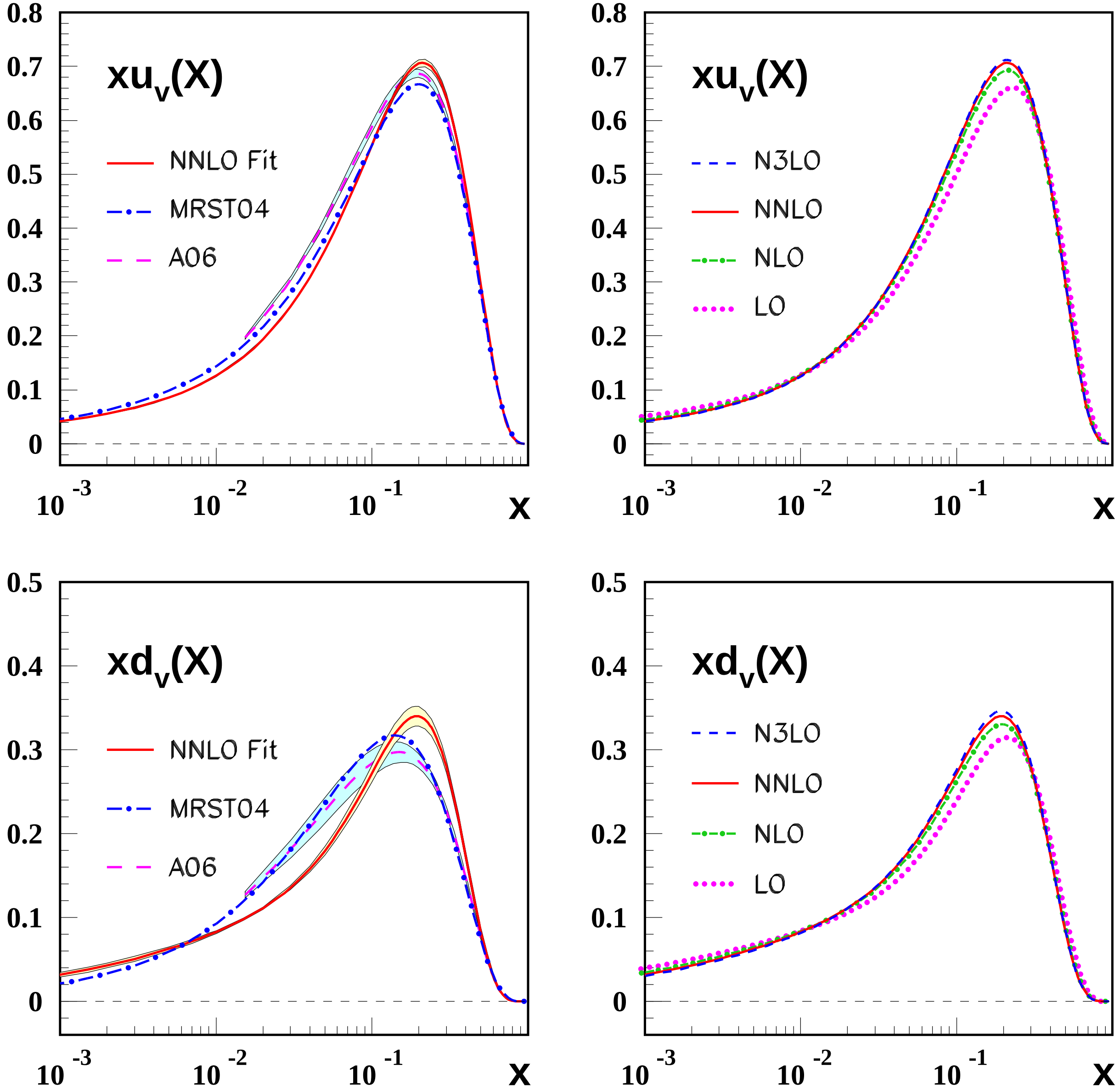
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

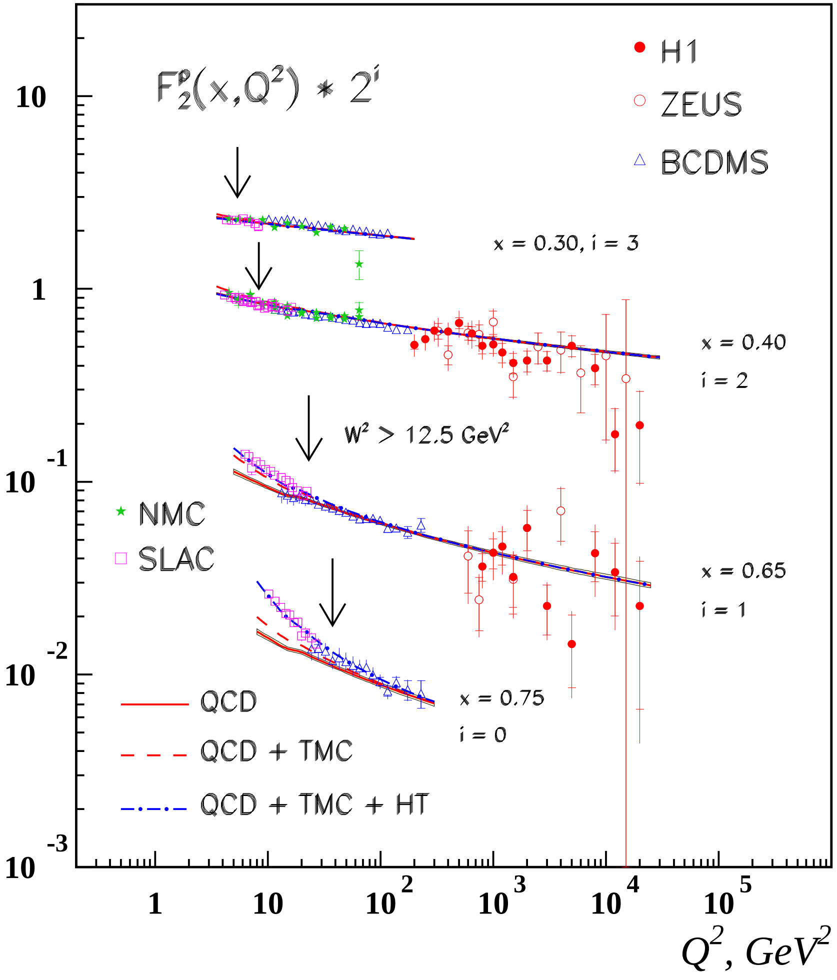
$$\begin{aligned}
\gamma_2^{3;NS} \quad &= \quad \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 \\
&\quad + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4
\end{aligned}$$

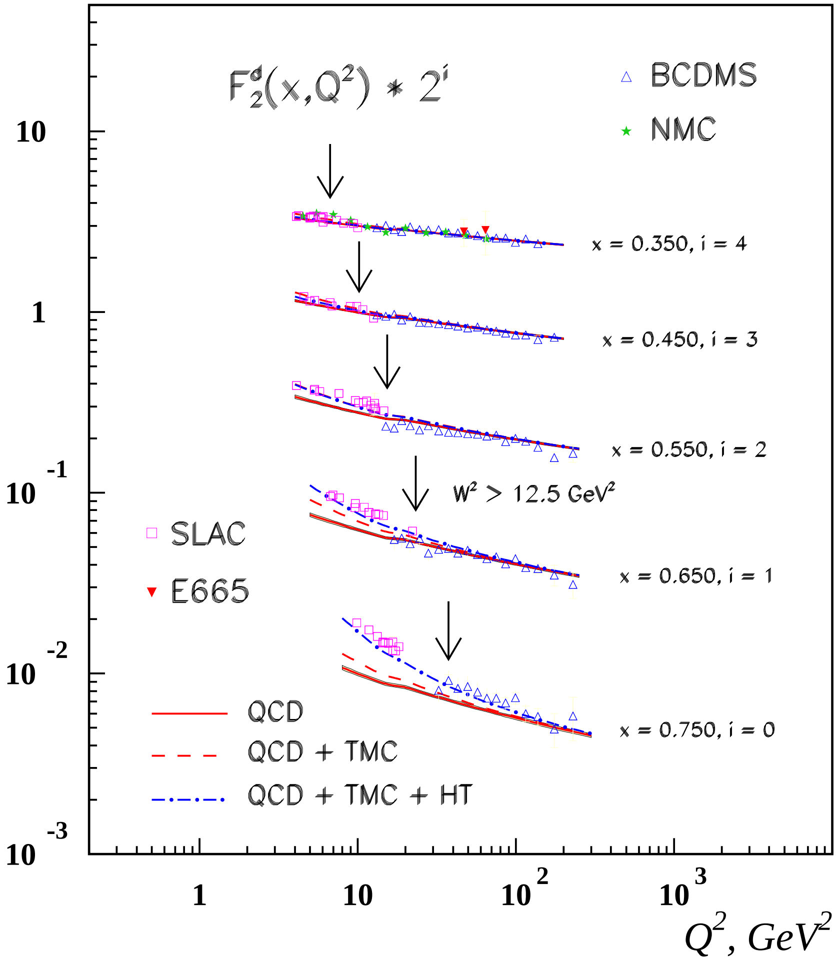
The results agree better than 20%.

Valence Distributions

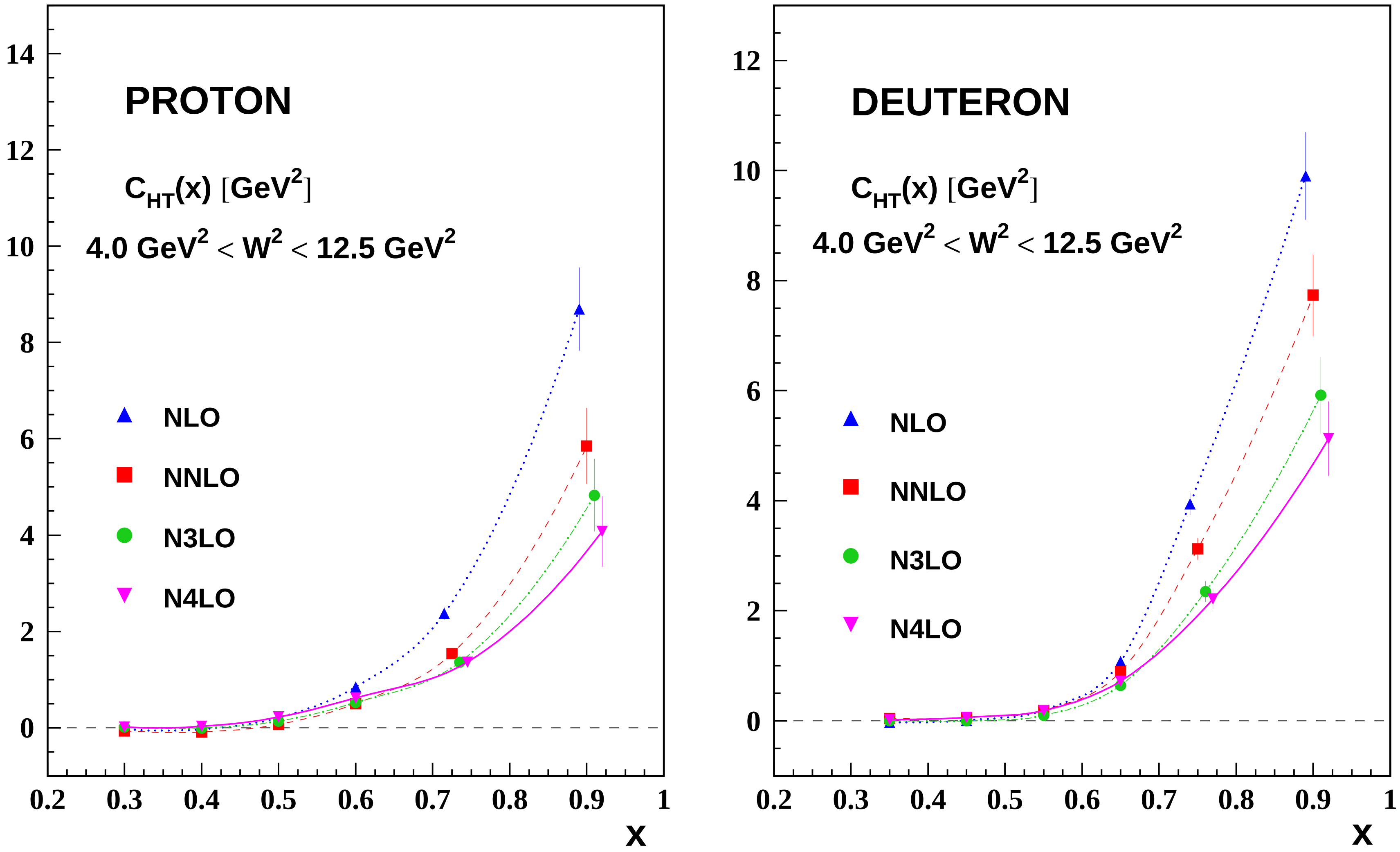


Valence Distributions



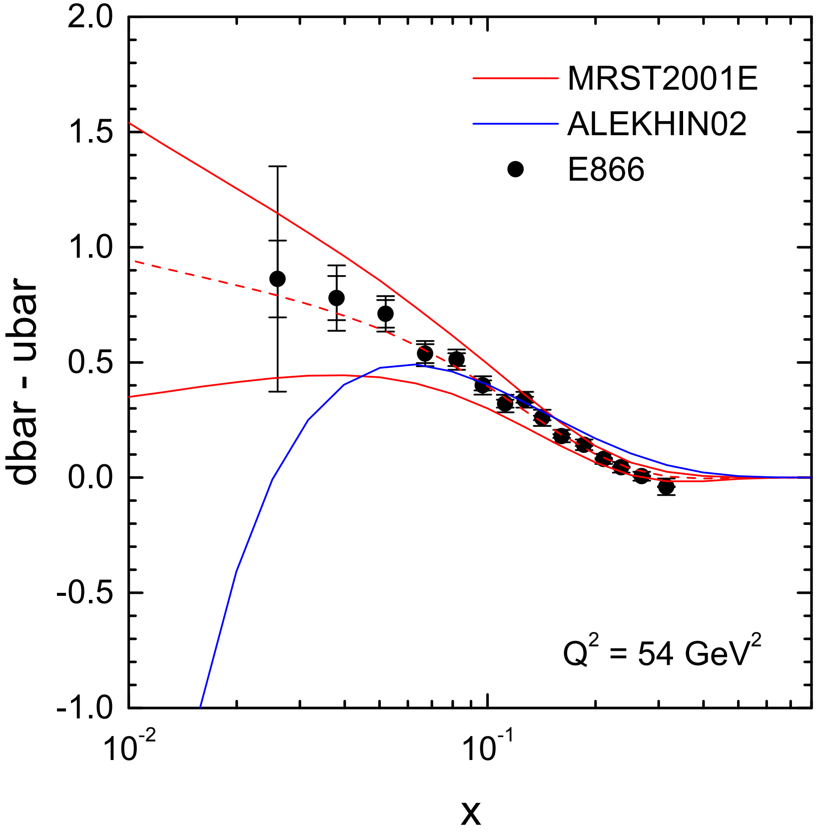


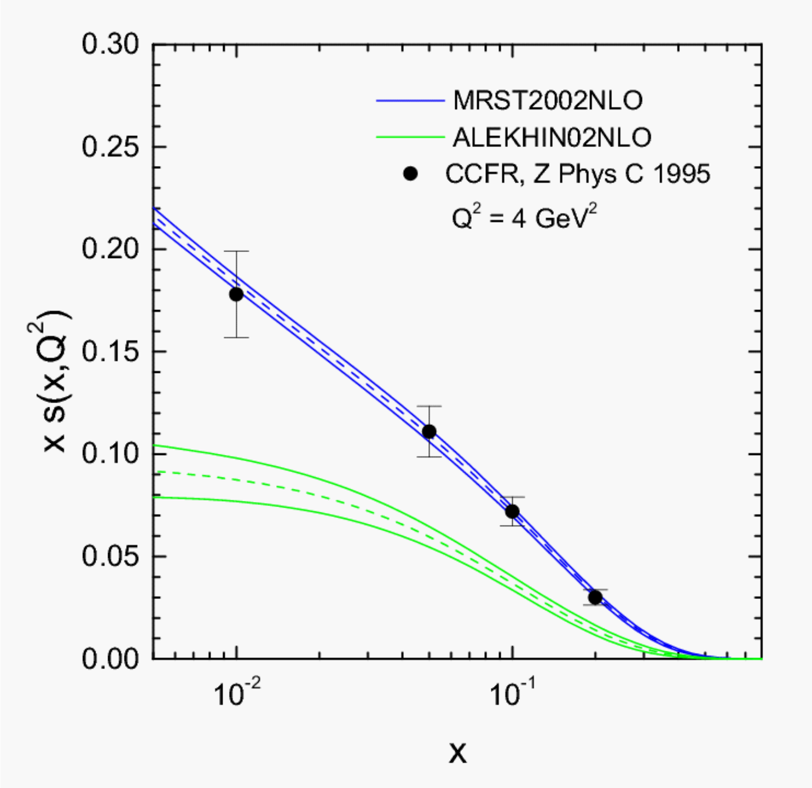
Valence Distributions: higher twist



- agreement between p and d analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

Flavor distributions: light quarks



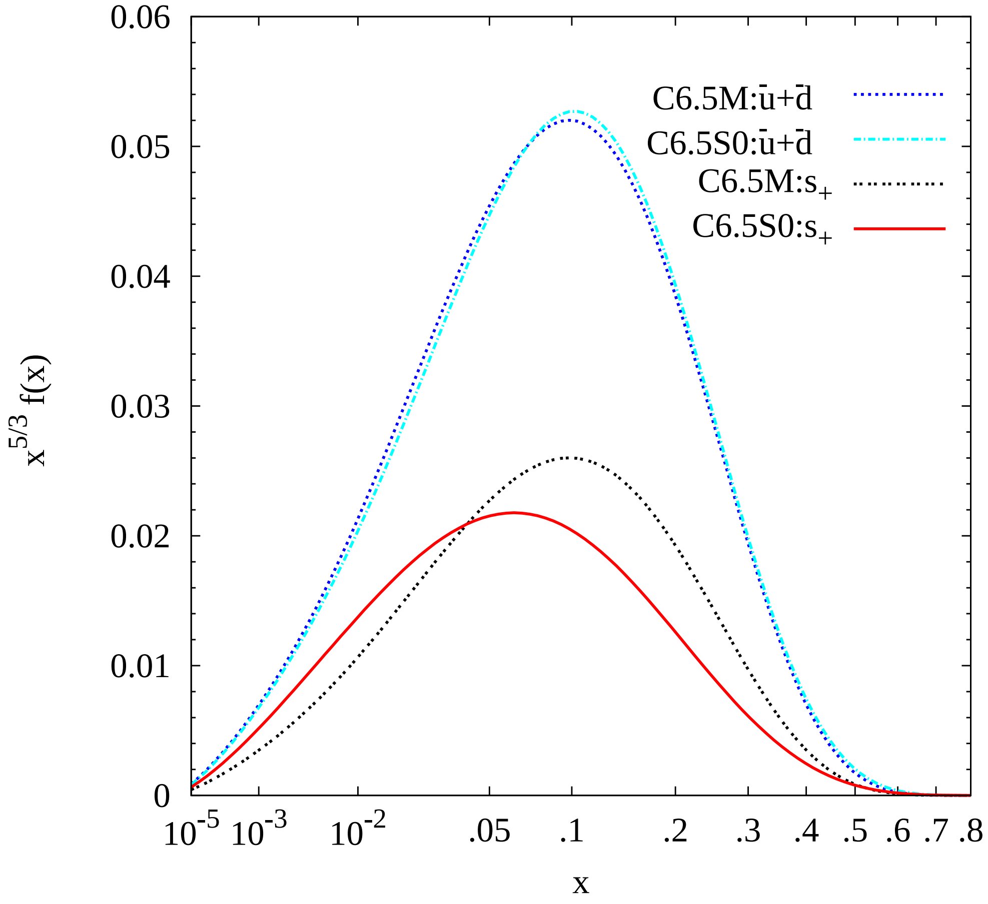


J. Stirling, 2004

More work needed.

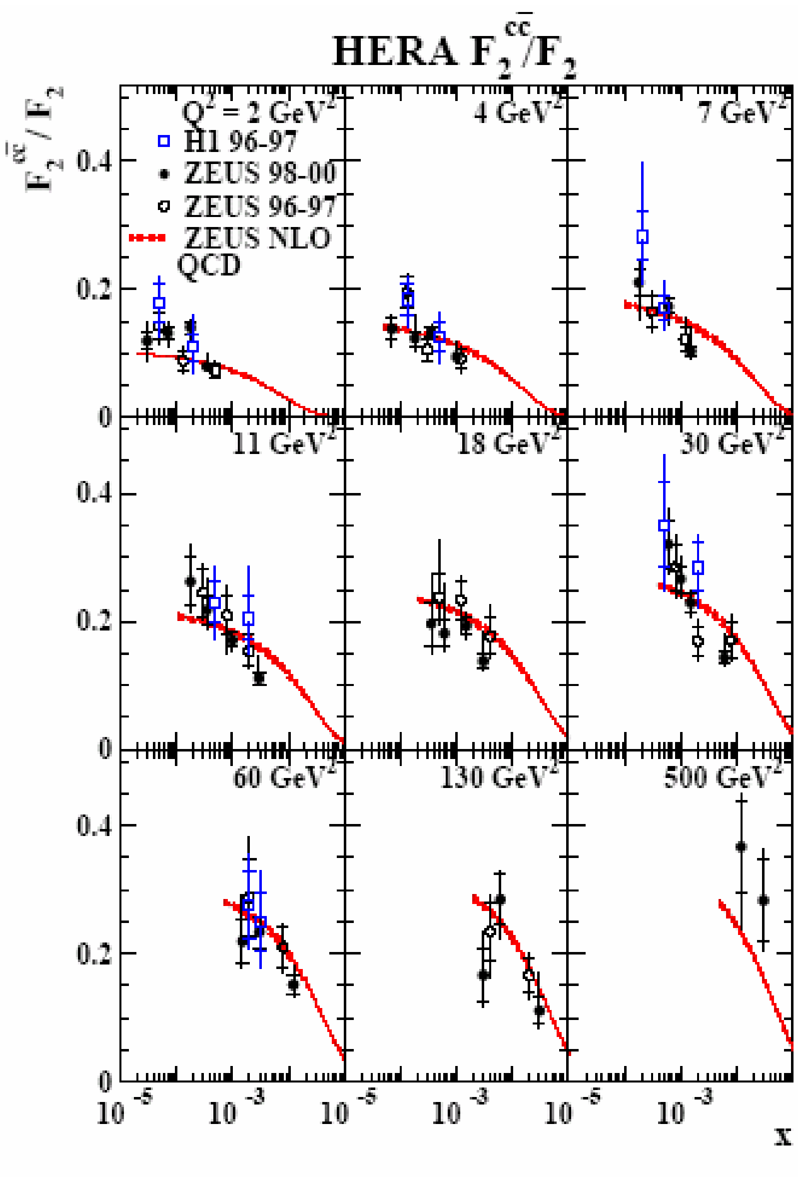
HERMES probably could measure $s(x,Q^2)$ in an independent way.

Flavor distributions: light quarks



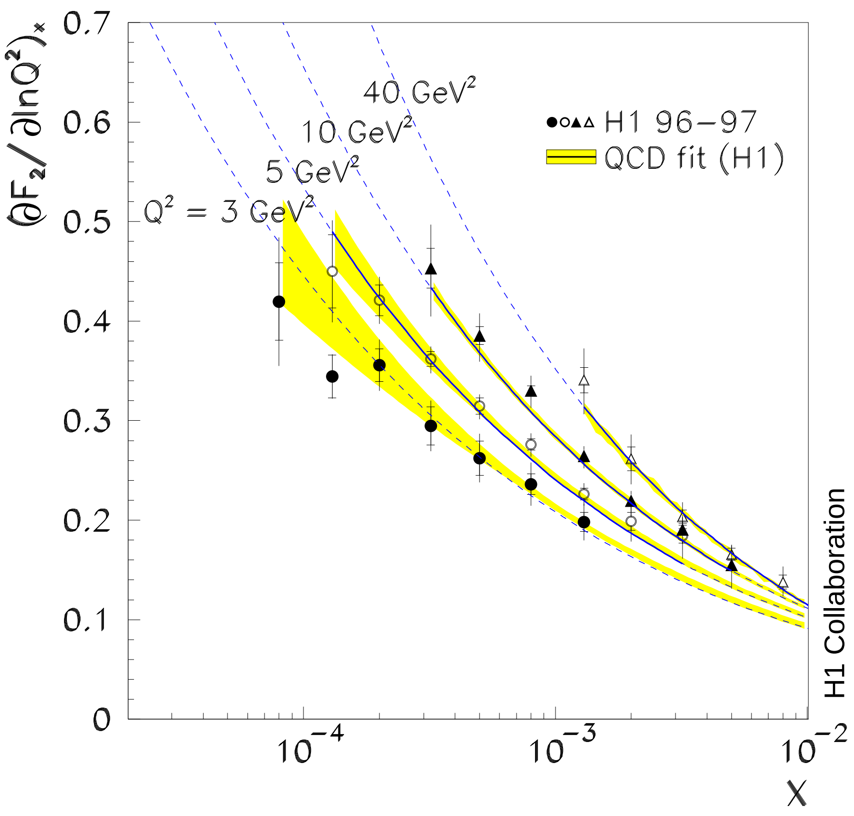
CTEQ 2007

Charm

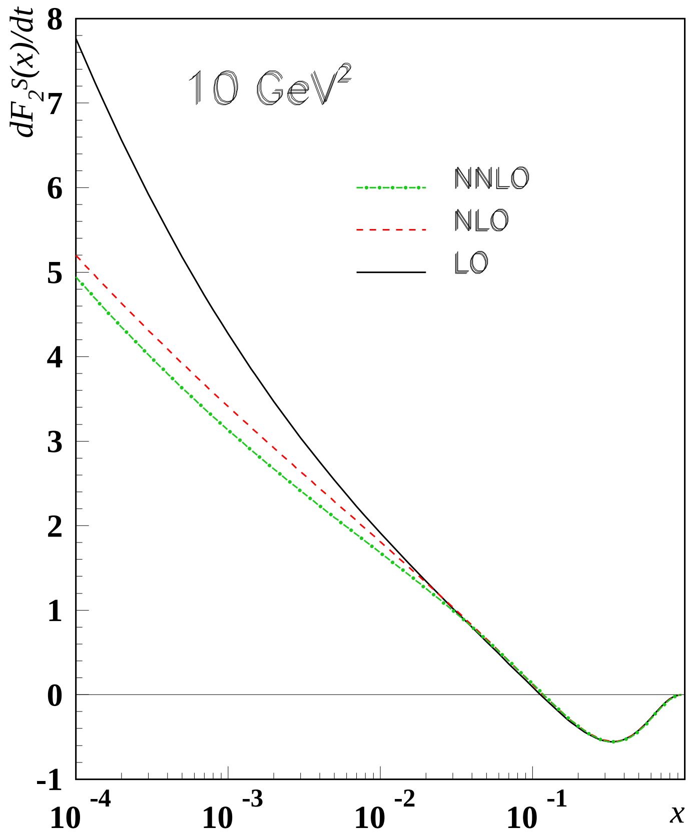


$F_2^{c\bar{c}}(x, Q^2)$ will be very well measured at HERA.

Slope of F_2 at low x



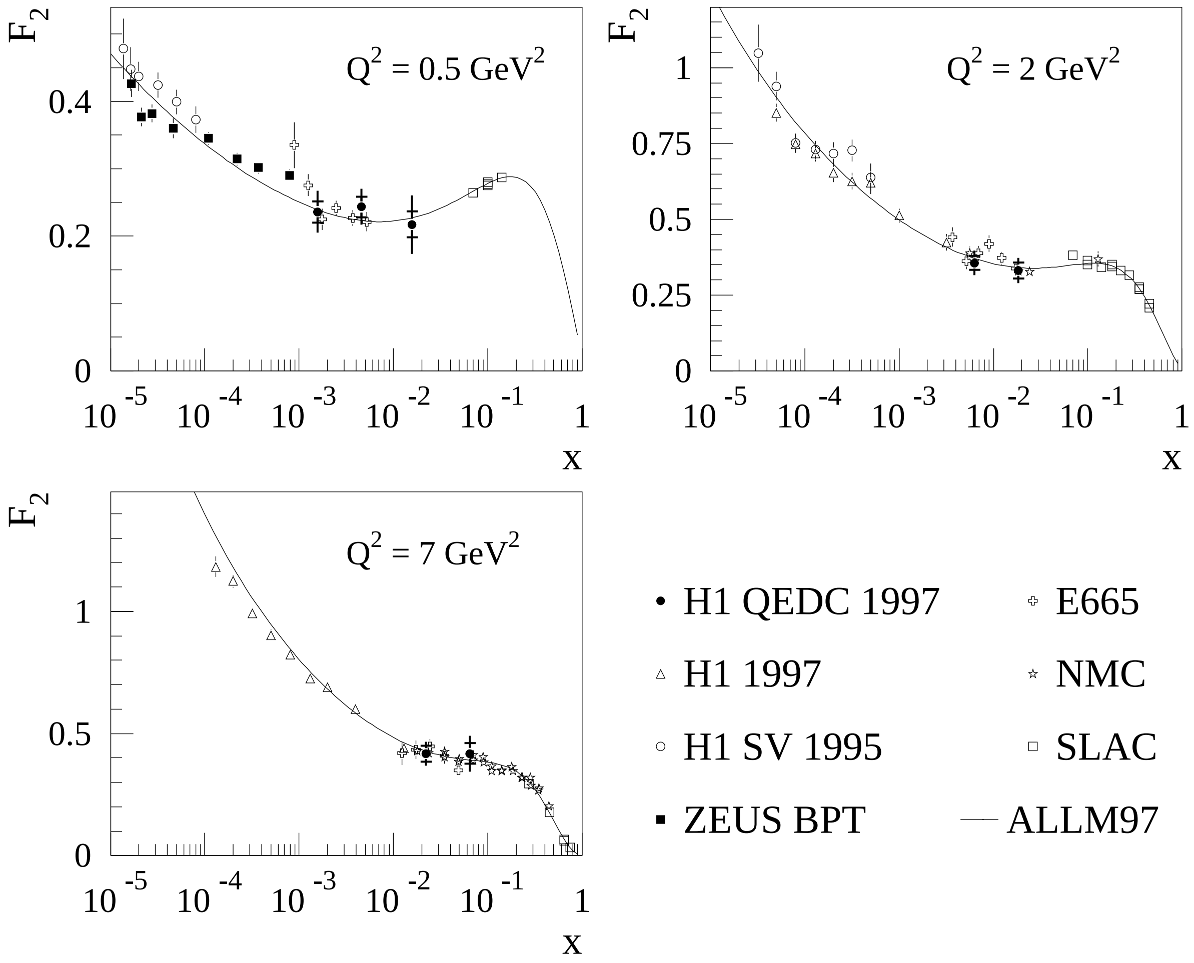
H1



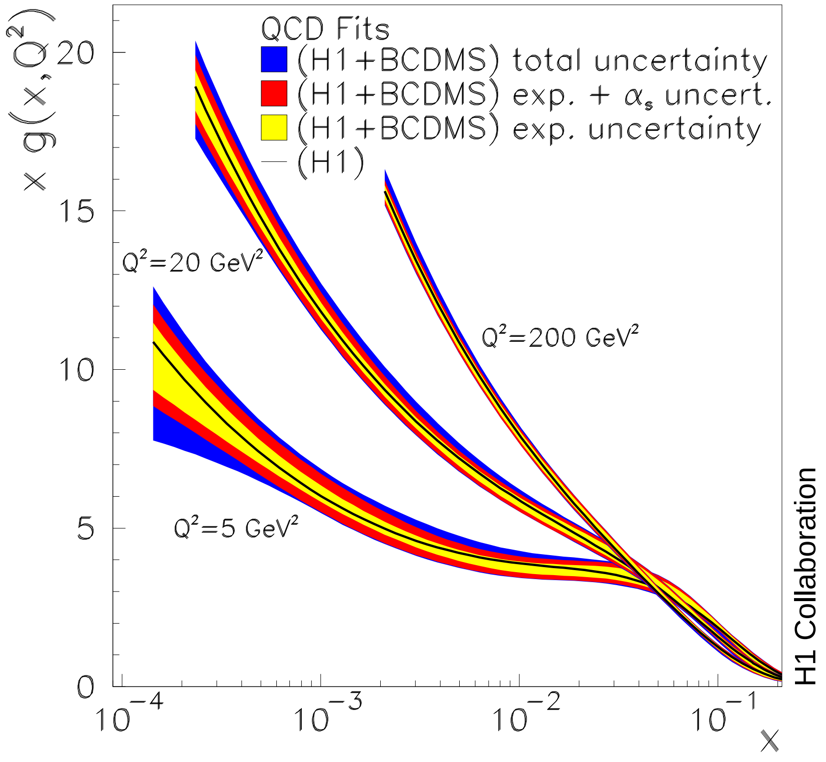
J.B., A. Guffanti 2005

Very likely, that the $\overline{\mathrm{MS}}$–gluon is remains positive!

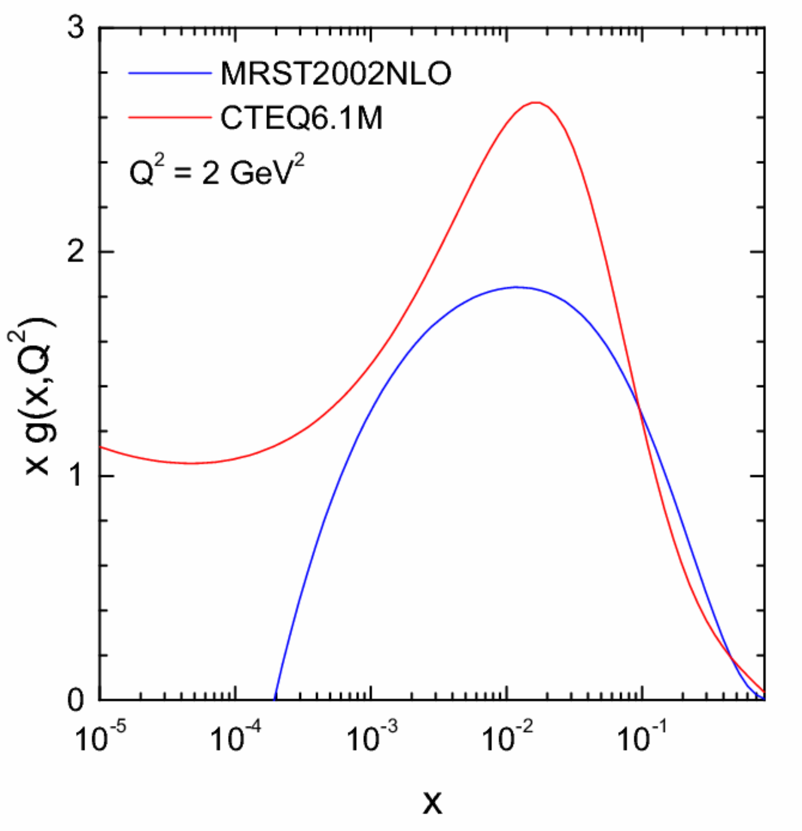
Perturbative or non-perturbative growth?



Gluon Density



H1

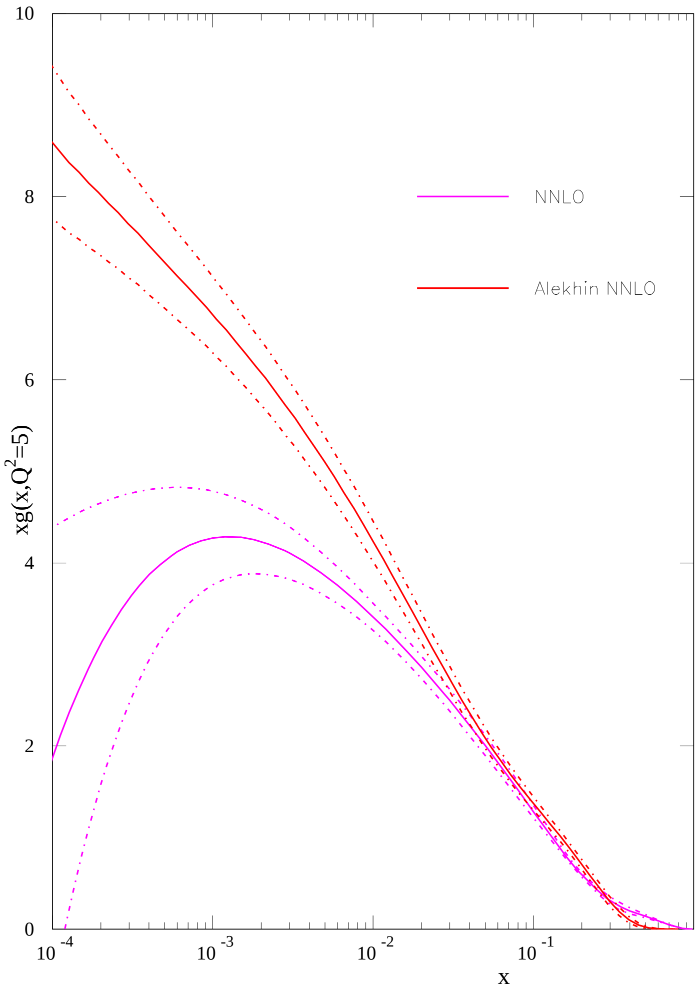


MRST 02 vs CTEQ 6

More work needed; MS– vs scheme-invariant evolution.

$F_L(x,Q^2)$ could be decisive.

Gluon Density



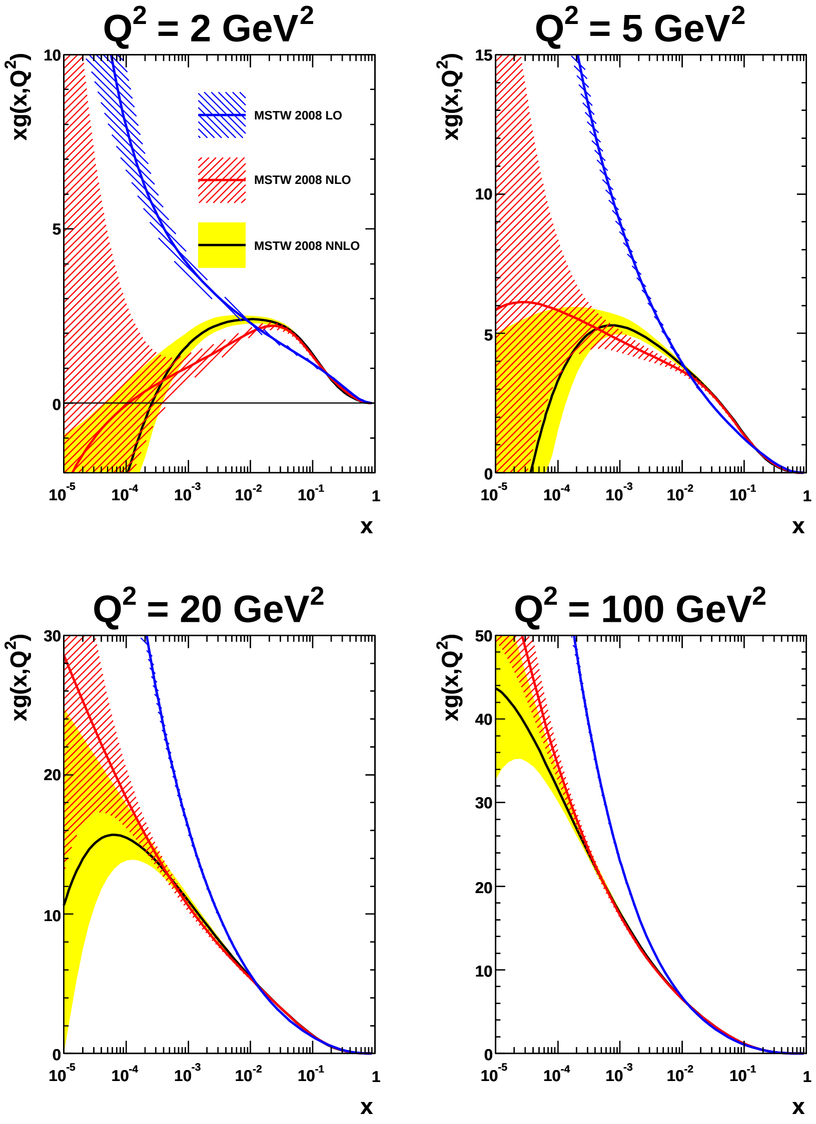
Not both distributions can be correct.

$F_L(x,Q^2)$ could be decisive.

MRST06 vs Alekhin: 2006

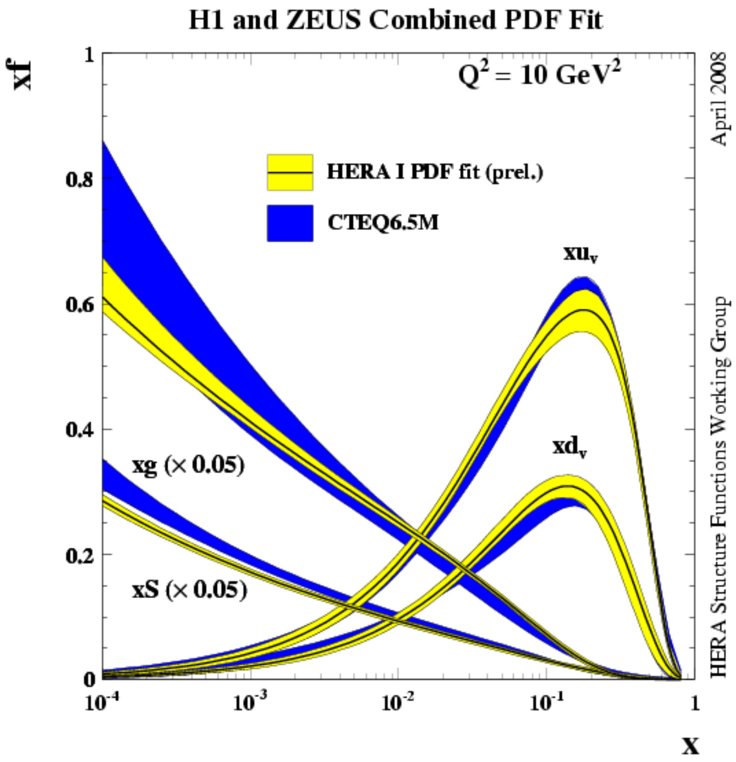
More work needed ! BB Analysis in progress.

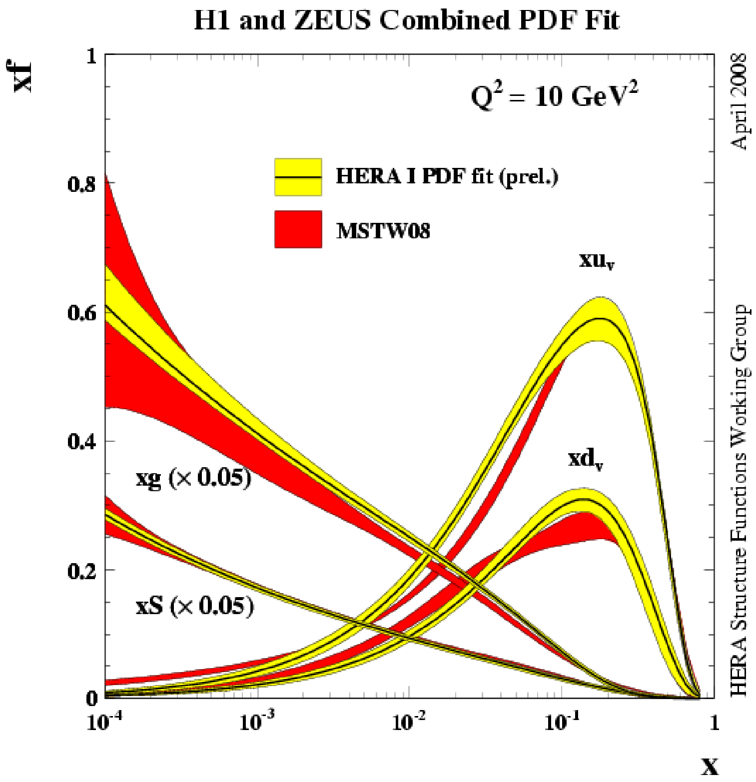
Gluon Density



MSTW 2008

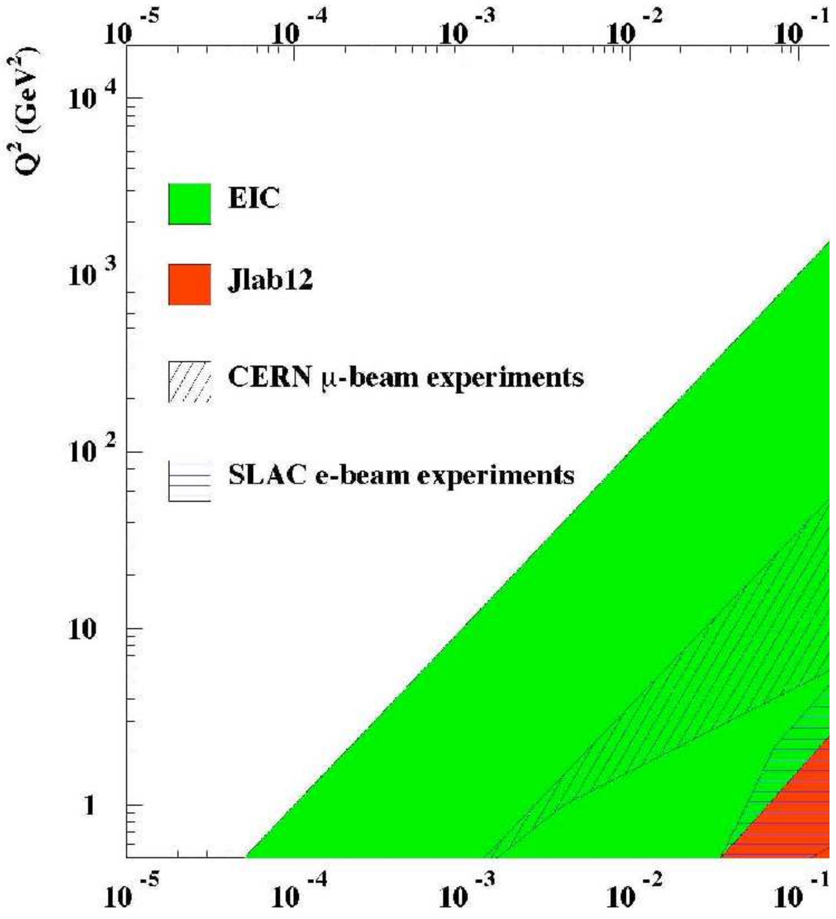
Gluon Density

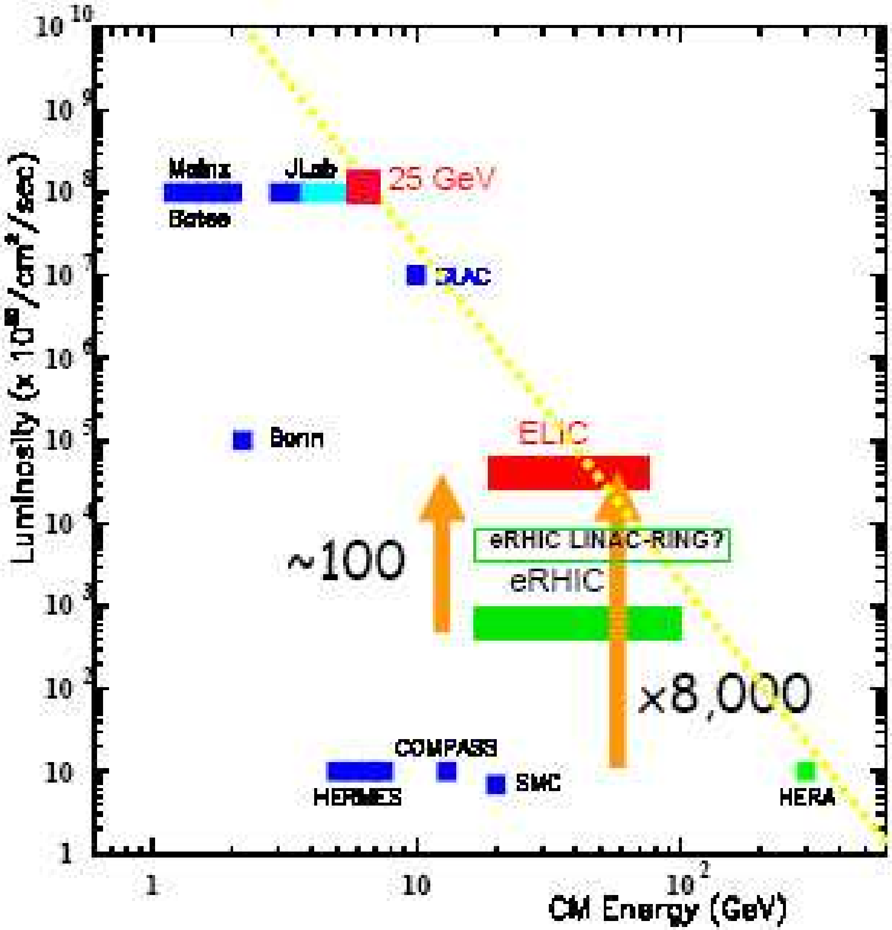




Recent Fits based on H1 + ZEUS combined data sets

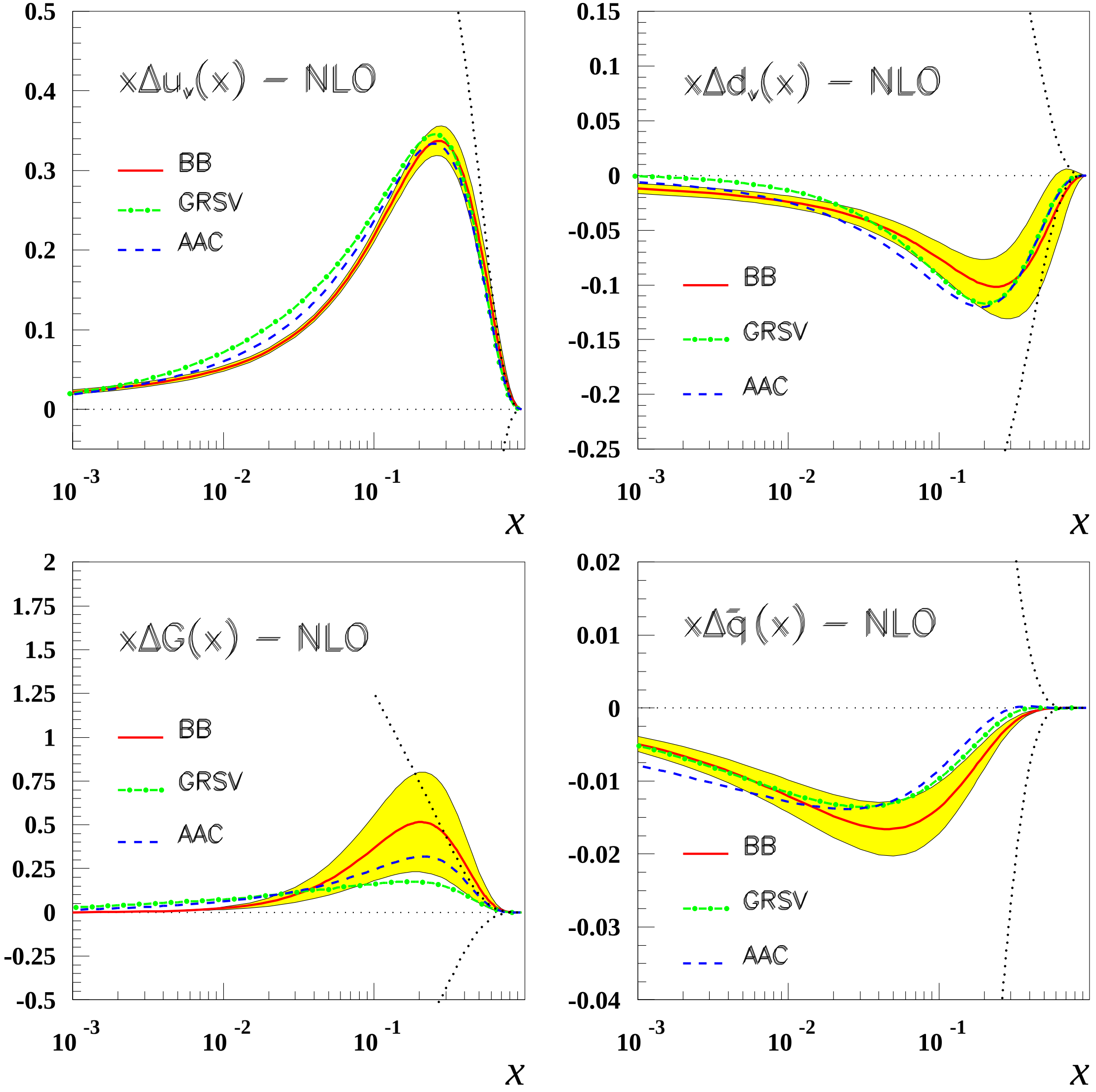
3. Polarized Structure Functions





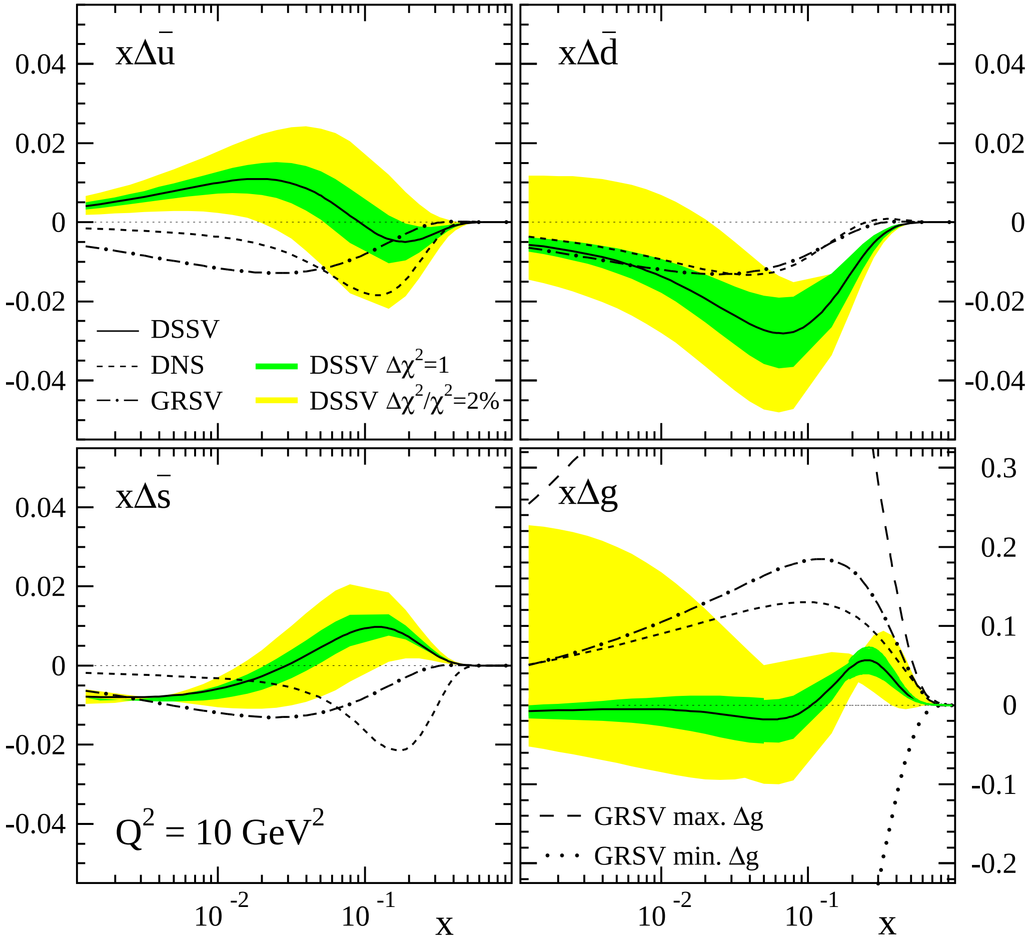
High Luminosity is most important: Various precision measurements.

Polarized Parton Densities at Present



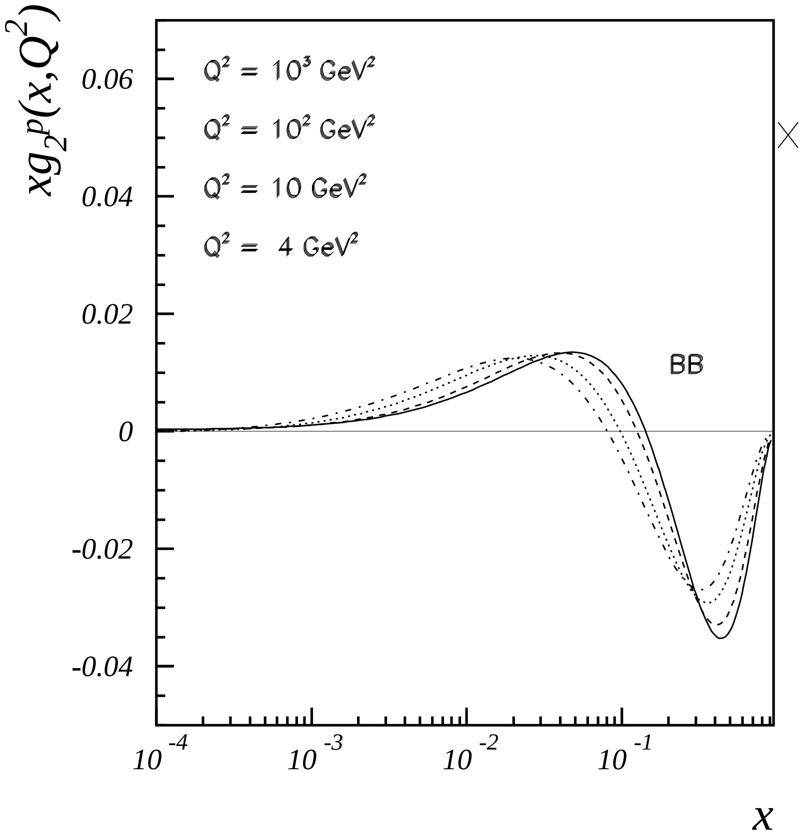
J.B., H. Böttcher (2002)

Unfolding the Sea Quarks

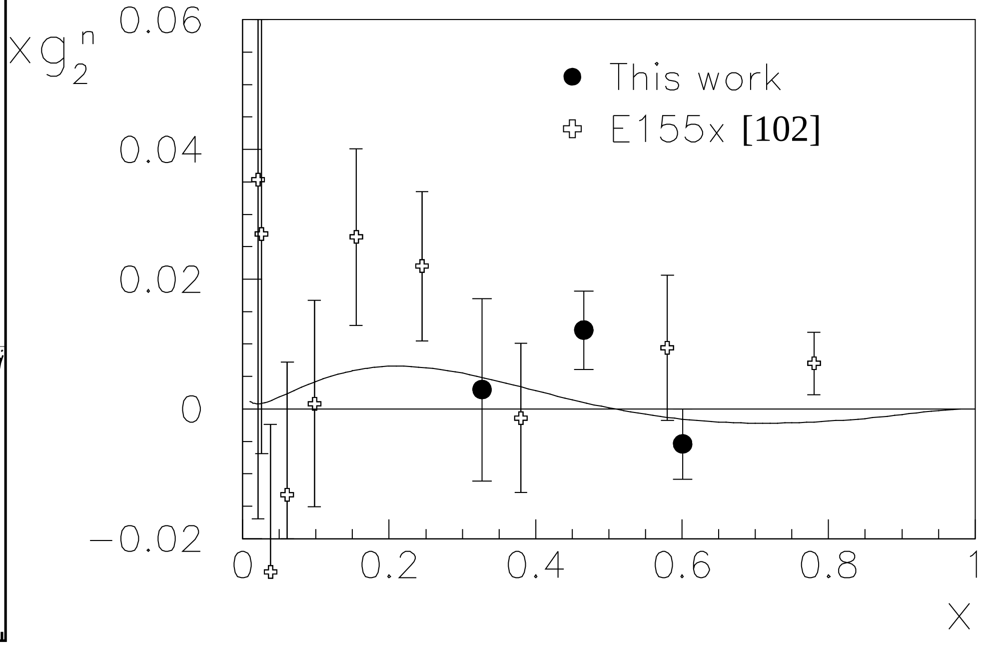


De Florian, Sassot, Stratmann, Vogelsang, 2008

$g_2(x,Q^2)$ - a Window to Higher Twist



$g_2^{\tau=2}(x,Q^2)$ (light partons)



JLAB Hall A, 2004

Accurate measurement highly desired.

How big is the $\tau = 3$ contribution ?

4. Moments of PDF's: PT + data

f	n	This Fit N^{3}LO	MRST04 NNLO	A02 NNLO
u_v	2	0.3006 ± 0.0031	0.285	0.304
	3	0.0877 ± 0.0012	0.082	0.087
	4	0.0335 ± 0.0006	0.032	0.033
d_v	2	0.1252 ± 0.0027	0.115	0.120
	3	0.0318 ± 0.0009	0.028	0.028
	4	0.0106 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1754 ± 0.0041	0.171	0.184
	3	0.0559 ± 0.0015	0.055	0.059
	4	0.0229 ± 0.0007	0.022	0.024

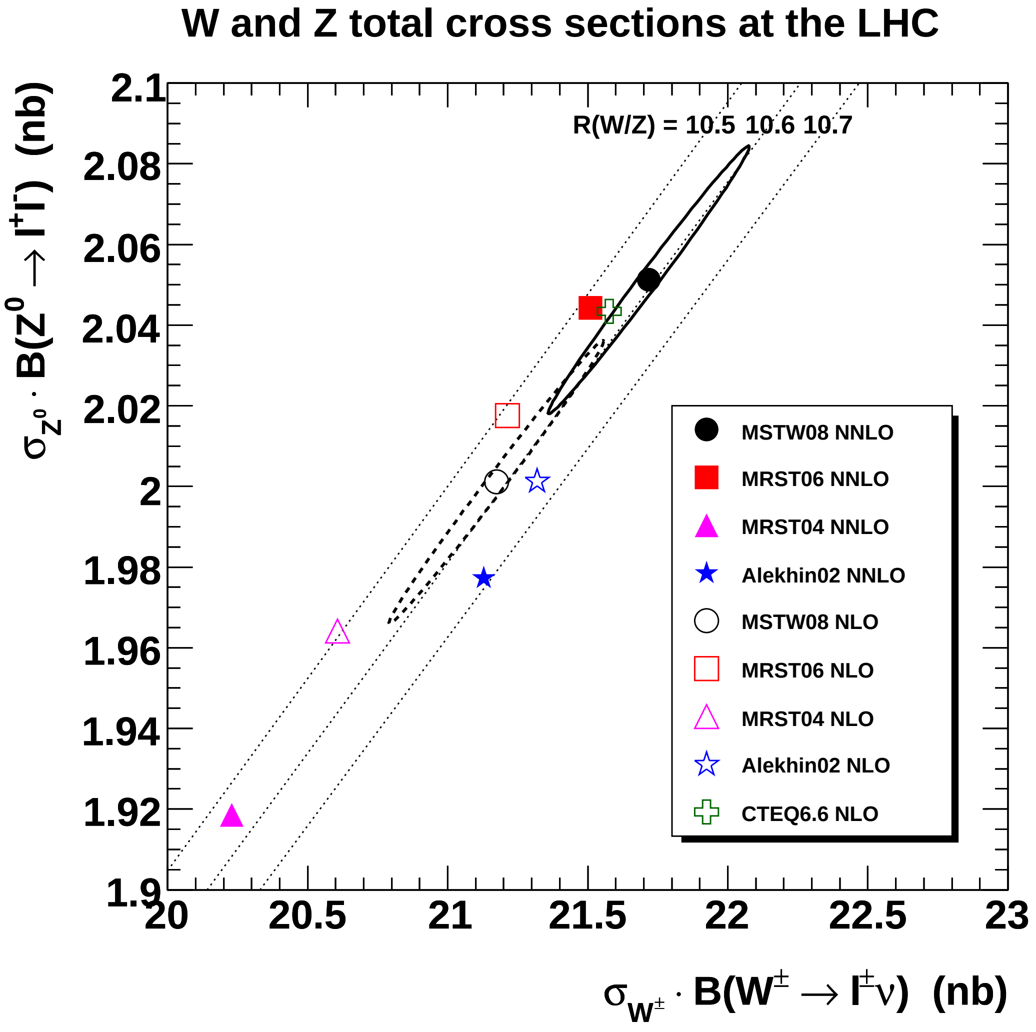
J.B., H. Böttcher, A. Guffanti, 2006

	Moment	BB, NLO
Δu_v	0	0.926
	1	0.163 ± 0.014
	2	0.055 ± 0.006
Δd_v	0	-0.341
	1	-0.047 ± 0.021
	2	-0.015 ± 0.009
$\Delta u_v - \Delta d_v$	0	1.267
	1	0.210 ± 0.025
	2	0.070 ± 0.011

J.B., H. Böttcher, 2002

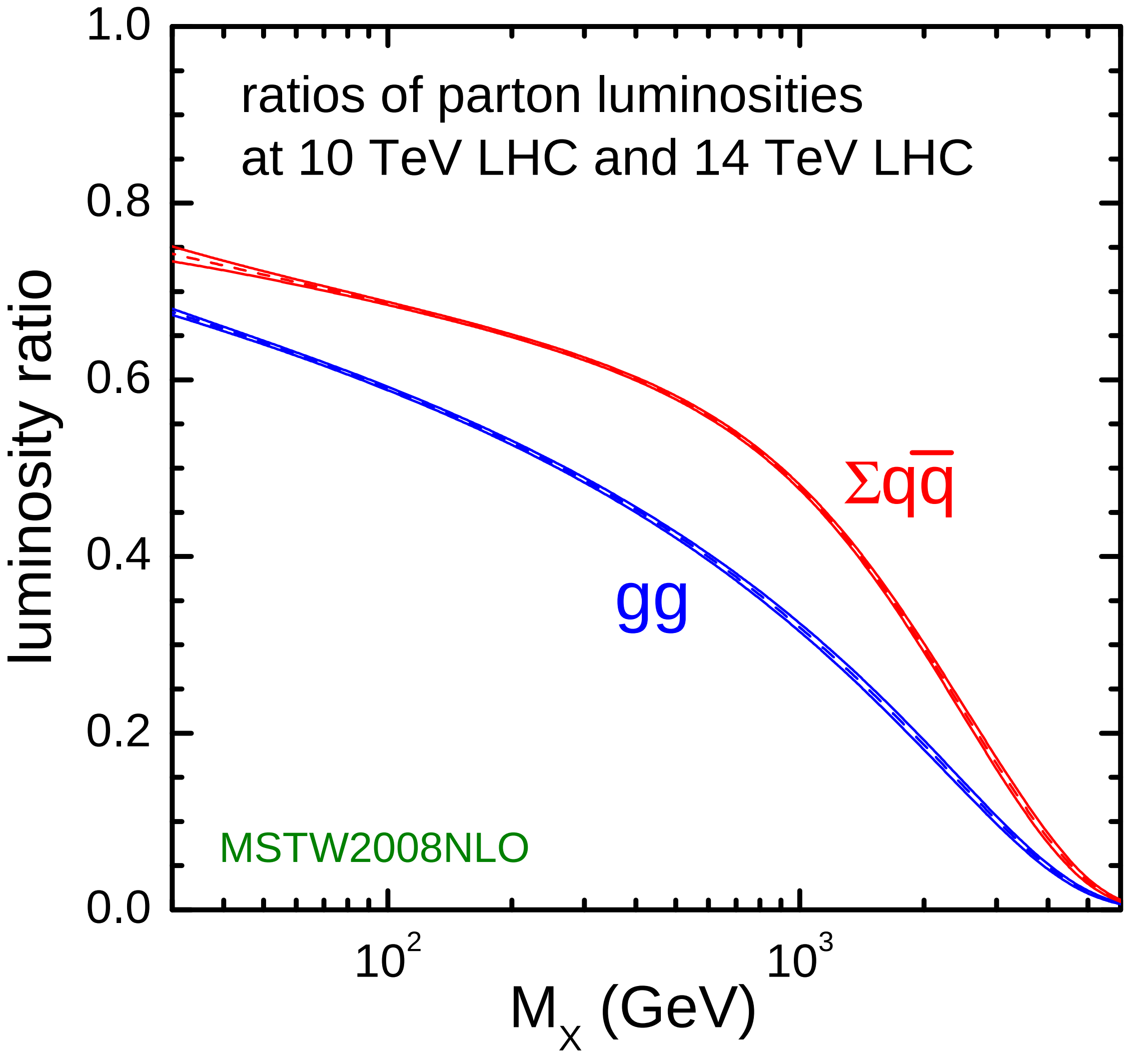
Lattice Results : developping; different fermion-types studied.
Low values of m_π crucial; values approach 270 MeV now.

Light Candle Processes at LHC



MSTW08

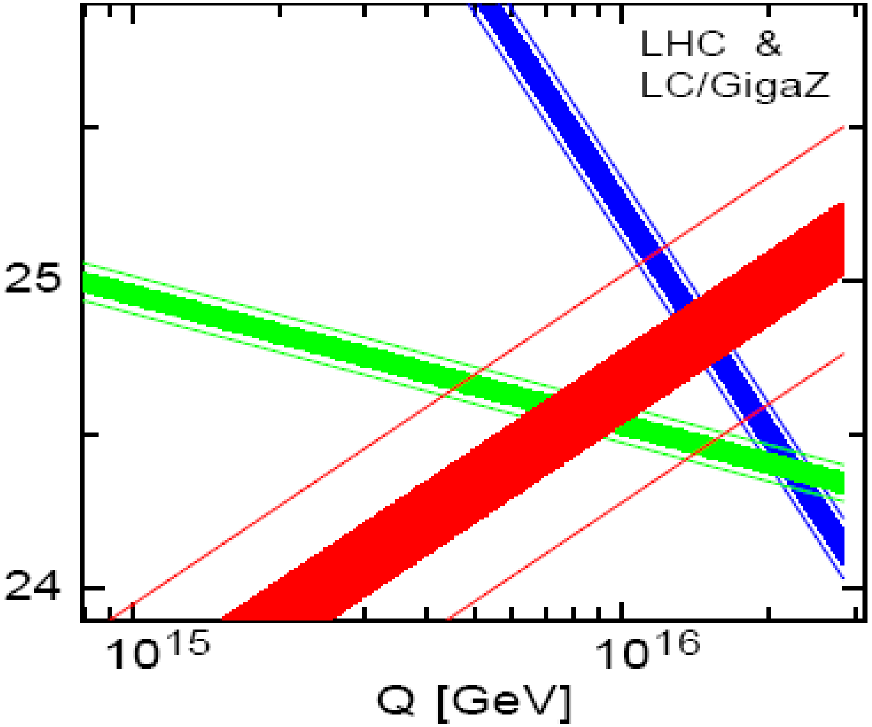
Parton Luminosities at LHC



MSTW08

5. Λ_{QCD} and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{\rm em}(0)}{\alpha_{\rm em}(0)} \sim 3\cdot 10^{-11} \qquad \frac{\delta\alpha_{\rm weak}}{\alpha_{\rm weak}} \sim 7\cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2\cdot 10^{-2}$$



P. Zerwas, 2004

Overview of the Analyses

- Various NLO analyses; ⇒ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N^{3}LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
GRS	0.112			[10]
BBG	0.1148	± 0.0019		[9]
BB (pol)	0.113	± 0.004	$^{+0.009}_{-0.006}$	[7]

NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]

N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

NNLO and N^{3}LO

BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

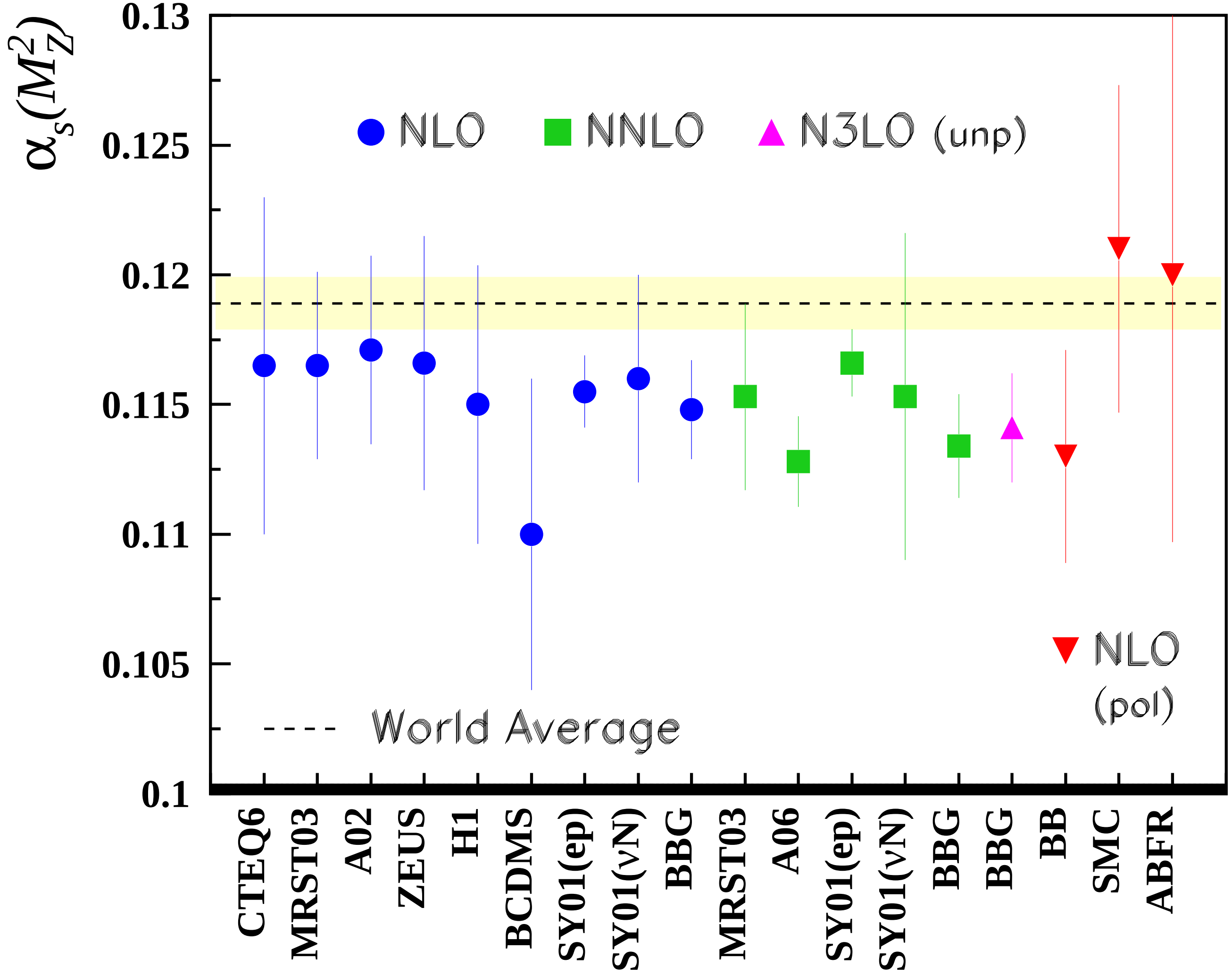
Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger Values, to be discussed.

$\alpha_s(M_Z^2)$

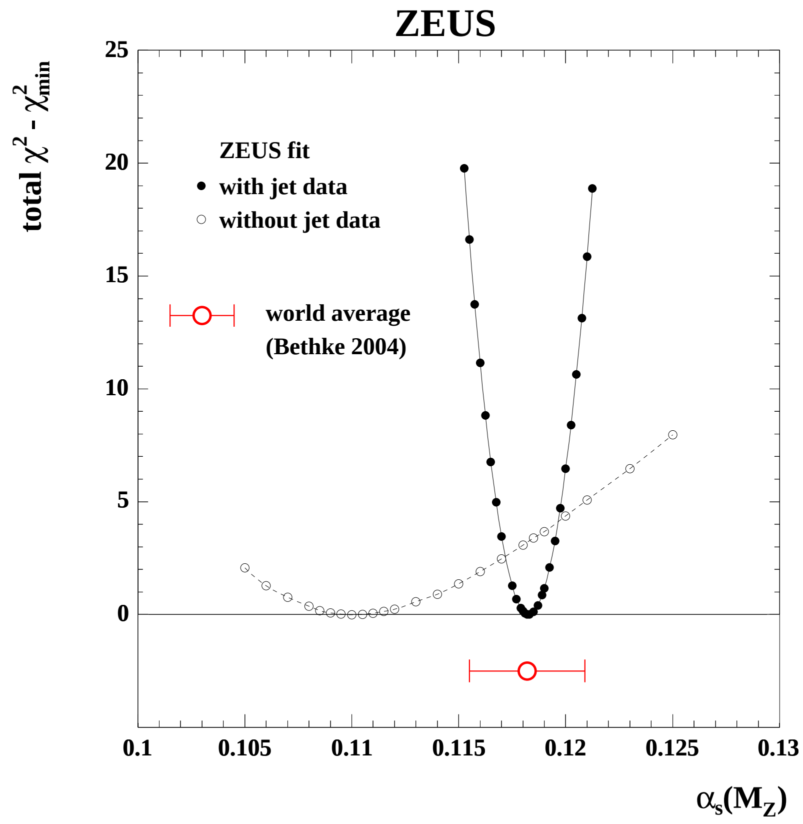


J.B., H. Böttcher, A. Guffanti, 2006

More Global Analyses

- $\alpha_s(M_Z^2)$ for different data sets included are too different !

⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet



M. Cooper-Sarkar, 2005

6. What would we like to know?

HERA:

- Analyze complete collected luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized. DIS PDF's $\Longleftrightarrow$ Collider PDF's

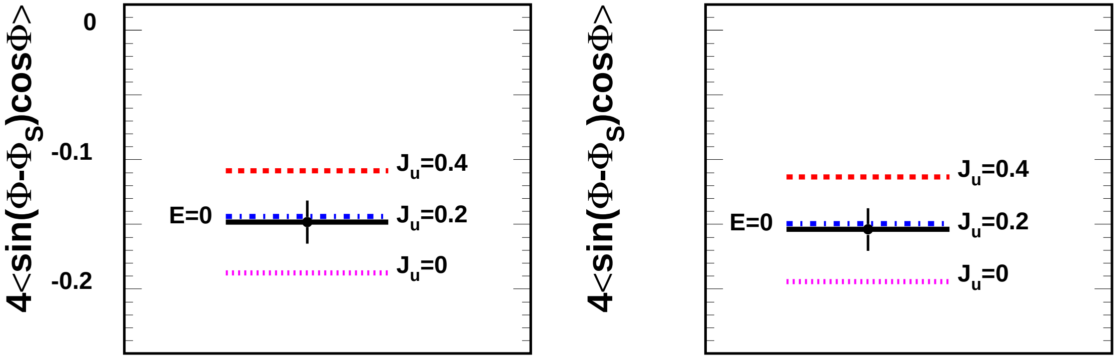
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

L_q from DVCS

- HERA and JLAB : Improve DVCS data

Theory widely developed, cf. rev. Belitsky & Radyushkin, 2005



Expected DVCS asymmetry $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ with $b_v = 1$, $b_s = \infty$, $J_u = 0.4(0.2, 0.0)$, $J_d = 0.0$ in the Regge (left panel) and factorized (right panel) ansatz, at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPD E. The projected statistical error for 8M DIS events is shown. The systematic error is expected to not exceed the statistical one.

F. Ellinghaus et al. 2005

The measurement of L_q off data is model–dependent at the moment.
Lattice calculations at low pion masses are needed to complete the picture

Graph Resummation and Saturation

Further study of proposed mechanisms needed: RHIC, LHC for nucleus-nucleus collisions.

ep scattering: partly different mechanisms

more studies would be welcome; link to higher twist contributions in gluon–dynamics

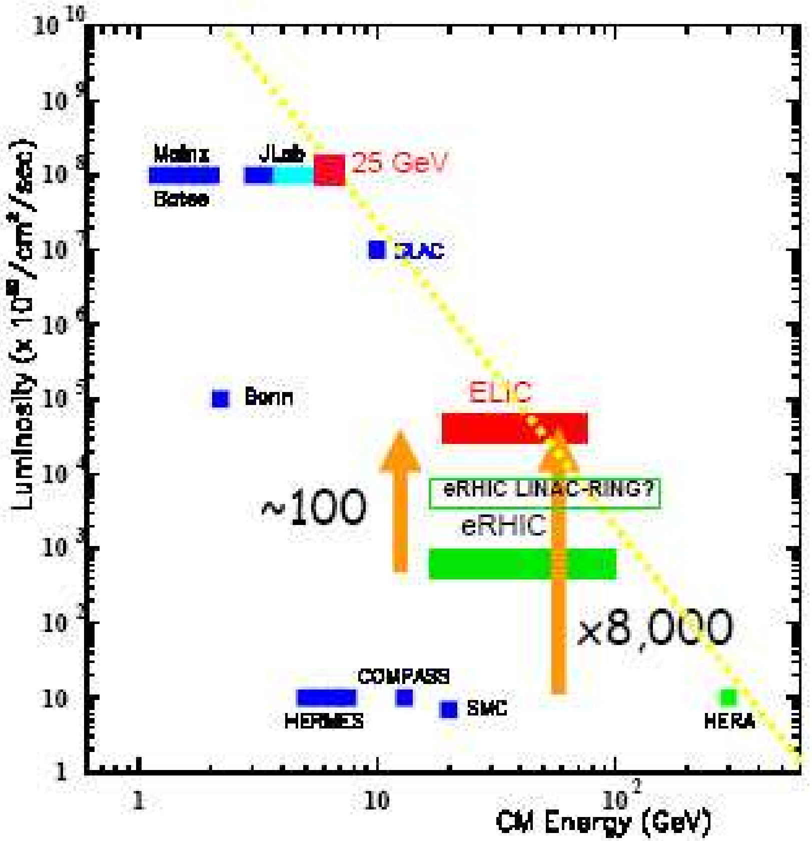
How do the non-perturbative and perturbative parts factorize ?

Conservation laws and interplay between the small x and medium x range behaviour

New DIS Machines

Where to go ?

- High energies : small x, large Q^2 desirable.
- High luminosities : ELIC/EIC: $\sqrt{s}$ between CERN and HERA energies



R. Ent,2004
high precision physics
polarized and unpolarized

Would be an important extension of the present programmes in many respects.

Enhancing Precision Further...

- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\rm MS}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor. (Theory & Experiment)
- Flavor Structure of Sea-Quarks: More studies needed.(All Experiments)
- Revisit polarized data upon completion of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed. (Theory)
- QCD at Twist 3: $g_2(x,Q^2)$, semi–exclusive Reactions, Transversity, diffraction in polarized scattering (HERMES, High Precision polarized experiments, JLAB, ELIC)
- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Angular Momentum.