

# Status of Deep-Inelastic Scattering and PDFs for the LHC

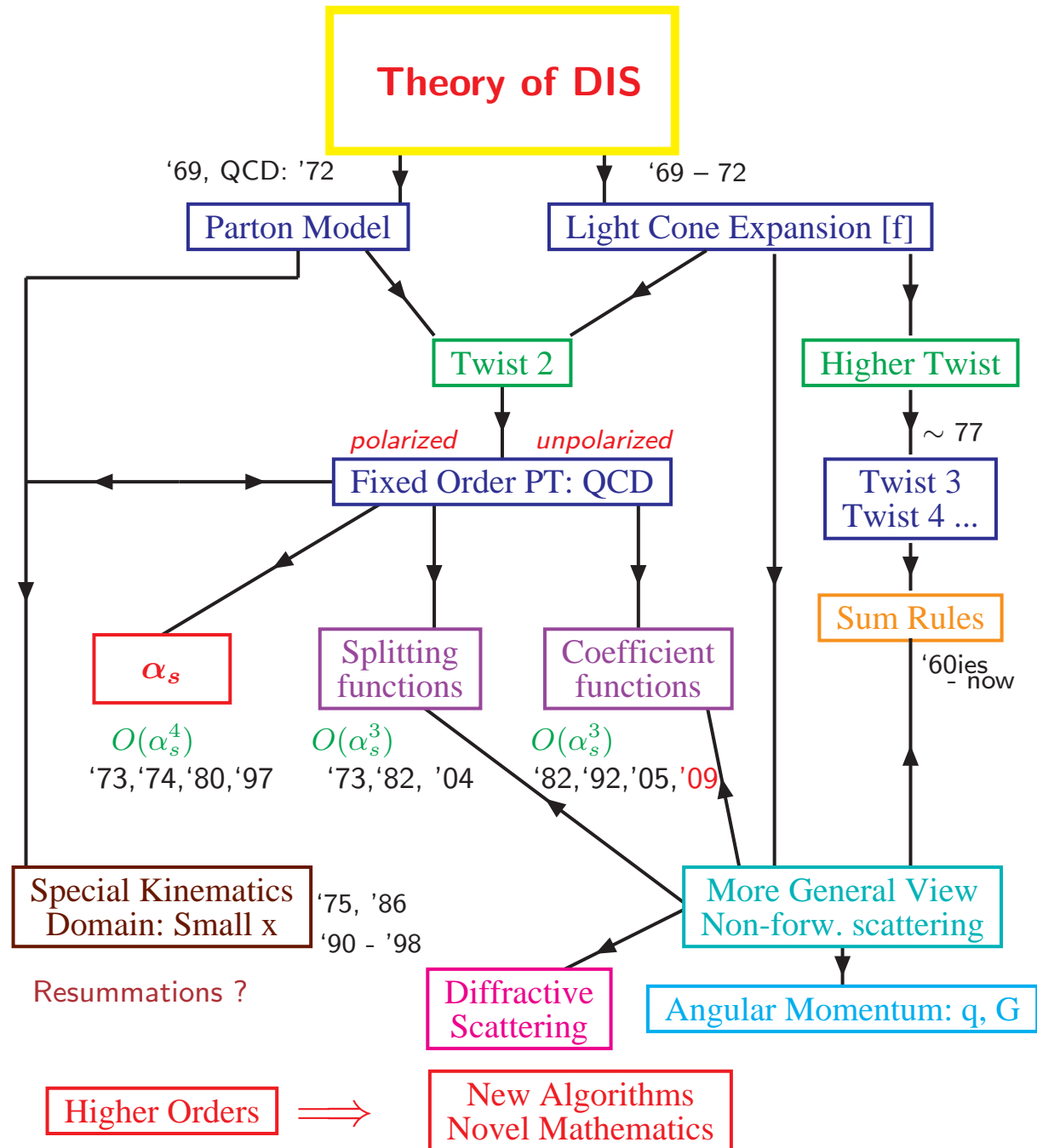
Johannes Blümlein  
DESY











- The Major Goals
- DIS Theory Status
- Unpolarized Parton Distribution Functions
- Polarized Parton Distribution Functions
- $\Lambda_{\text{QCD}}$  and  $\alpha_s(M_Z^2)$
- PDFs and Inclusive Cross Sections at LHC
- Advanced Technologies for Feynman Diagrams @ 3 Loops
- Outlook

# 1. The Major Goals

- Precision Measurement of the Strong Coupling Constant  $\alpha_s(M_Z^2)$
- Precision Measurement of the Unpolarized Parton Densities
- Precision Measurement of the Polarized Parton Densities
- Who Carries the Spin of the Proton?
- Higher Twist Effects
- Is there Saturation in DIS at small  $x$  ?  $\implies$  answered by experiment.



# Status of Highest Order Calculations

- Running  $\alpha_s$ :  $O(\alpha_s^4)$  Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients:  $O(\alpha_s^3)$   
Moch, Vermaseren, Vogt 2004/05 
- Unpol. NS anomalous dimension 2nd Moment:  $O(\alpha_s^4)$  Baikov, Chetyrkin 2006
- Pol. anomalous dimension:  $O(\alpha_s^2)$ ; Mertig, van Neerven, 1995; Vogelsang 1995;  
 $\Delta P^{qq} \Delta P_{qG}$ :  $O(\alpha_s^3)$  Moch, Rogal, Vermaseren, Vogt 2008 
- Pol. Wilson coefficients:  $O(\alpha_s^2)$ ;  $\Delta C_{NS}^{qq}, \Delta C_{qG}$ : van Neerven, Zijlstra 1994
- Transversity:  $O(\alpha_s^2)$ , some moments anom. dim.:  $O(\alpha_s^3)$ , Hayashigaki, Kanazawa, Koike;  
Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Klein, B. Tödtli 2008 
- Unpol. Heavy Flavor Wilson Coefficients:  $O(\alpha_s^2)$  Laenen, van Neerven, Riemersma, Smith, 1993  
Fast Mellin Space code: Blümlein & Alekhin, 2003 
- Pol. Heavy Flavor Wilson Coefficients:  $O(\alpha_s^1)$  Watson 1982
- $Q^2 \gg m^2$  Unpol. Heavy Flavor Wilson Coefficient  $F_L$ :  $O(\alpha_s^3)$   
Blümlein, De Freitas, van Neerven, S. Klein 2005 
- $Q^2 \gg m^2$  Pol. Heavy Flavor Wilson Coefficient :  $O(\alpha_s^2)$  van Neerven, Smith et al. 1996,  
Bierenbaum, Blümlein & Klein 2007 
- $Q^2 \gg m^2$  Unpol. Heavy Flavor Wilson Coefficient  $F_2$ :  $O(\alpha_s^2 \varepsilon)$ : all operators  
(also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008;   $O(\alpha_s^3)$ : Moments 2–10(12,14)  
of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008 

 = done at DESY (or in DESY collab.).



# DIS Structure Functions @ Twist 2

$$F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma_j^i \left( \alpha_s, \frac{Q^2}{\mu^2}, x \right)$$

↑ bare pdf    ↑ sub – system cross – sect.

$$= \underbrace{\hat{f}_i(x, \mu^2) \otimes \Gamma_k^i \left( \alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k \left( \alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right)}_{\text{finite Wilson coefficient}}$$

**Move to Mellin space :**

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions  $\otimes$  into ordinary products.

# Evolution Equations

$$\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_\psi(g) \right] F_i(N) = 0$$

$$\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + \gamma_\kappa^N(g) - 2\gamma_\psi(g) \right] f_k(N) = 0$$

$$\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - \gamma_\kappa^N(g) \right] C_j^k(N) = 0$$

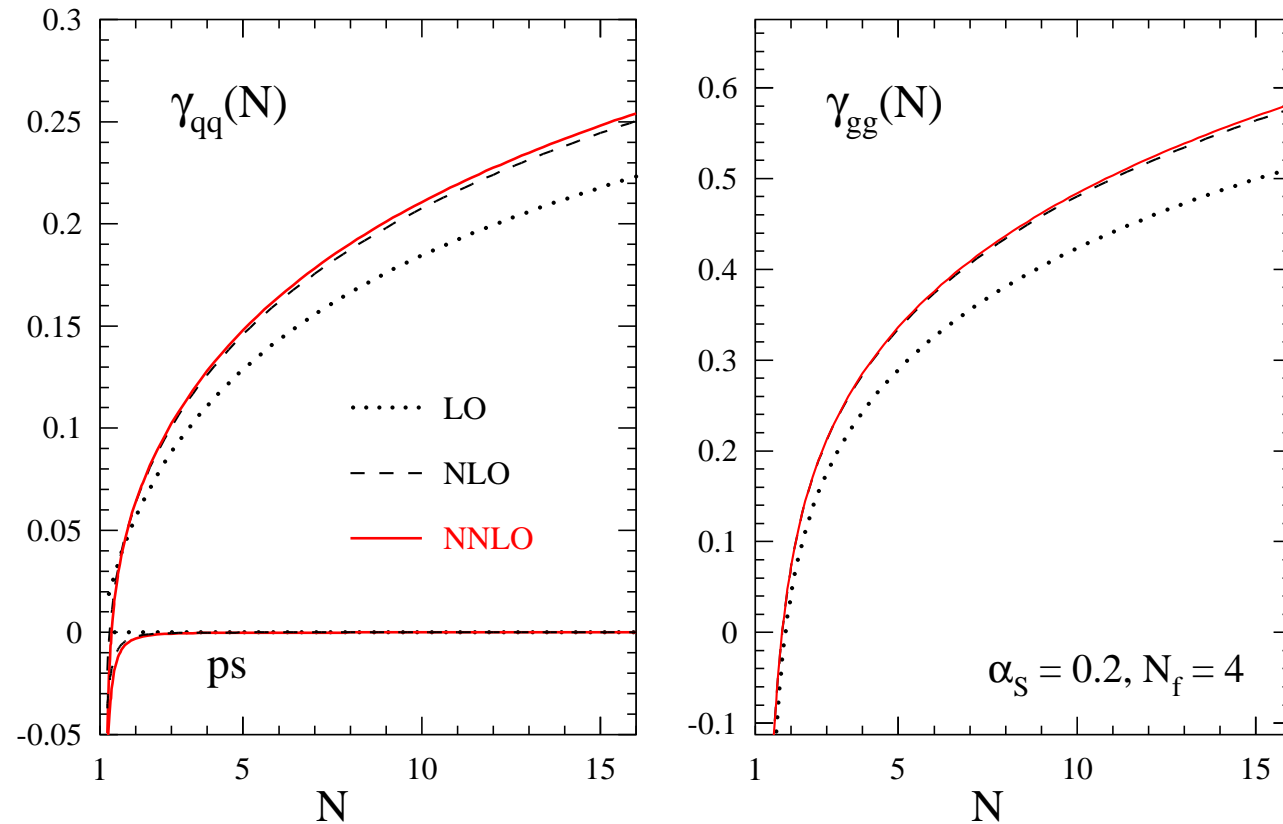
CALLAN–SYMANZIK equations for mass factorization   $\equiv$   
 ALTARELLI–PARISI evolution equations

**x-space :**

$$\frac{d}{d \log(\mu^2)} \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \mathbf{P}(x, \alpha_s) \otimes \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix}$$

$$\mathbf{P}(x, \alpha_s) = \mathbf{P}^{(0)}(x) + \frac{\alpha_s}{2\pi} \mathbf{P}^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 \mathbf{P}^{(2)}(x) + \dots$$

# Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004 

# The Basic Functions of massless QCD to $w=5:\equiv 3$ Loops

Representative :  $S_1(N) = \psi(N + 1) + \gamma_E$  and its derivatives.

Weight  $w=3$  : 
$$F_1(N) = \mathbf{M} \left[ \frac{\ln(1+x)}{1+x} \right] (N)$$

$$F_2(N) = \mathbf{M} \left[ \frac{\text{Li}_2(x)}{1+x} \right] (N), \quad F_3(N) = \mathbf{M} \left[ \left( \frac{\text{Li}_2(x)}{1-x} \right)_+ \right] (N)$$

Yndurain et al., 1981:  $F_2(N)$

Weight  $w=4$  :

$$F_4(N) = \mathbf{M} \left[ \frac{S_{1,2}(x)}{1+x} \right] (N), \quad F_5(N) := \mathbf{M} \left[ \left( \frac{S_{1,2}(x)}{1-x} \right)_+ \right] (N)$$

$F_3(N) - F_5(N)$ : J.B., 2003; J.B., V. Ravindran ,2004



Weight w=5 :

$$F_{6,7}(N) = \mathbf{M} \left[ \left( \frac{\text{Li}_4(x)}{1 \pm x} \right)_{(+)} \right] (N), \quad F_8(N) = \mathbf{M} \left[ \frac{S_{1,3}(x)}{1+x} \right] (N),$$

$$F_{9,10}(N) = \mathbf{M} \left[ \left( \frac{S_{2,2}(x)}{1 \pm x} \right)_{(+)} \right] (N), \quad F_{11}(N) = \mathbf{M} \left[ \frac{\text{Li}_2^2(x)}{1+x} \right] (N),$$

$$F_{12,13}(N) := \mathbf{M} \left[ \left( \frac{\ln(x)S_{1,2}(-x) - \text{Li}_2^2(-x)/2}{1 \pm x} \right)_{(+)} \right] (N)$$


$F_6(N) - F_{13}(N)$  : J.B., S. Moch, 2004.

**Massless QCD to 3 Loops depends on 14 Functions.**

Weight w=6 :

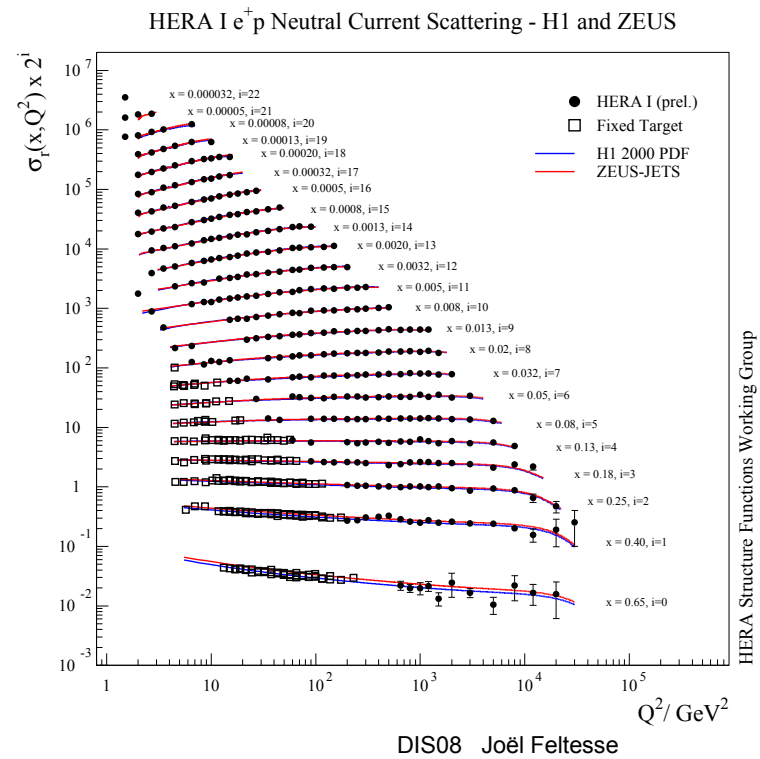
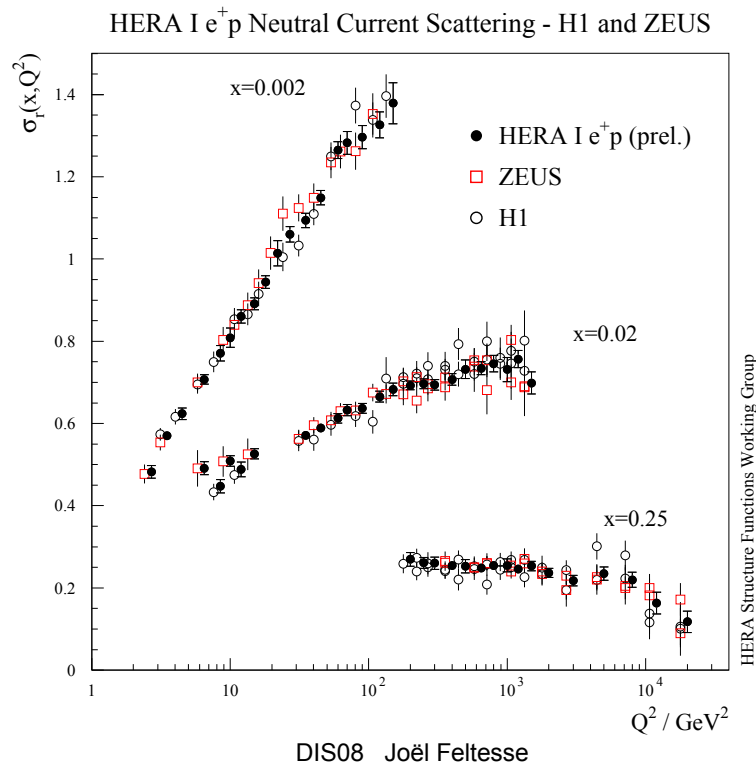
⇒ Representation for 3 Loop Wilson Coeff.: 35 Functions, J.B., 2009. 

# Complex Analysis of these Functions

- Construct exact analytic continuations to **complex  $N$**
- The functions are meromorphic  
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion  $z + 1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an **analytic expression** for the Structure functions in Mellin Space at all  $Q^2$
- Include the **heavy flavor** Wilson coefficients in Mellin Space  
 $\Rightarrow$  nearly accomplished to  $O(a_s^3)$  I. Bierenbaum, JB, S. Klein (2009) 
- Perform a **single** fast, numerical Mellin inversion  
(at high precision)

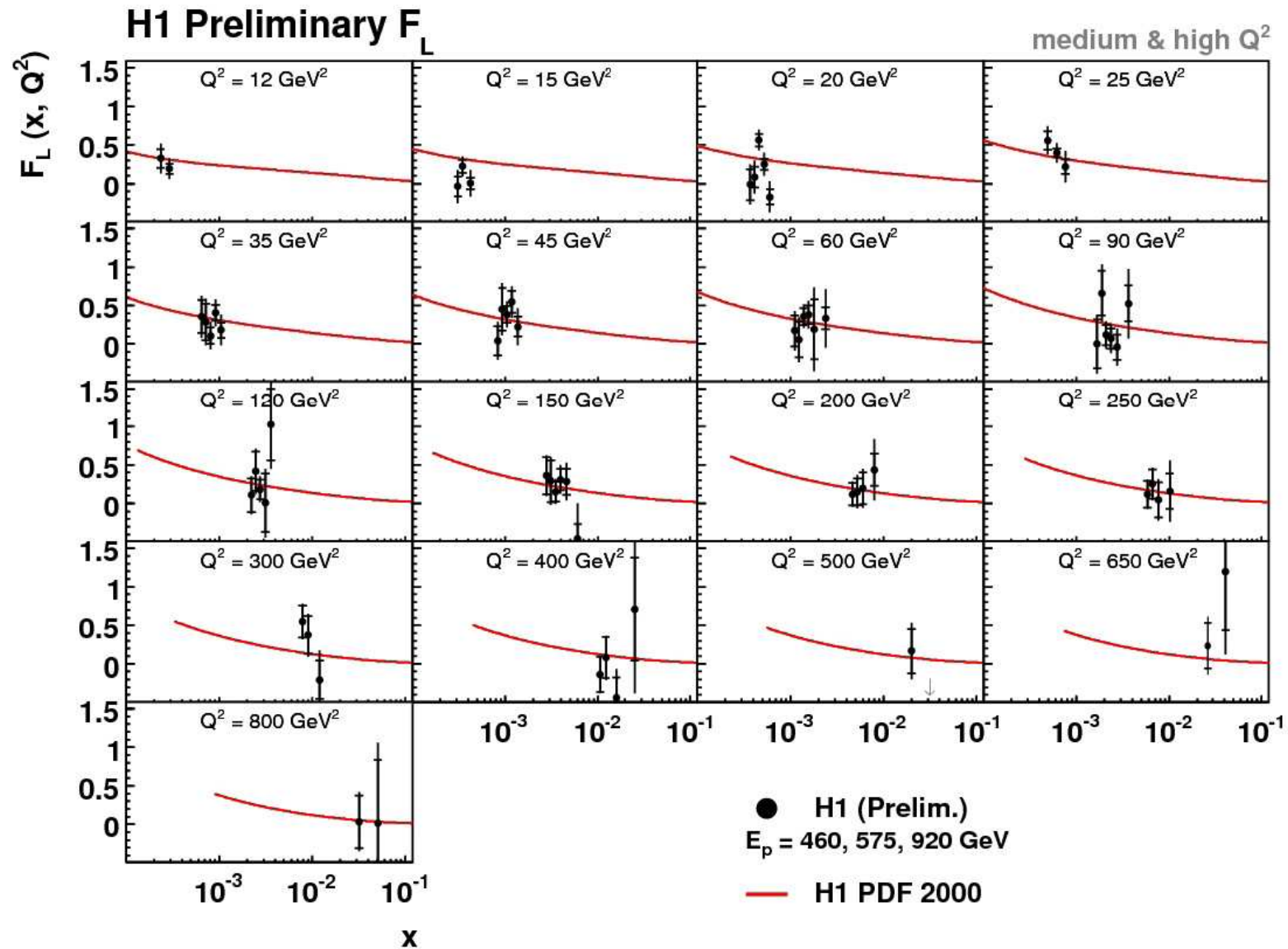
$\Rightarrow$  **Fastest and most Precise Way of Analysis**

# 3. Unpolarized Parton Distribution Functions

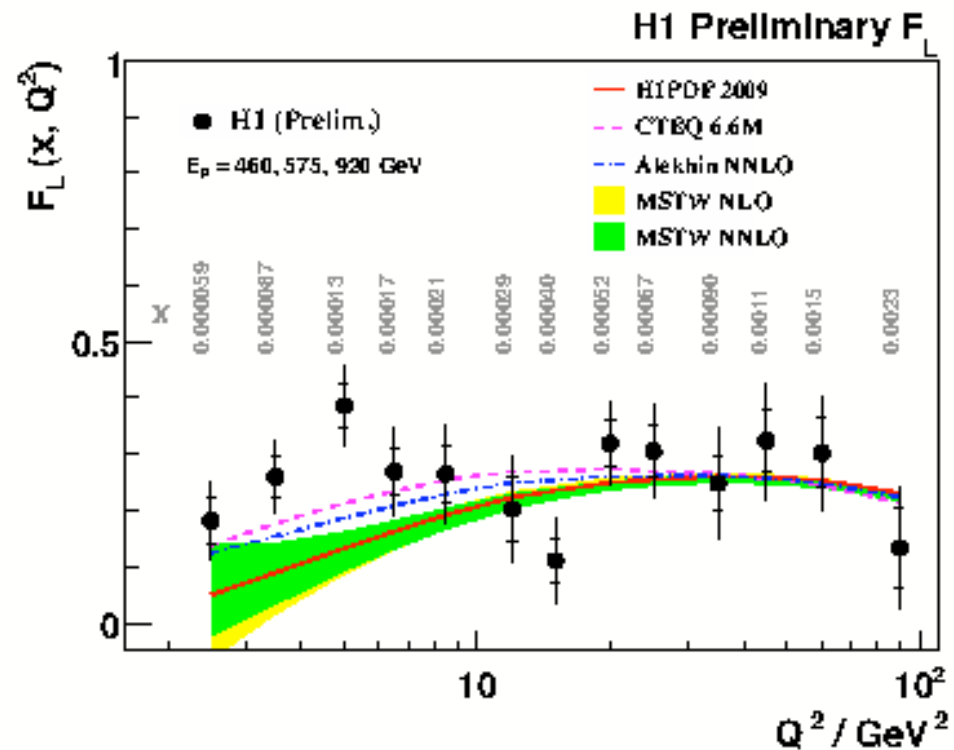


New ZEUS + H1 averaged  $F_2(x, Q^2)$

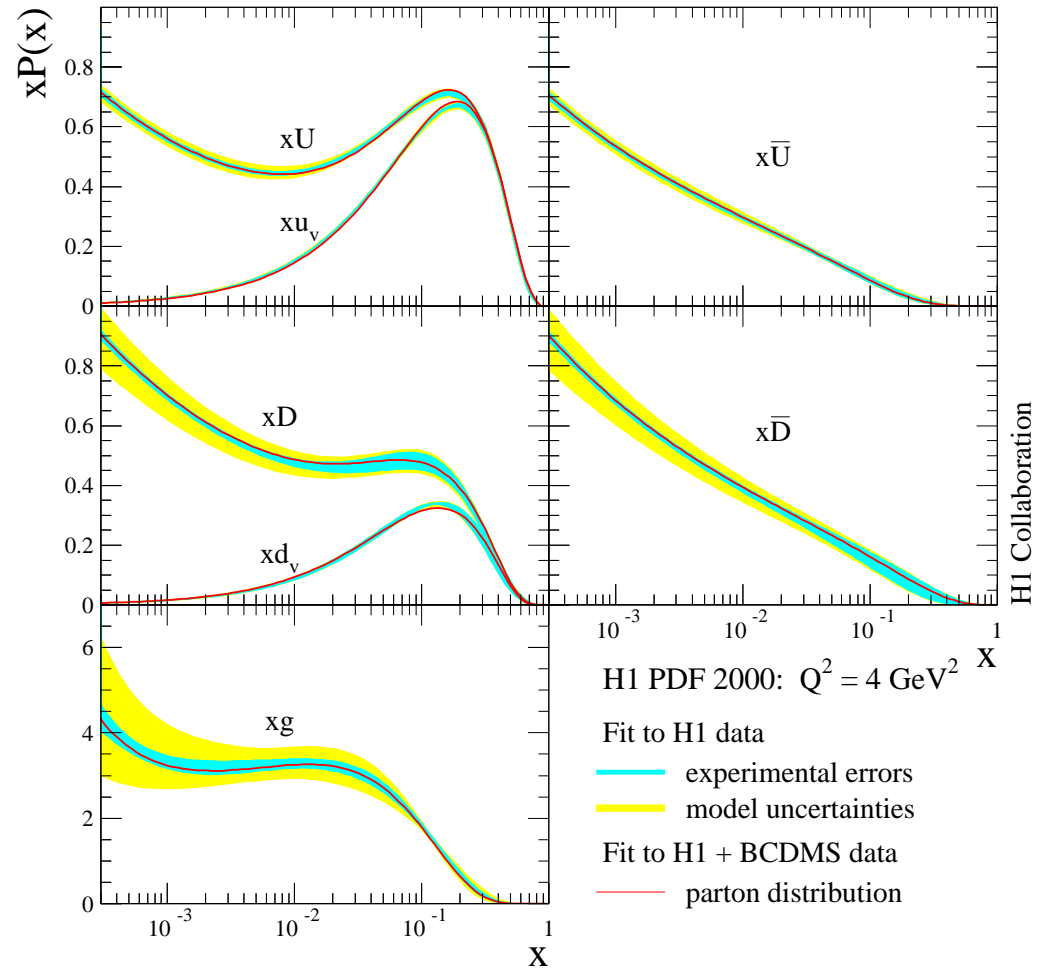
# Direct $F_L(x, Q^2)$ Measurement at HERA



# Direct $F_L(x, Q^2)$ Measurement at HERA (H1-prel.)

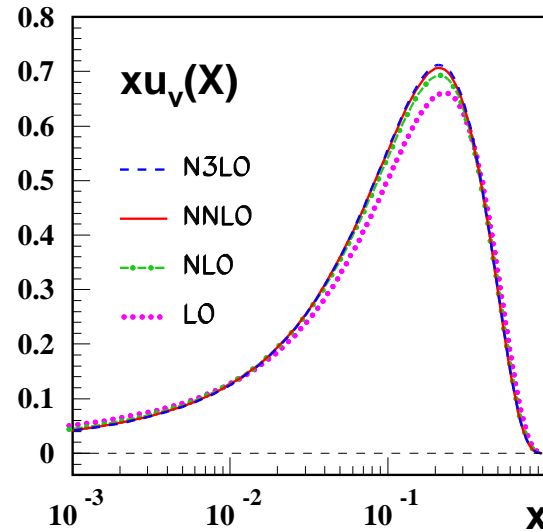
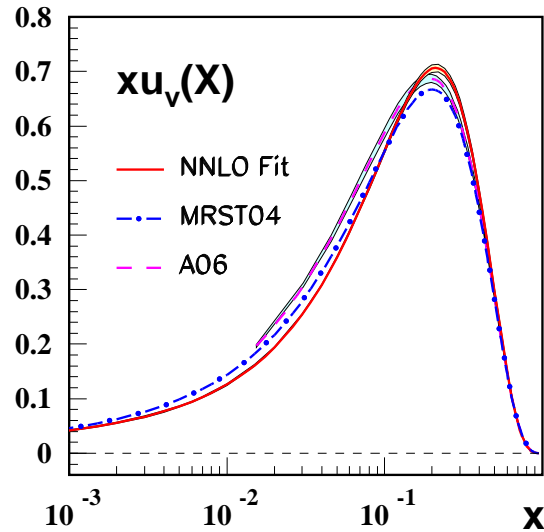


# Parton Distributions: Overview



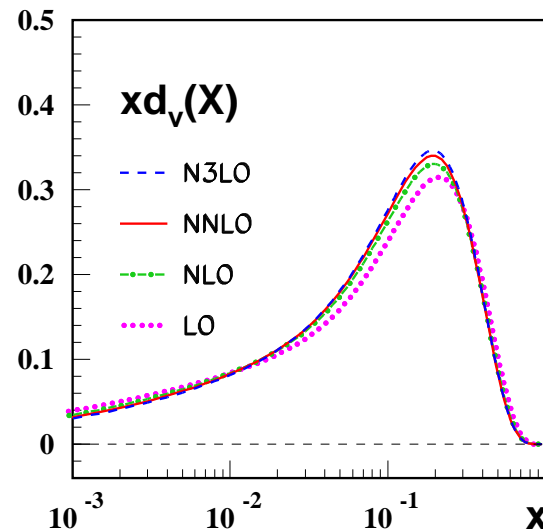
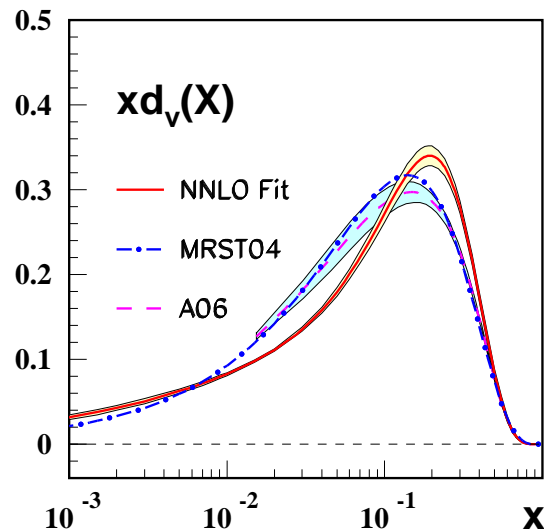
H1

# World Data Analysis: Valence Distributions



World data:  
NS-analysis

$$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$$



$N^3LO$  :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,  
A. Guffanti,  
(hep-ph/0607200)

# Why an $O(\alpha_s^4)$ analysis can be performed?

assume an  $\pm 100\%$  error on the Padé approximant  $\longrightarrow \pm 2 \text{ MeV}$  in  $\Lambda_{QCD}$

$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)2}}{\gamma_n^{(1)}}$$

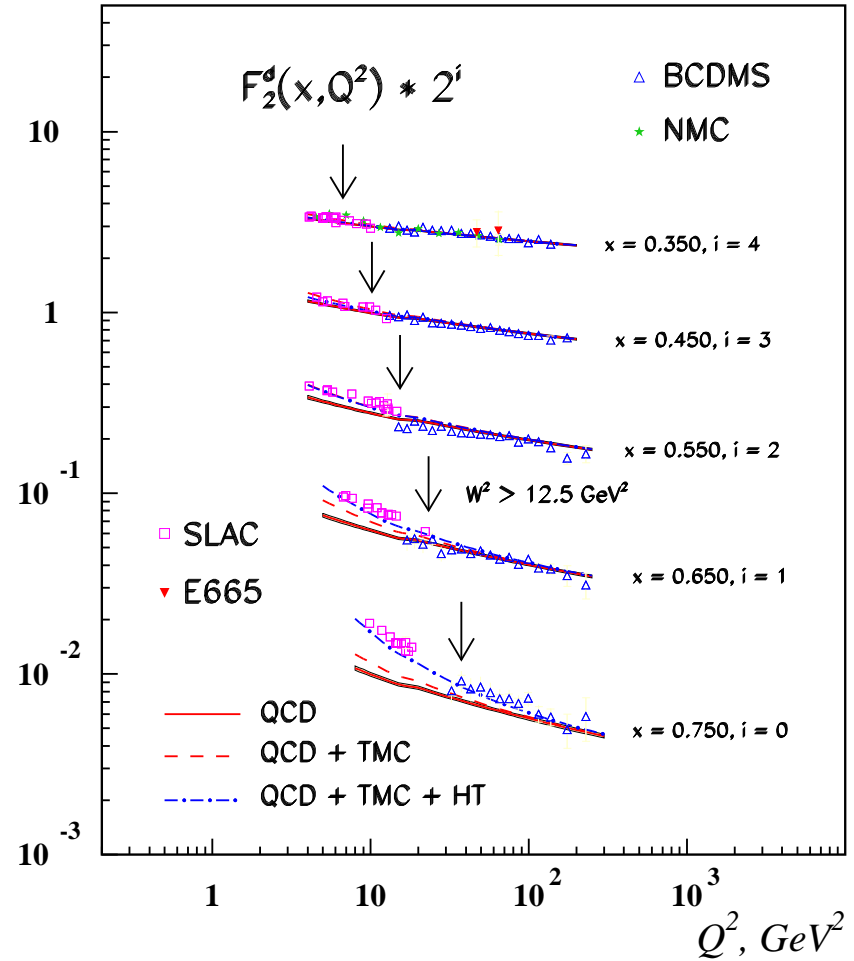
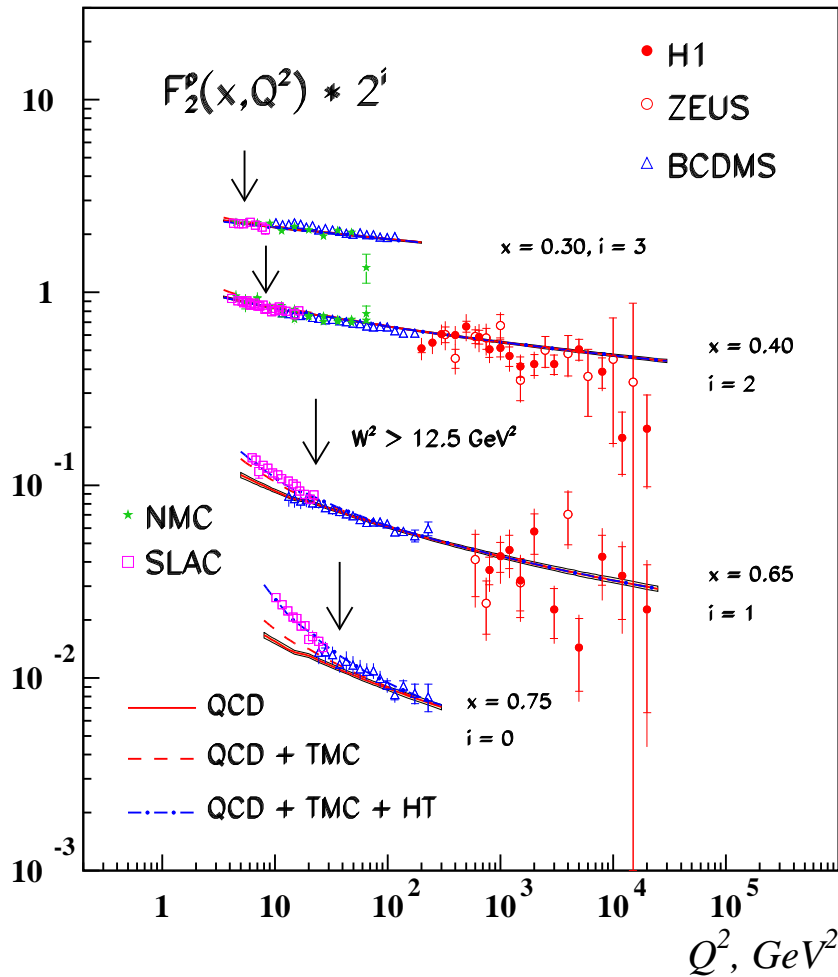
Baikov & Chetyrkin, April 2006:

$$\begin{aligned} \gamma_2^{3;NS} &= \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[ \frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3 \\ &+ \left[ \frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4 \end{aligned}$$

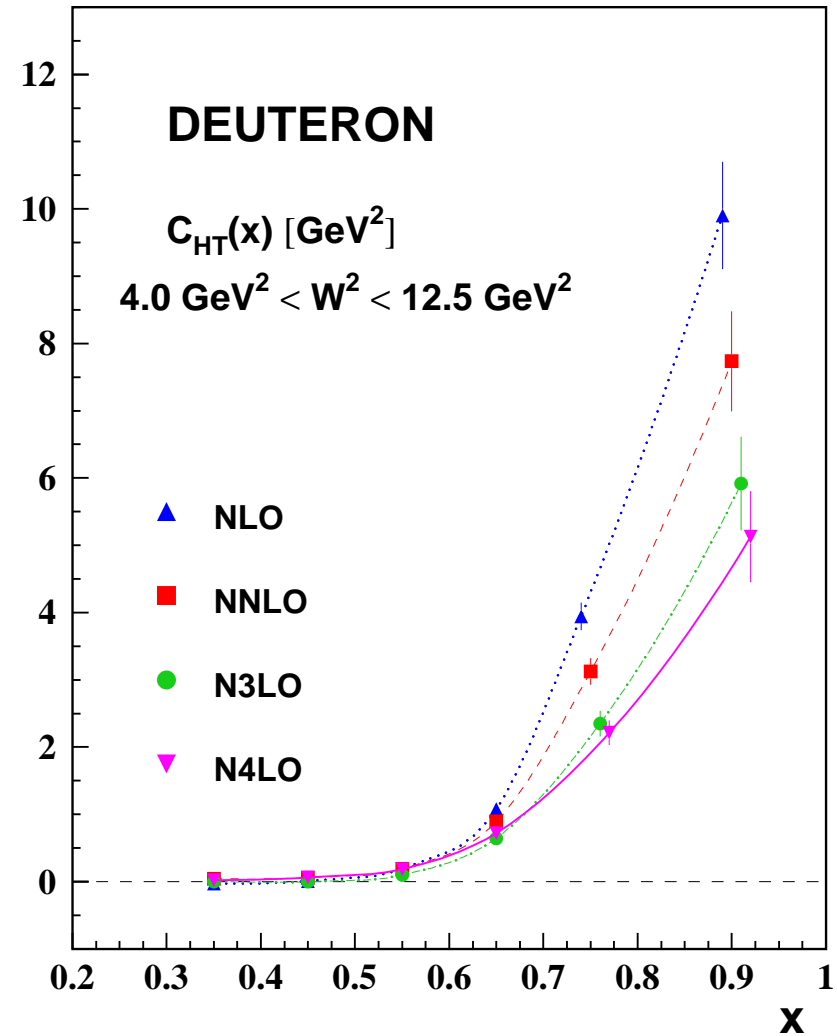
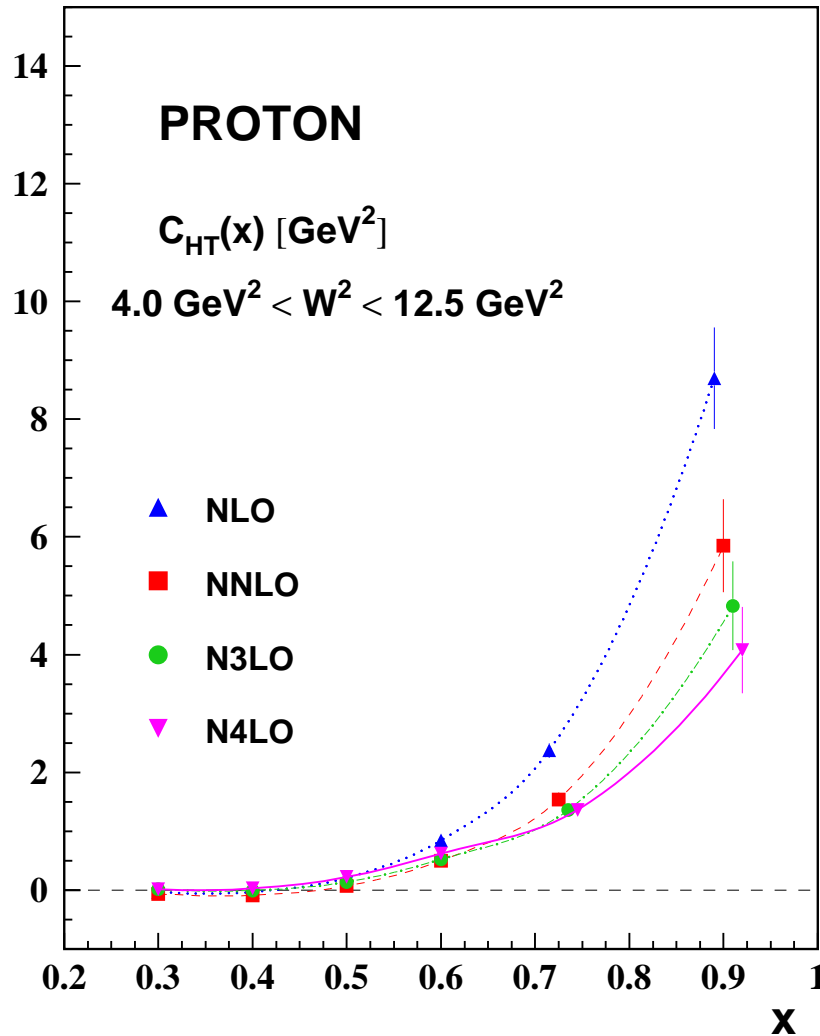
The results agree better than 20%.



# Valence Distributions

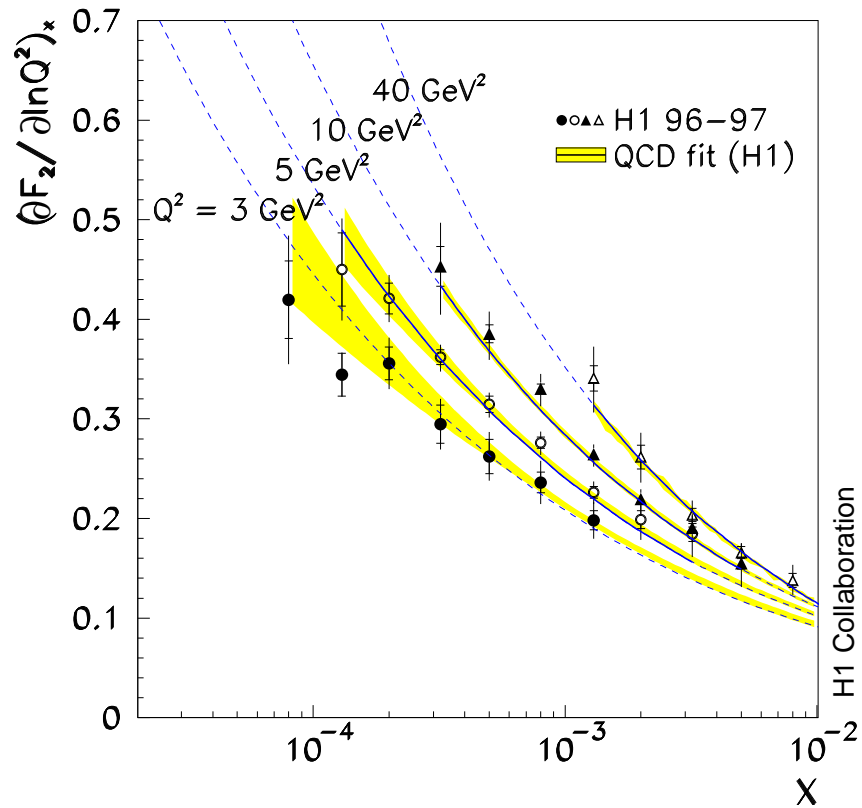


# Valence Distributions: higher twist

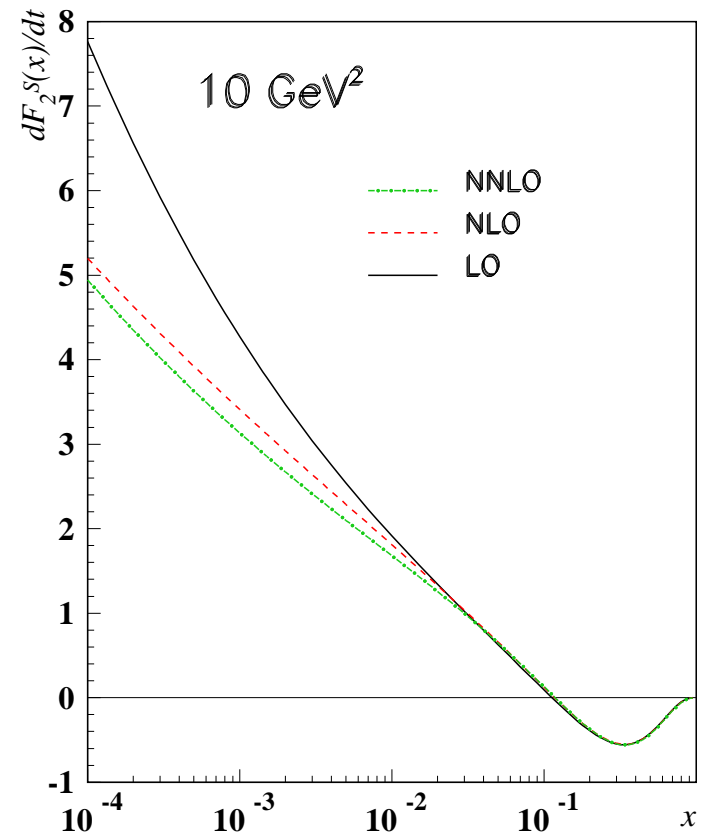


- agreement between  $p$  and  $d$  analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

# Slope of $F_2$ at low $x$



H1



J.B., A. Guffanti 2005

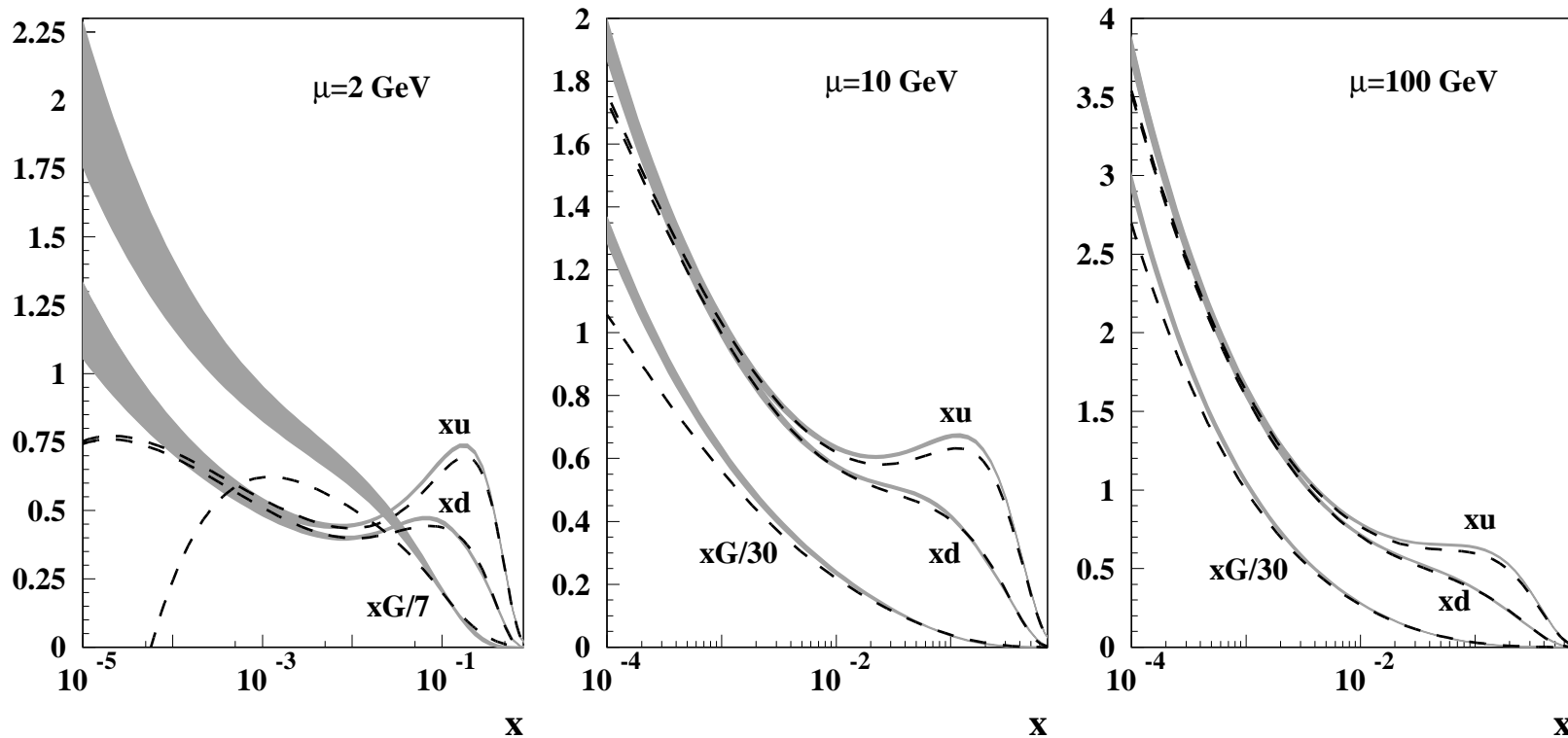
Very likely, that the  $\overline{\text{MS}}$ -gluon is remains positive!

# Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO):



+ Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...

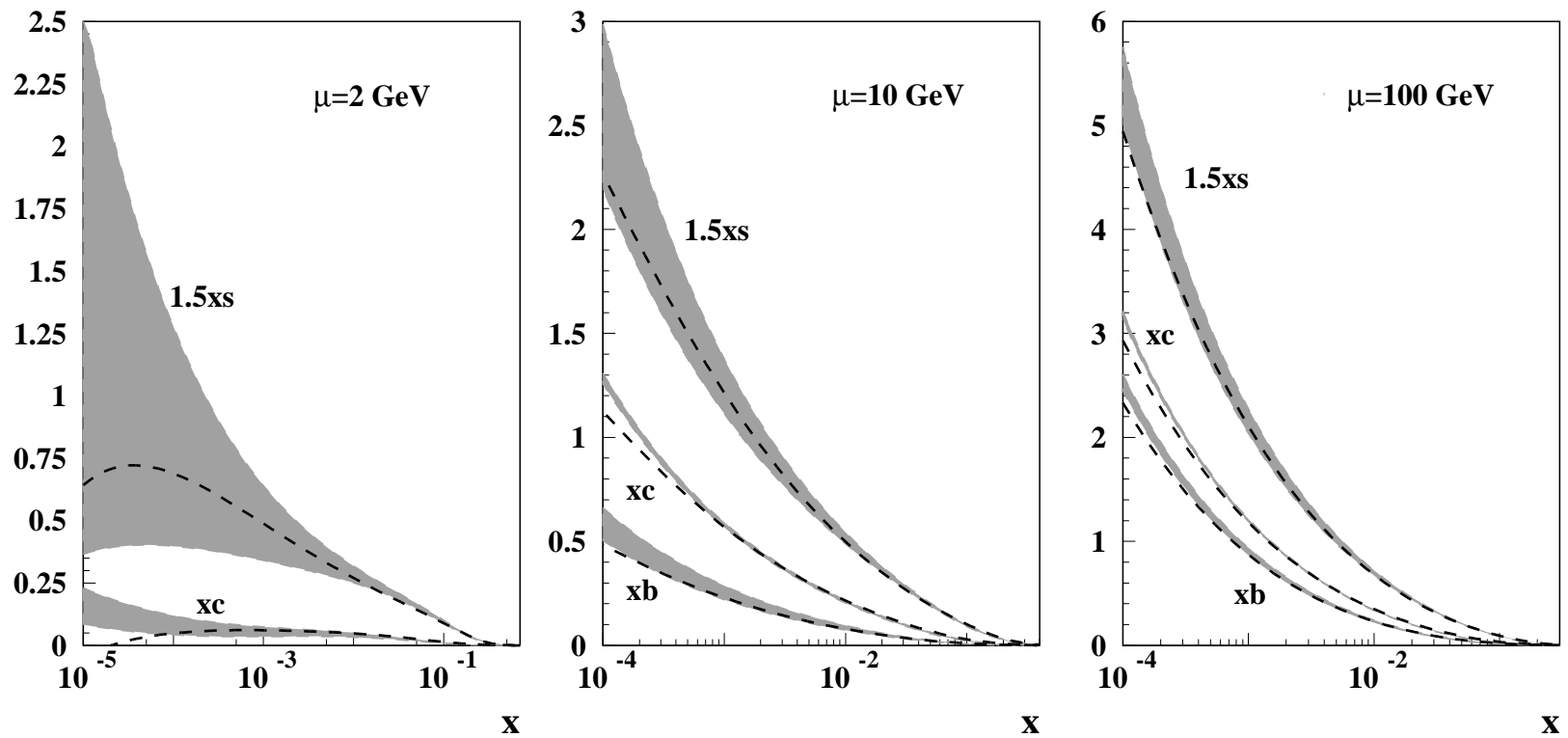


S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

Correct treatment of HQ very essential: FFNS, BSMN-schemes.

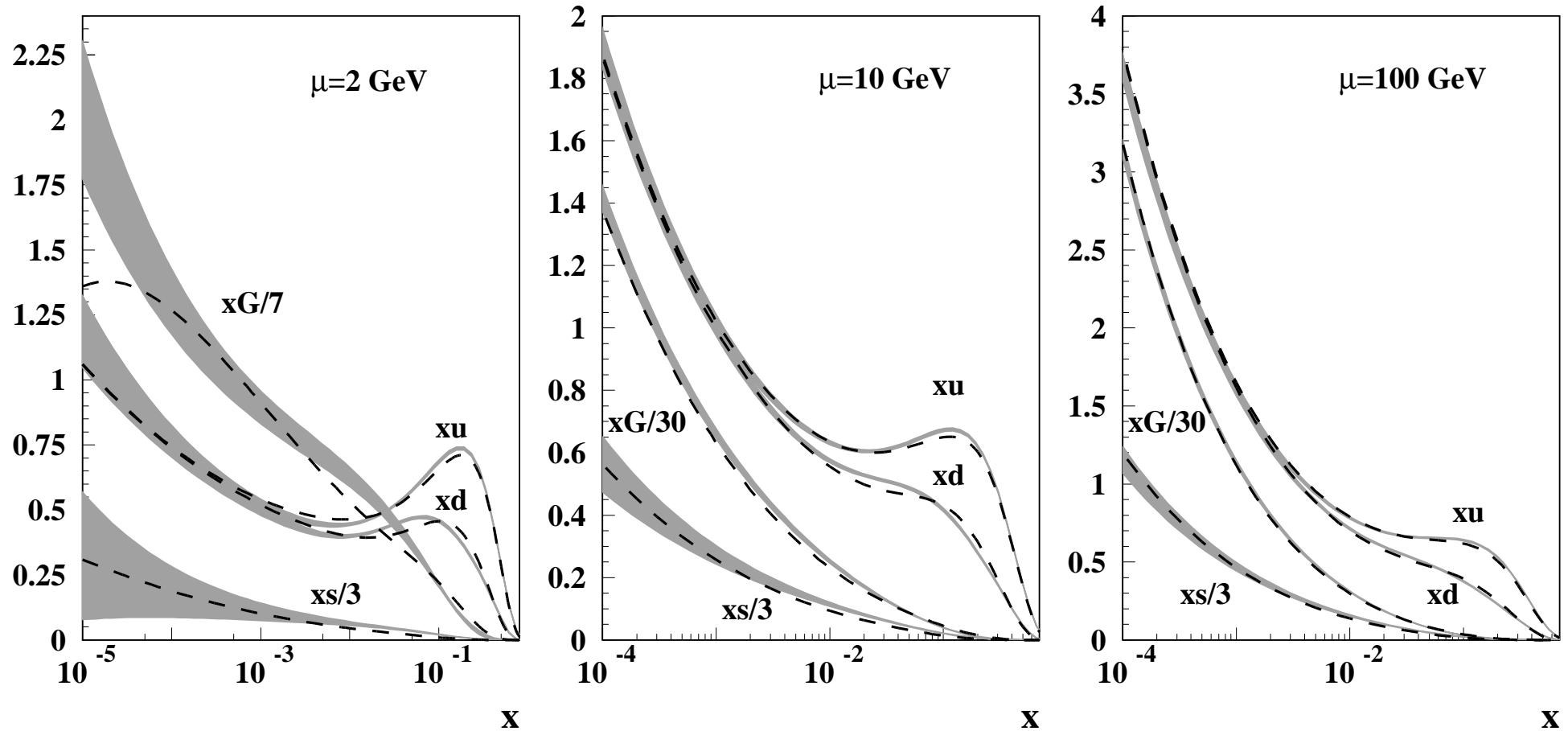
full lines: ABKM error band; dashed lines: MSTW08

# Heavy quarks and gluon (NNLO)



S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102  
full lines: ABKM error band; dashed lines: MSTW08

# FFNS, $N_f = 3$



comparison: ABKM (2009) vs. Jimenez-Delgado/Reya (2008)

# Flavor distributions: strangeness

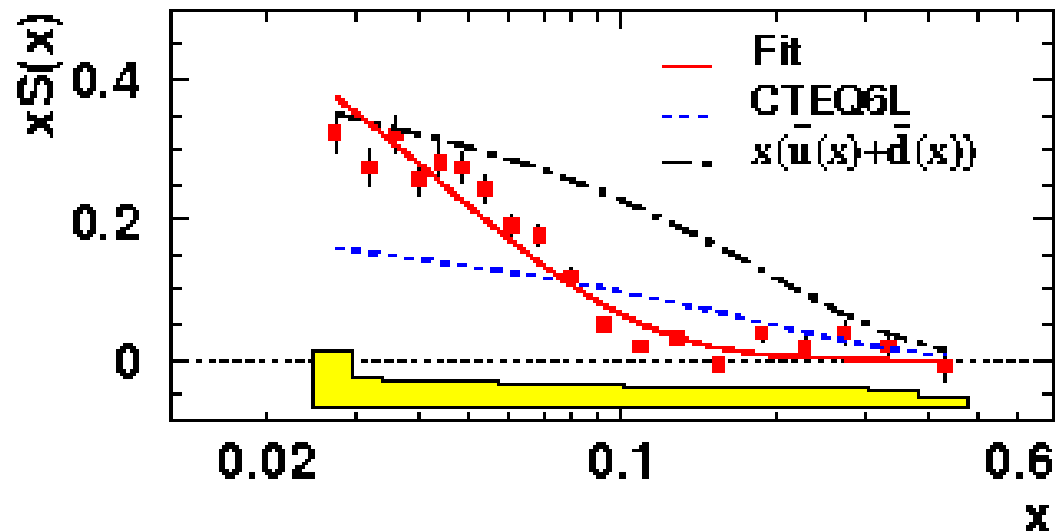
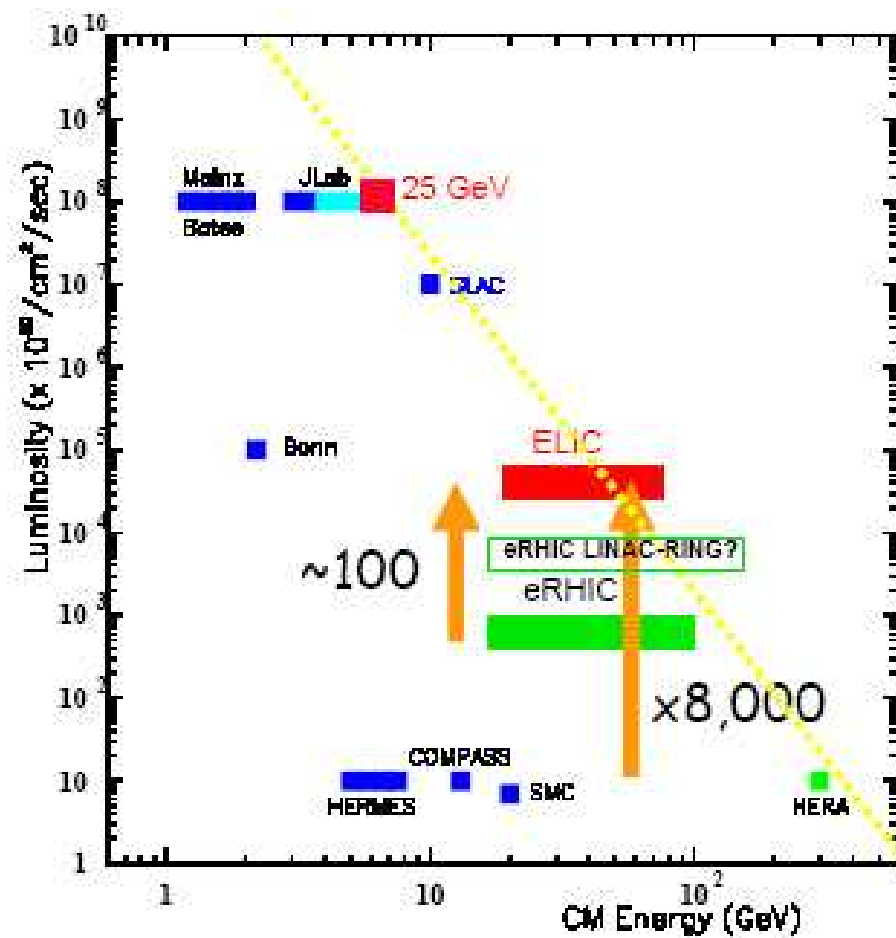
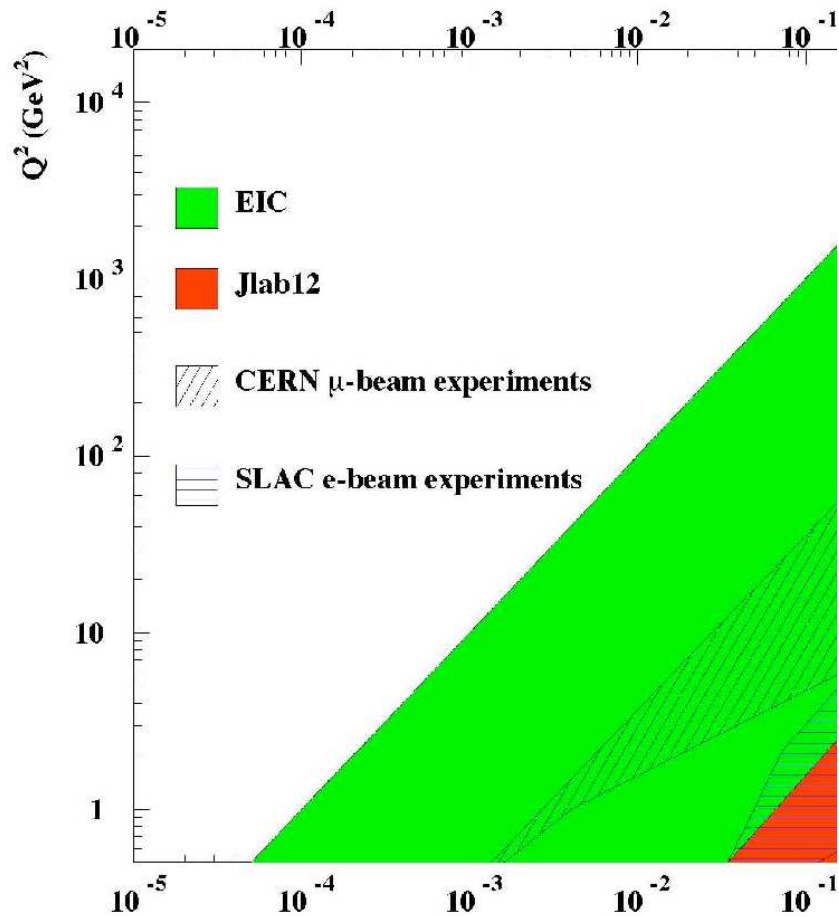


FIG. 3: The strange parton distribution  $xS(x)$  from the measured HERMES multiplicity for charged kaons evolved to  $Q_0^2 = 2.5 \text{ GeV}^2$  assuming  $\int \mathcal{D}_S^K(z) dz = 1.27 \pm 0.13$ . The solid curve is a 3-parameter fit for  $S(x) = x^{-0.924} e^{-x/0.0404} (1-x)$ , the dashed curve gives  $xS(x)$  from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

Nice HERMES measurement (hep-ex/0803.2993); still to be understood.

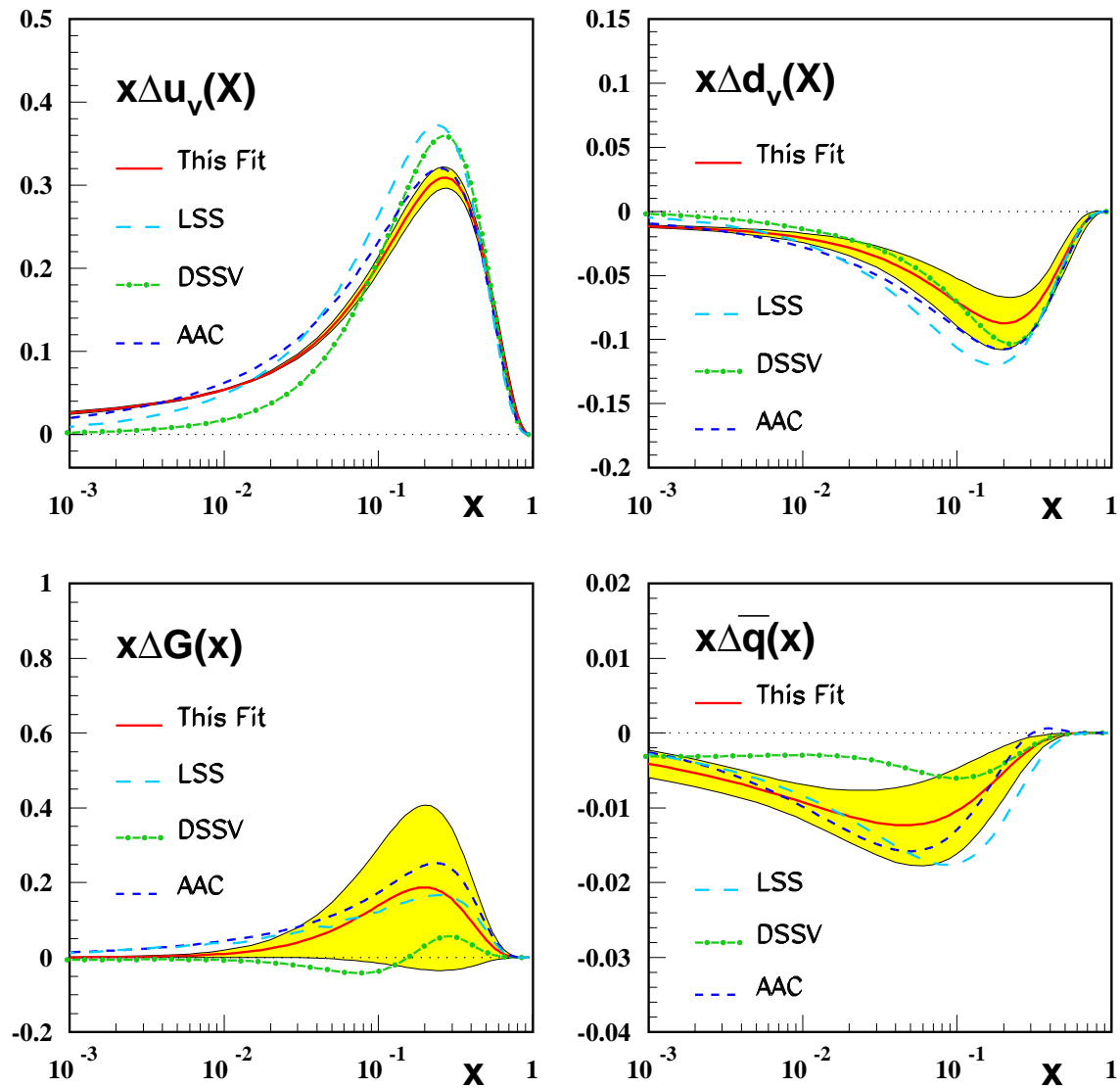
# 4. Polarized Structure Functions



High Luminosity is most important: Various precision measurements.

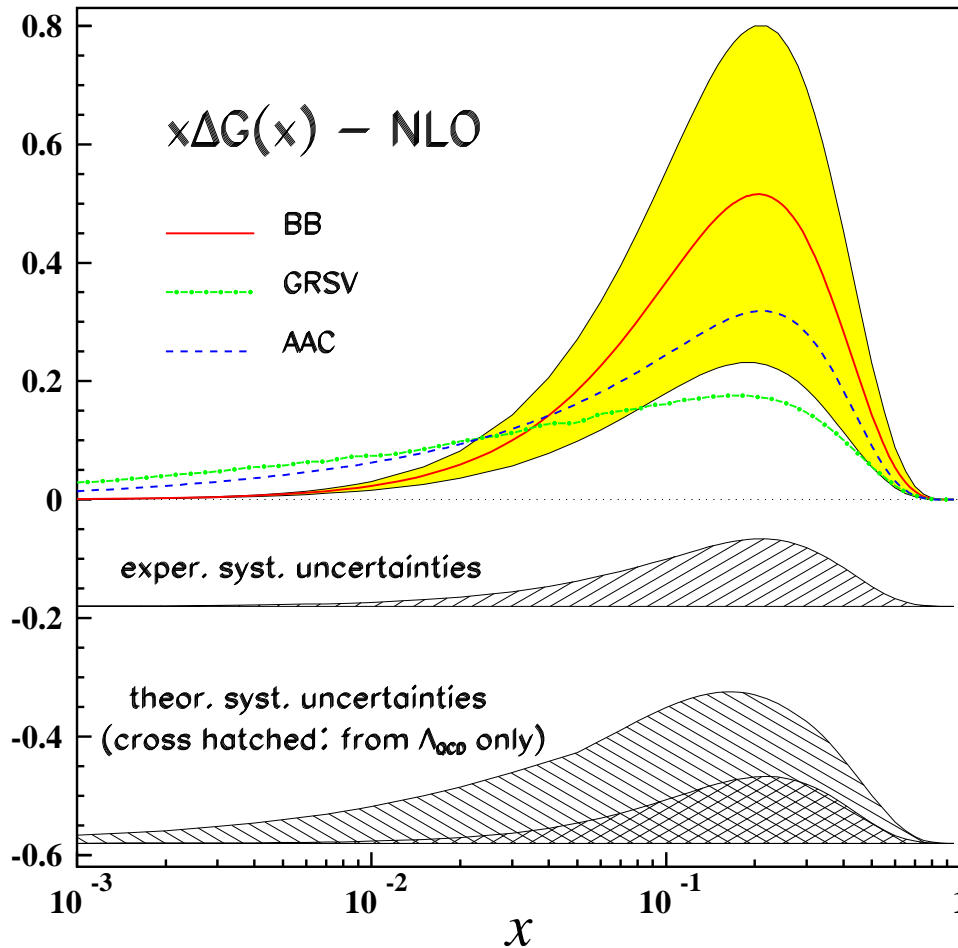


# Polarized Parton Densities at Present

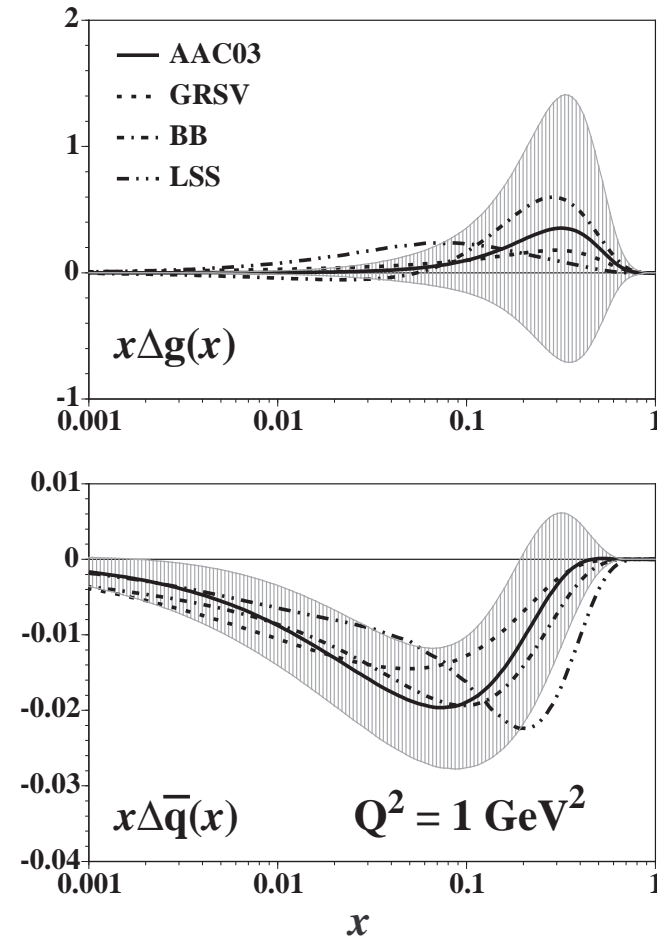


J.B., H. Böttcher (2002, 2009)

# The Polarized Gluon Distribution at Present



J.B., H. Böttcher (2002)

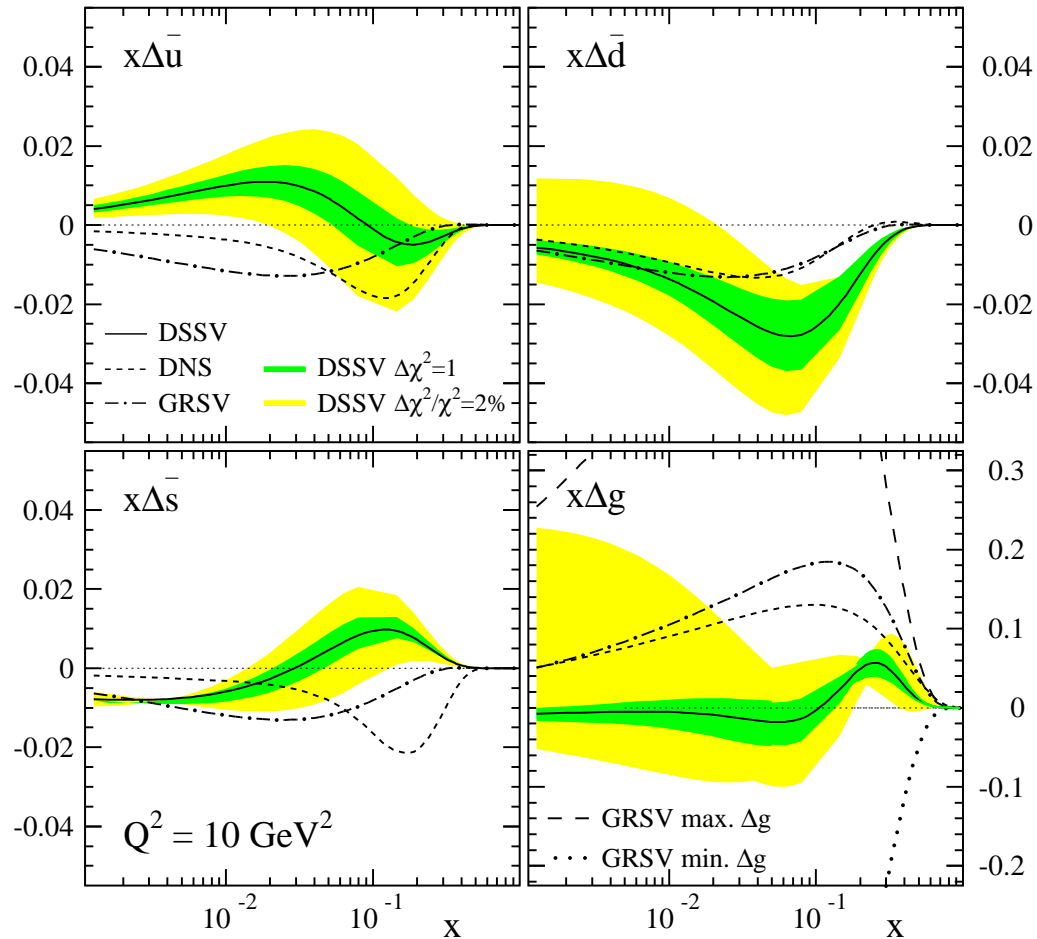


AAC

⇒ Currently slight move of  $\Delta G$  towards lower values

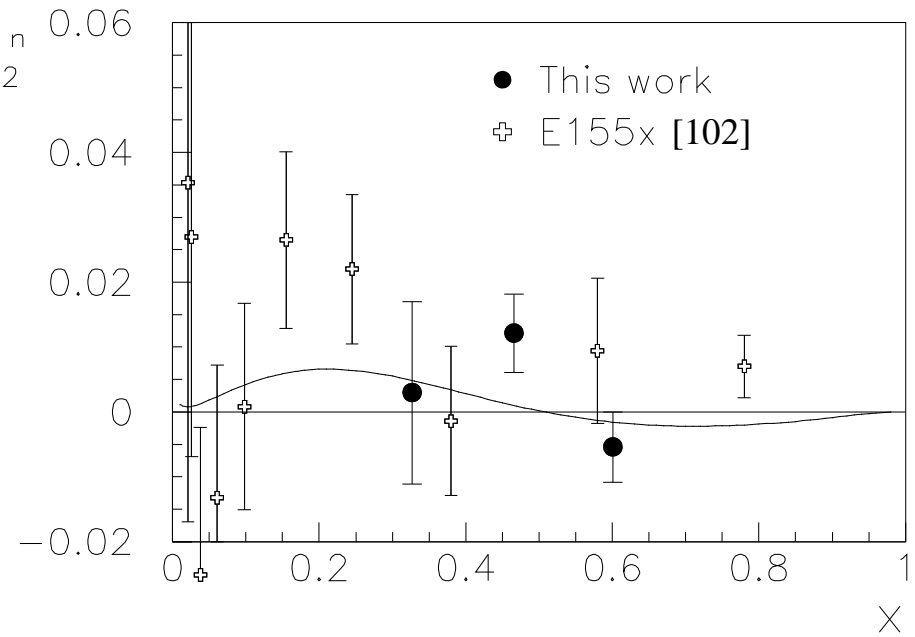
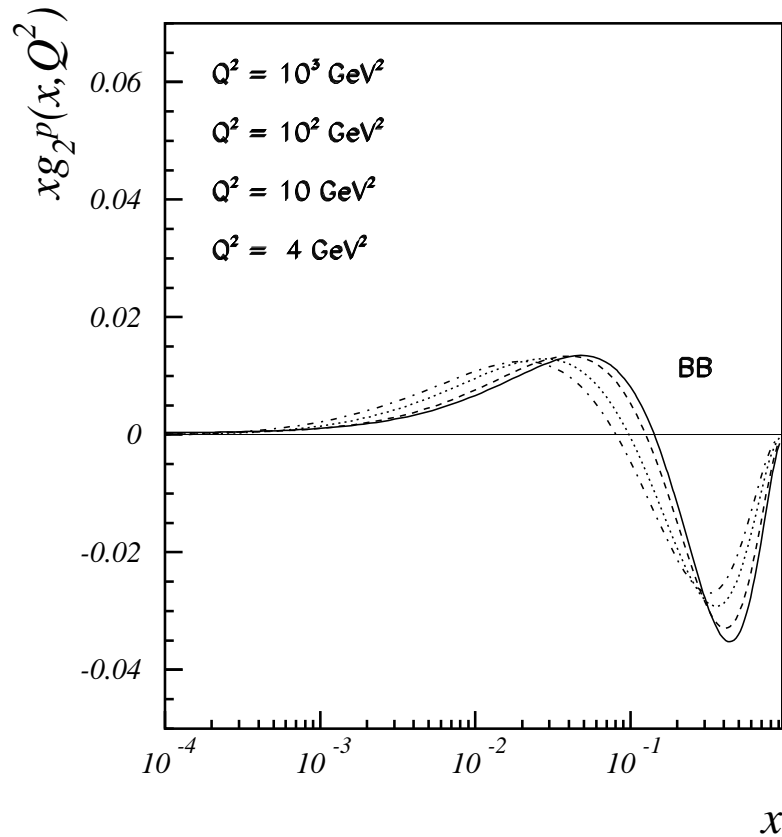
⇒ 3-loop analysis would settle theory error.

# Unfolding the Sea Quarks



De Florian, Sassot, Stratmann, Vogelsang, 2008

# $g_2(x, Q^2)$ - a Window to Higher Twist



JLAB Hall A, 2004

$g_2^{\tau=2}(x, Q^2)$  (light partons)

**Accurate measurement highly desired.  
How big is the  $\tau = 3$  contribution ?**

# Moments of PDF's: PT + data

$f$	$n$	This Fit N <sup>3</sup> LO	MRST04 NNLO	A02 NNLO		Moment	BB, NLO
$u_v$	2	$0.3006 \pm 0.0031$	0.285	0.304	$\Delta u_v$	0	0.926
	3	$0.0877 \pm 0.0012$	0.082	0.087		1	$0.163 \pm 0.014$
	4	$0.0335 \pm 0.0006$	0.032	0.033		2	$0.055 \pm 0.006$
$d_v$	2	$0.1252 \pm 0.0027$	0.115	0.120	$\Delta d_v$	0	-0.341
	3	$0.0318 \pm 0.0009$	0.028	0.028		1	$-0.047 \pm 0.021$
	4	$0.0106 \pm 0.0004$	0.009	0.010		2	$-0.015 \pm 0.009$
$u_v - d_v$	2	$0.1754 \pm 0.0041$	0.171	0.184	$\Delta u_v - \Delta d_v$	0	1.267
	3	$0.0559 \pm 0.0015$	0.055	0.059		1	$0.210 \pm 0.025$
	4	$0.0229 \pm 0.0007$	0.022	0.024		2	$0.070 \pm 0.011$

J.B., H. Böttcher, A. Guffanti, 2006

J.B., H. Böttcher, 2002

**Lattice Results** : developing; different fermion-types studied. Low values of  $m_\pi$  crucial; values approach 270 MeV now.

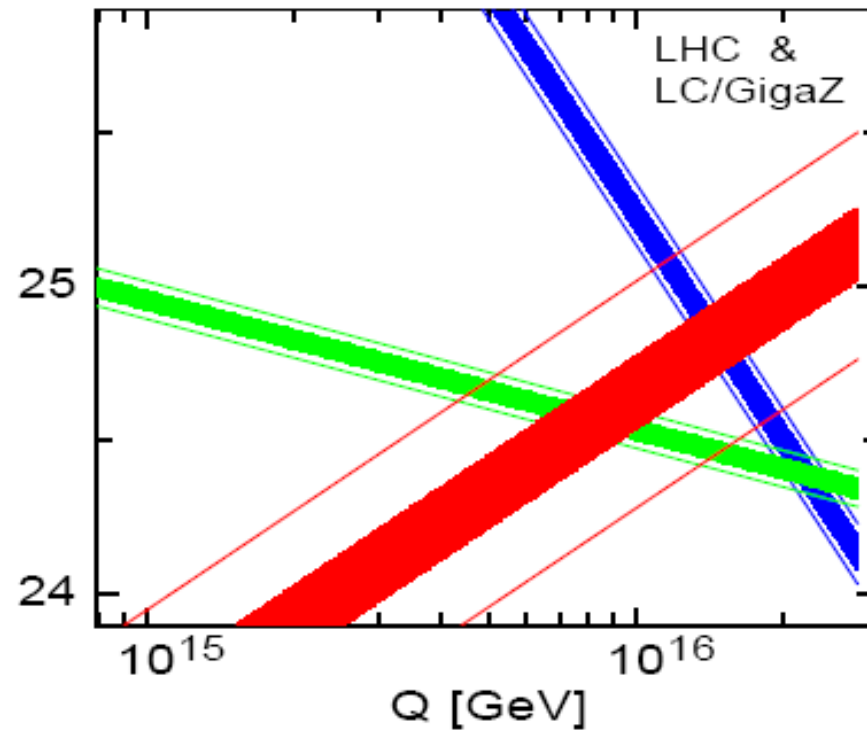
# 5. $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{em}(0)}{\alpha_{em}(0)} \sim 3 \cdot 10^{-11}$$

$$\frac{\delta\alpha_{weak}}{\alpha_{weak}} \sim 7 \cdot 10^{-4}$$

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}$$

(until recently)



P. Zerwas, 2004

# $\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	$\pm 0.0065$		[1]
MRST03	0.1165	$\pm 0.0020$	$\pm 0.0030$	[2]
A02	0.1171	$\pm 0.0015$	$\pm 0.0033$	[3]
ZEUS	0.1166	$\pm 0.0049$		[4]
H1	0.1150	$\pm 0.0017$	$\pm 0.0050$	[5]
BCDMS	0.110	$\pm 0.006$		[6]
GRS	0.112			[10]
BBG	0.1148	$\pm 0.0019$		[9]
BB (pol)	0.113	$\pm 0.004$	$+0.009$ $-0.006$	[7]

## NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	$\pm 0.0020$	$\pm 0.0030$	[2]
A02	0.1143	$\pm 0.0014$	$\pm 0.0009$	[3]
SY01(ep)	0.1166	$\pm 0.0013$		[8]
SY01( $\nu$ N)	0.1153	$\pm 0.0063$		[8]
GRS	0.111			[10]
A06	0.1128	$\pm 0.0015$		[11]
BBG	0.1134	$+0.0019 / - 0.0021$		[9]
N <sup>3</sup> LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020 / - 0.0022$		[9]

## NNLO and N<sup>3</sup>LO

 BBG:  $N_f = 4$ ; non-singlet data-analysis at  $O(\alpha_s^4)$ :  $\Lambda = 234 \pm 26$  MeV

Lattice results :

 Alpha Collab:  $N_f = 2$  Lattice; non-pert. renormalization  $\Lambda = 245 \pm 16 \pm 16$  MeV

 QCDSF Collab:  $N_f = 2$  Lattice, pert. reno.  $\Lambda = 261 \pm 17 \pm 26$  MeV

Lepage et al.: Larger, but no quenched result.

$$\alpha_s(M_Z^2)$$

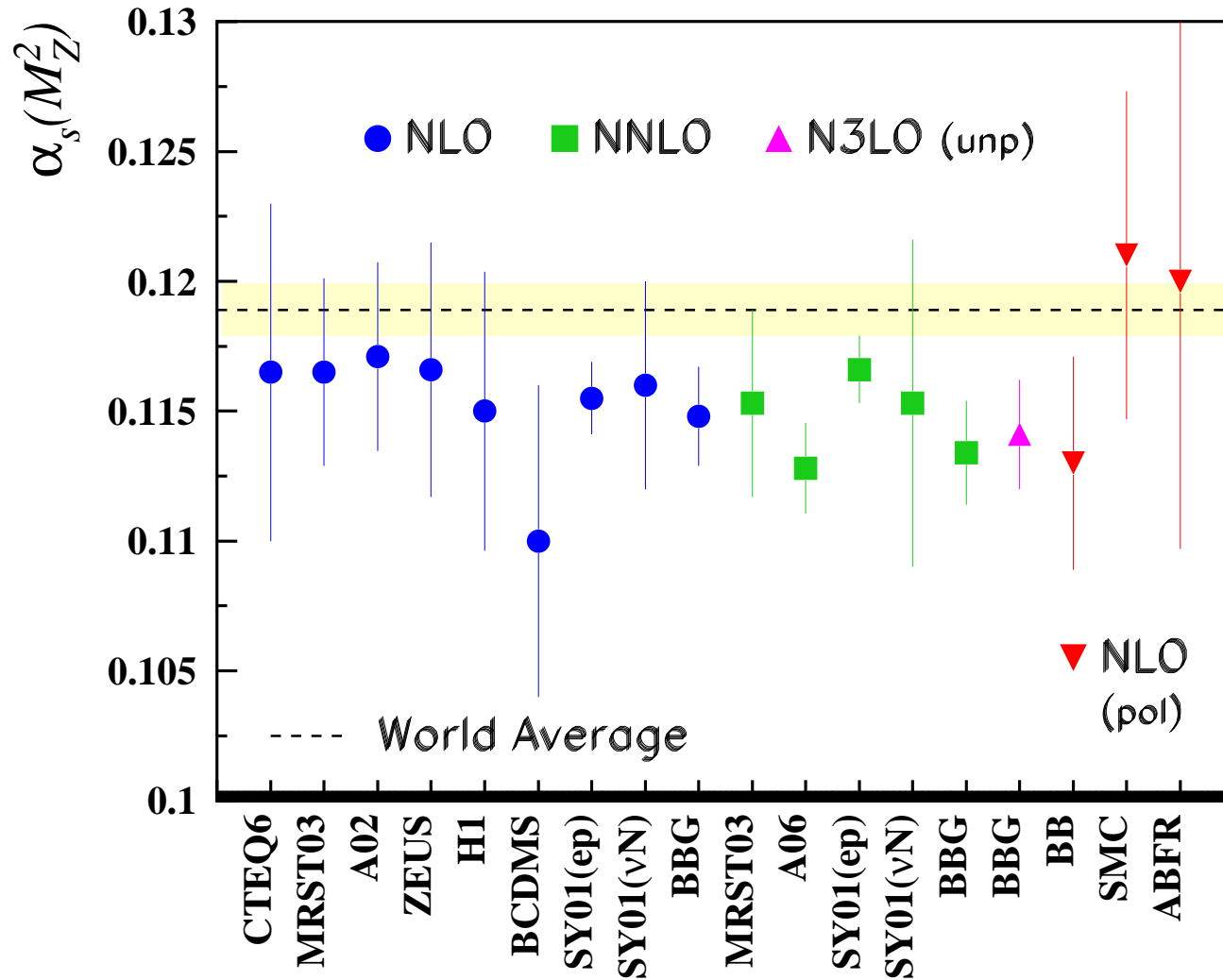
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \approx 1.2\%$$

	$\alpha_s(M_Z^2)$	
ABKM	$0.1135 \pm 0.0014$	HQ: FFS $N_f = 3$
A.Hoang et al.	$0.1135 \pm 0.0011 \pm 0.0006$	$e^+e^-$ thrust
ABKM	$0.1129 \pm 0.0014$	HQ: BSMN-approach
BBG (2006)	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO
JR (2008)	$0.1124 \pm 0.0020$	dynamical approach
MSTW (2008)	$0.1171 \pm 0.0014$	
BBG (2006)	$0.1141^{+0.0020}_{-0.0022}$	valence analysis, N <sup>3</sup> LO



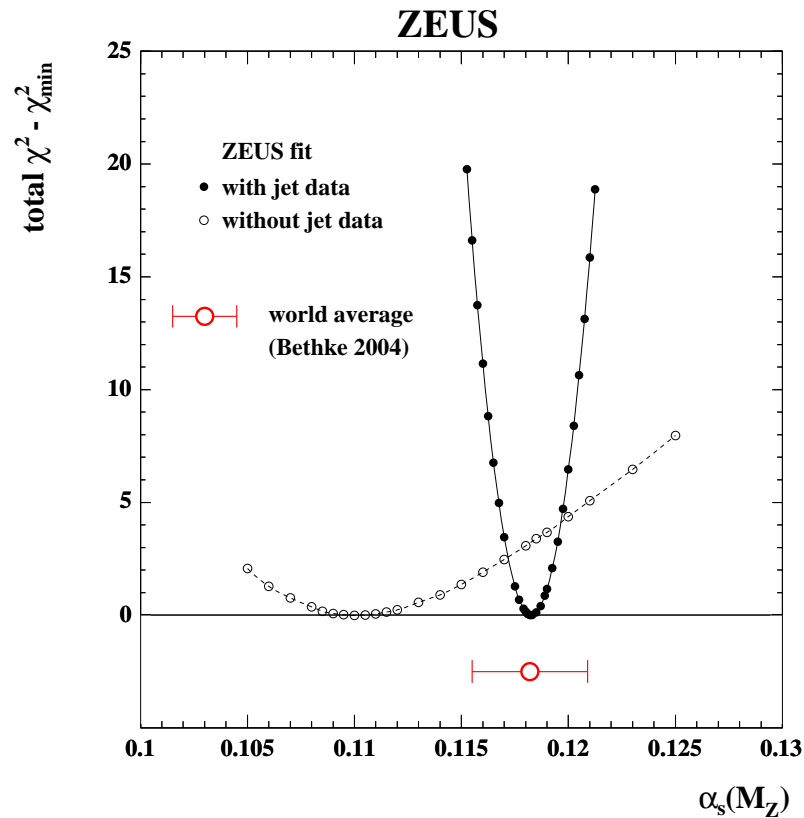
$$\alpha_s(M_Z^2)$$



J.B., H. Böttcher, A. Guffanti, 2006

# More Global Analyses

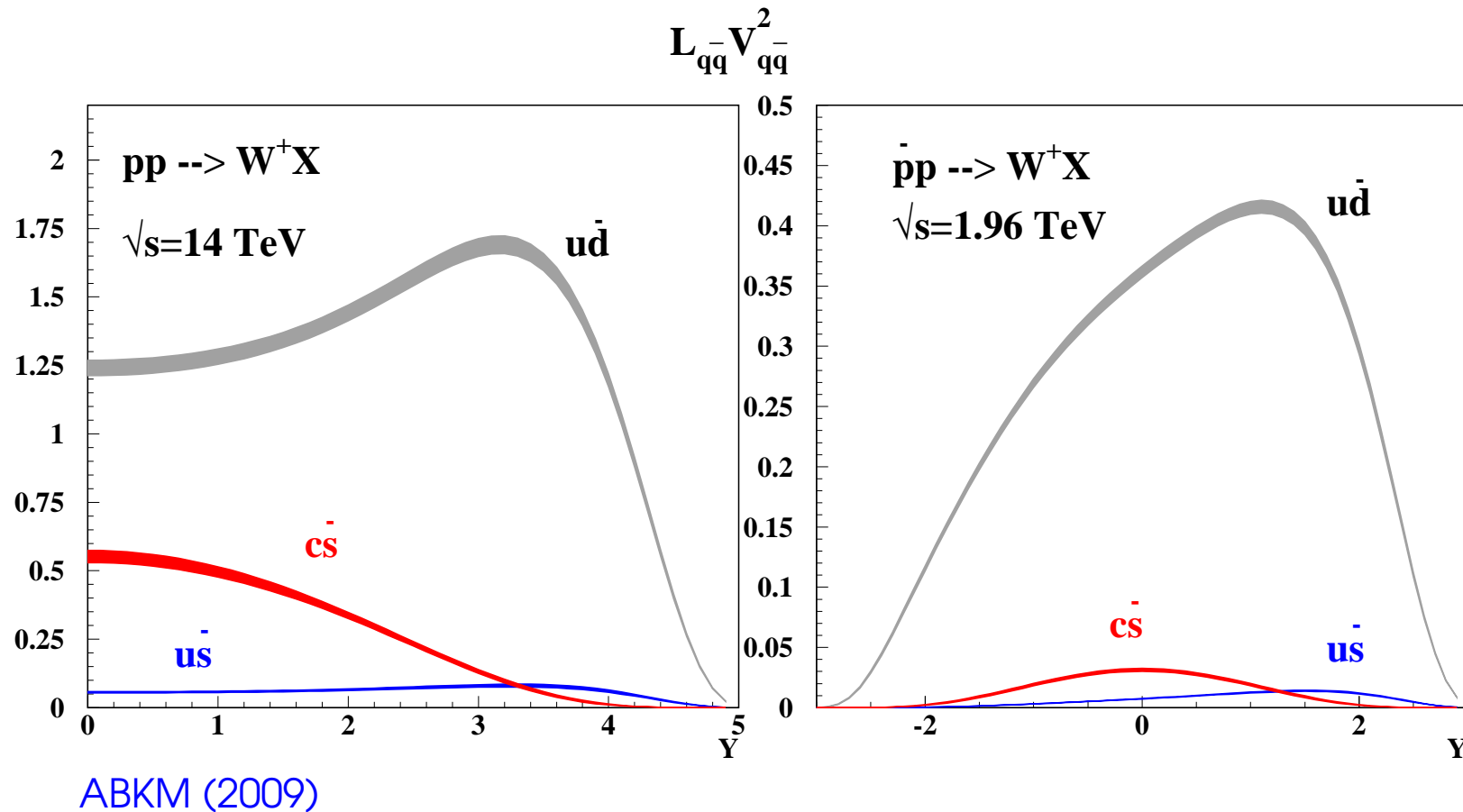
- $\alpha_s(M_Z^2)$  for different data sets included are too different !  
⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet



M. Cooper-Sarkar, 2005

# 6. Some Predictions for Tevatron and the LHC

## Drell-Yan Process (NNLO)

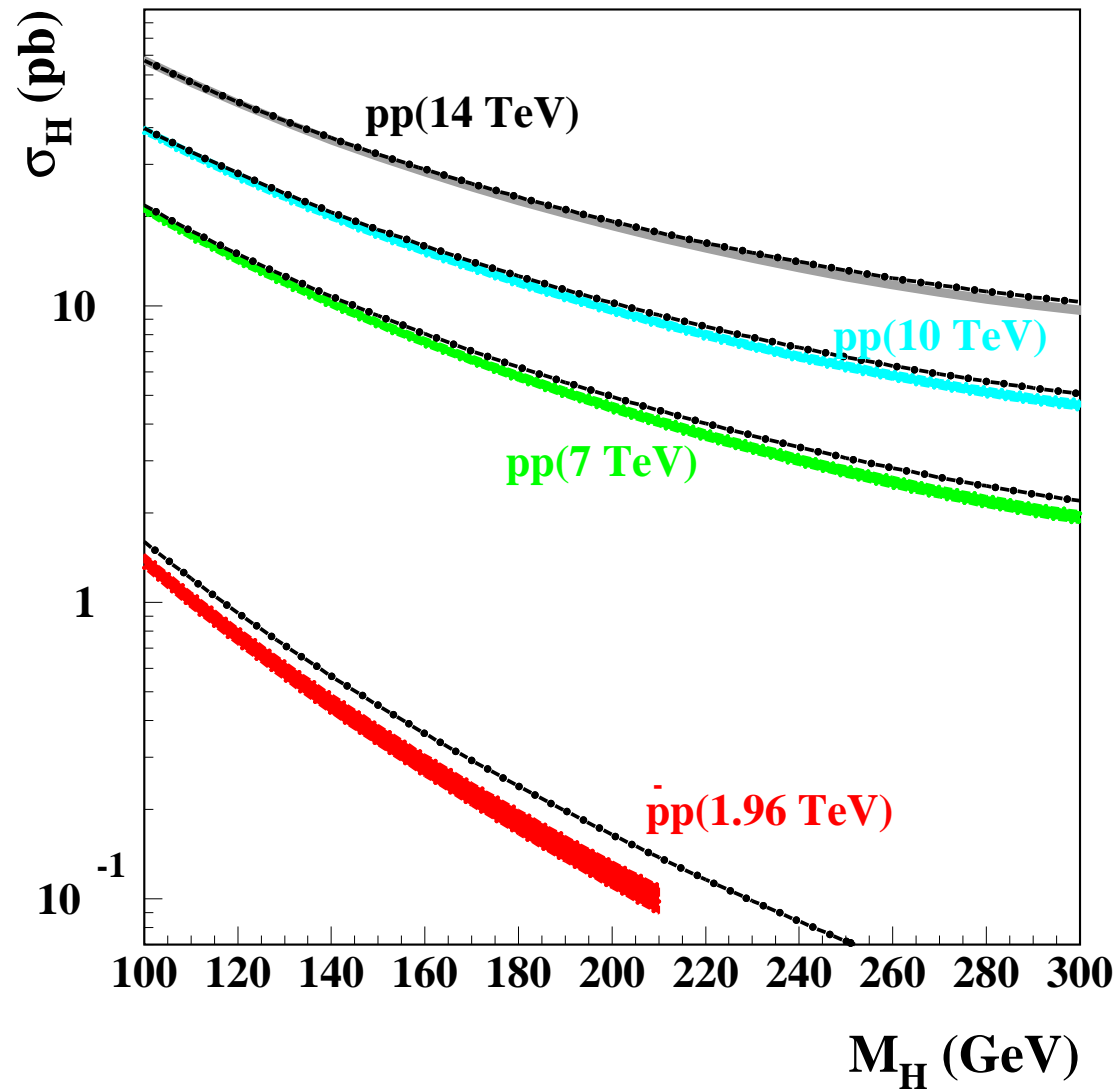


# $t\bar{t}$ Cross Section in $pp(\bar{p})$ scattering at (NNLO)

$\sqrt{s}$ (TeV)	this paper	MSTW2008
1.96 ( $\bar{p}p$ )	$6.91 \pm 0.17$	7.04
7 ( $pp$ )	$131.3 \pm 7.5$	160.5
10 ( $pp$ )	$343 \pm 15$	403
14 ( $pp$ )	$780 \pm 28$	887

ABKM (2009) vs MSTW08

# Higgs Cross Section in $pp(\bar{p})$ scattering at (NNLO)



bands: ABKM (2009); lines: MSTW08

# 7. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation **mandatory**: QGRAF   
# 2500 - 15000 diagrams
- The '**Only**' problem: Calculation of Feynman Parameter Integrals;  
everything else automated: FORM-codes
- Renormalization still not always trivial:  $\gamma_5$ , mass(es), ...
- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier  
values, etc. - **Avoids the problem of Babel**  in analytic integration
- Generalized** Hypergeometric Functions and their **Generalizations** are to the  
Heart of the Matter. M. Kalmykov et al., JB et al.
- Need: advanced Difference Equation Establishers & Solvers: Sigma 
- Do not proliferate !**, i.e. avoid IBP, MB, and other methods causing **gigantic** Zeroes.
- What remains is : **Integrating the hard way.**

# Advanced Technologies to Evaluate Feynman Diagrams

## Some Examples:

- **Zero-scale Problems** : Euler-Zagier and Multiple Zeta Values  
JB, D. Broadhurst, J. Vermaseren, DESY 09-03  
find all relations :  $\implies$  **Tera-Terms** to be processed  
alternating: all relations up to  $w = 12$  (6-loop level);  
non-alternating: all relations up to  $w = 22$ ; determined.  
Interesting relations: to  $w = 30$ ;
- **Reconstructing recurrent quantities** from Mellin Moments  
JB, M. Kauers, S. Klein, C. Schneider DESY 09-02  
Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments ? **Yes** - recurrent quantities in Mellin space.  
 $\leq 5114$  Moments; difference equation fills 440 books  
Complete computation: 5 CPU Months
- **Massive Wilson coefficients at 3 Loops**  
I. Bierenbaum, JB, S. Klein, DESY 09-57  
first analytic massive 1-scale calculation @ 3-loops  
Moments 2–10 (12/14) have been calculated for all unpolarized channels  
Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers

## 8. Outlook

### Theory:

- **Polarized** Anomalous Dimensions & massless Wilson coefficients @ 3 Loops
- **Unpolarized** Heavy Flavor Wilson coefficients @ 3 Loops : **general  $N$**
- **Polarized** Heavy Flavor Wilson coefficients @ 3 Loops
- **Along with this:** Development of efficient analytic calculation methods being suited for 3-Loops and higher
- **$ep$  &  $pp$**  jet cross sections at HO; progress in **pdf Lattice calculations**

### Code:

- Creation of an **Open Source Code** for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

### Experiment:

- Precision Data from **LHC, JLAB and EIC.**

Can we get  $\delta\alpha_s$  even smaller ?