Deep Inelastic Non-Forward Scattering

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1. Introduction

Deeply Virtual Compton Scattering: A New Test Ground for QCD

- Scaling Violations: Non-Forward
- New Evolution Equations
- Generalization of the Light Cone Expansion
- New Integral Relations Specific to the Non-Forward Case

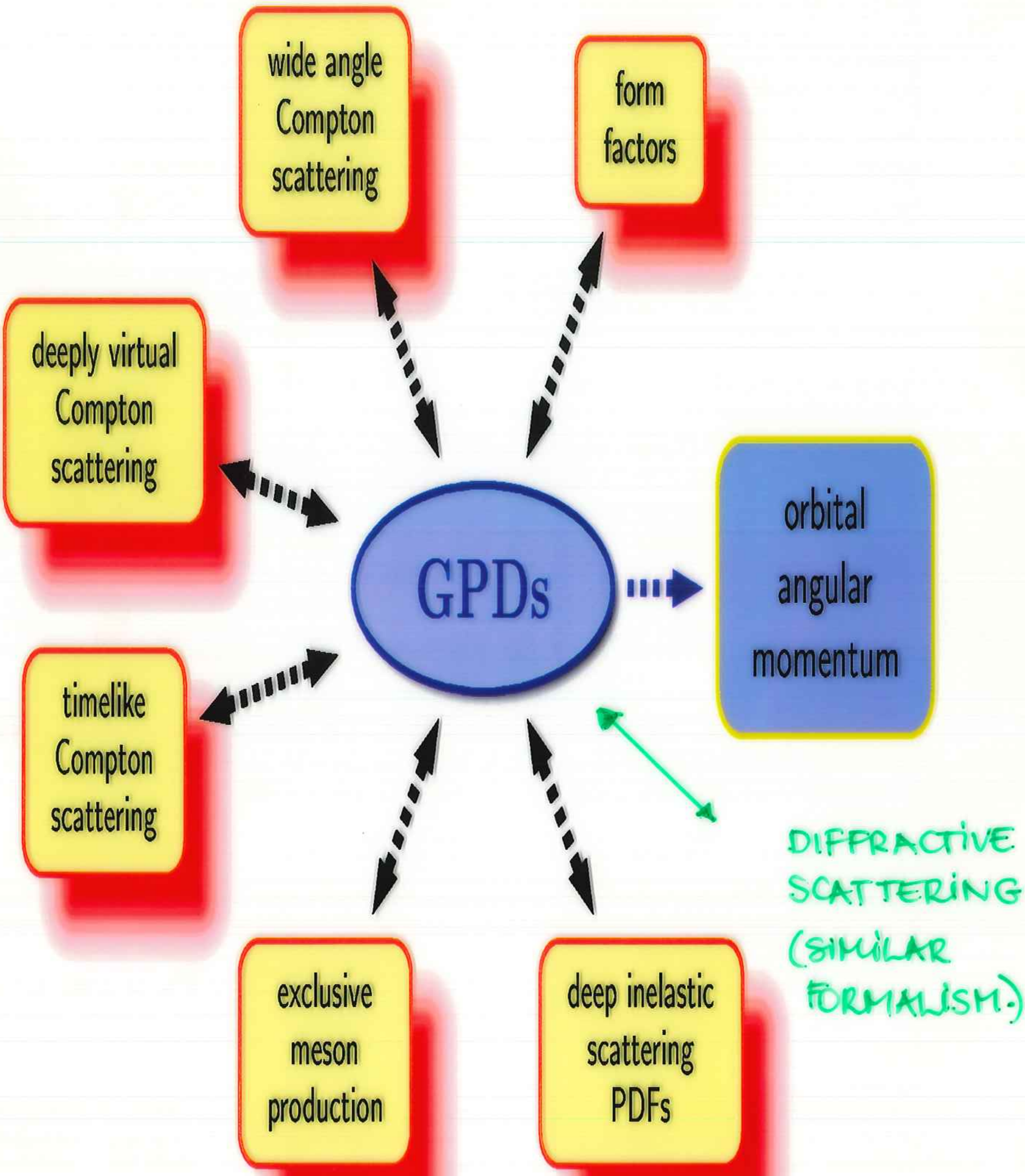
Generalization of the CALLAN–GROSS and WANDZURA–WILCZEK Relations to the Amplitude Level

Conceptional Problem: Non-Forward Light Cone Expansion & Current Conservation

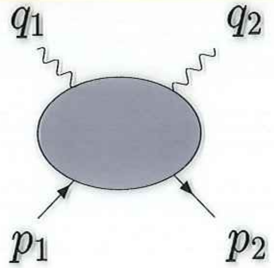
How to extract the Twist–2 Contributions ?

How to resum the Spin Towers ? $\rightarrow$ Also a Problem for the Higher Twist Operators for Forward-Scattering!

Application: Operator Approach to Diffractive ep Scattering.


wide angle Compton scattering
form factors
deeply virtual Compton scattering
GPDs
orbital angular momentum
timelike Compton scattering
exclusive meson production
deep inelastic scattering PDFs
DIFFRACTIVE SCATTERING (SIMILAR FORMALISM.)

2. The Compton Amplitude



$$T_{\mu\nu}(p_+, p_-, q) = i \int d^4x e^{iqx} \langle p_2, S_2 | T(J_\mu(x/2) J_\nu(-x/2)) | p_1, S_1 \rangle$$

$$p_+ = p_2 + p_1 \qquad p_- = p_2 - p_1 = q_1 - q_2$$

$$q = \frac{1}{2}(q_1 + q_2) \qquad p_1 + q_1 = p_2 + q_2,$$

Generalized Bjorken Limit:

$$\nu = q.p_+ \to \infty, \qquad -q^2 \to \infty,$$

fixing

$$\xi = -\frac{q^2}{q.p_+} \qquad \eta = \frac{q.p_-}{q.p_+} = \frac{q_1^2 - q_2^2}{2\nu}$$

$$\begin{aligned} q_1 &= q + \frac{1}{2}p_- \\ q_2 &= q - \frac{1}{2}p_- \end{aligned}$$

2.1 Operator Structure

$$\hat{T}_{\mu\nu}(x) = iRT\left[J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right) S\right]$$

$$\hat{T}^{\mu\nu}(x) = -e^2 \frac{\tilde{x}^\lambda}{2\pi^2(x^2 - i\epsilon)^2} RT\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu\psi\left(-\frac{\tilde{x}}{2}\right) - \overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu\psi\left(\frac{\tilde{x}}{2}\right)\right.$$

$$\tilde{x} = x + \frac{\zeta}{\zeta^2}\left[\sqrt{x.\zeta^2 - x^2\zeta^2} - x.\zeta\right]$$

unpol. pol.

$$\hat{T}_{\mu\nu}(x) = -e^2 \frac{\tilde{x}^\lambda}{i\pi^2(x^2 - i\epsilon)^2}\left[\underset{\text{unpol.}}{S_{\alpha\mu\lambda\nu} O^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right)} + \underset{\text{pol.}}{i\varepsilon_{\mu\lambda\nu\sigma} O_5^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right)}\right],$$

$$S_{\alpha\mu\lambda\nu} = g_{\alpha\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\alpha\nu} - g_{\mu\nu}g_{\lambda\alpha} .$$

$$\begin{aligned}
O^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) &= \frac{i}{2}\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma^\alpha\psi\left(-\frac{\tilde{x}}{2}\right) - \overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma^\alpha\psi\left(\frac{\tilde{x}}{2}\right)\right], \\
O_5^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) &= \frac{i}{2}\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(-\frac{\tilde{x}}{2}\right) + \overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(\frac{\tilde{x}}{2}\right)\right]
\end{aligned}$$

$$\begin{aligned}
O\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) &= \tilde{x}_\alpha O^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \\
O_5\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) &= \tilde{x}_\alpha O_5^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right)
\end{aligned}$$

$$\Box O^{q,\,\mathrm{traceless}}_{(5)}(-\kappa x, \kappa x) = 0\ .$$

$$\begin{aligned} O^{q,\,\mathrm{twist2}}_{\sigma}(-\kappa\tilde{x}, \kappa\tilde{x}) &= \int_0^1 d\tau\, \partial_\sigma\, O^q_{\mathrm{traceless}}(-\kappa\tau x, \kappa\tau x)\|_{x\to\tilde{x}} \\ &= \int_0^1 d\tau \left[\partial_\sigma + \tfrac{1}{2}(\ln\tau) x_\sigma \Box\right] O^q(-\kappa\tau x, \kappa\tau x)\|_{x=\tilde{x}} \end{aligned}$$

$$\partial^\sigma O^{q,\,\mathrm{traceless}}_{\sigma}(-\kappa x, \kappa x) = 0\ , \quad \Box O^{q,\,\mathrm{traceless}}_{\sigma}(-\kappa x, \kappa x) = 0\ .$$

2.2 Operator Matrix Elements

$$e^2\left\langle p_2, S_2 \middle| O\left(\frac{x}{2}, -\frac{x}{2}\right) \middle| p_1, S_1 \right\rangle$$

$$\begin{aligned} &= i\,\overline{u}(p_2, S_2)\gamma x u(p_1, S_1) \int Dz e^{-ixp_z/2} f(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2) \\ &+ i\,\overline{u}(p_2, S_2) x\sigma p_- u(p_1, S_1) \int Dz e^{-ixp_z/2} g(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2) \end{aligned}$$

SCALAR GDP'S

$$\Box e^2\left\langle p_2, S_2 \middle| O\left(\frac{x}{2}, -\frac{x}{2}\right) \middle| p_1, S_1 \right\rangle\|_{x\to\tilde{x}} \simeq 0\ .$$

3. Non–Forward Anomalous Dimensions

Non–local Operators ≡ Taylor Summed-up Local Operators

$$
\begin{aligned}
O^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_1\tilde{x})\right] \\
O_5^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right] \\
O^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\gamma_\mu\psi(\kappa_1\tilde{x})\right] \\
O^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \tilde{x}^\nu \, \frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})F_{\nu\rho}^a(\kappa_2\tilde{x}) + F_\mu^{a\,\rho}(\kappa_2\tilde{x})F_{\nu\rho}^a(\kappa_1\tilde{x})\right] \\
O_5^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right] \\
O_5^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \tilde{x}^\nu \, \frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_2\tilde{x}) - F_\mu^{a\,\rho}(\kappa_2\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_1\tilde{x})\right]
\end{aligned}
$$

Renormalization Group Equation:

$$
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1\tilde{x},\kappa_2\tilde{x};\mu^2) = \int_{\kappa_2}^{\kappa_1} d\kappa_1' d\kappa_2' \gamma^{AB}(\kappa_1,\kappa_2,\kappa_1',\kappa_2';\mu^2) O^B(\kappa_1'\tilde{x},\kappa_2'\tilde{x};\mu^2) \, .
$$

Argument-relations of the anomalous dimension

$$
\begin{aligned}
\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(\kappa_1-\kappa,\kappa_2-\kappa;\kappa_1'-\kappa,\kappa_2'-\kappa) \\
&= \lambda^{d_{AB}}\gamma^{AB}(\lambda\kappa_1,\lambda\kappa_2;\lambda\kappa_1',\lambda\kappa_2') \, ,
\end{aligned}
$$

$$
d_{AB} = 2 + d_A - d_B,
$$

$$
d_q = 1 \quad \text{and} \quad d_G = 2 \, .
$$

$$
\begin{aligned}
(\kappa_2-\kappa_1)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(0,1;\alpha_1,1-\alpha_2) \\
&\equiv \widehat{K}^{AB}(\alpha_1,\alpha_2), \\
4(\kappa_-)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= 4\gamma^{AB}(-1,+1;w_1,w_2) \\
&\equiv \widetilde{K}^{AB}(w_1-w_2,w_1+w_2)
\end{aligned}
$$

$$
\alpha_1 = \frac{\kappa_1'-\kappa_1}{\kappa_2-\kappa_1}, \qquad -\alpha_2 = \frac{\kappa_2'-\kappa_2}{\kappa_2-\kappa_1},
$$

$$
w_1 = \alpha_1-\alpha_2 = \frac{\kappa'_+-\kappa_+}{\kappa_-}, \qquad w_2 = 1-\alpha_1-\alpha_2 = \frac{\kappa'_-}{\kappa_-},
$$

Evolution Equations for Operators:

$$
\begin{aligned}
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int D\alpha\,(\kappa_2-\kappa_1)^{d_B-d_A}\,\widehat{K}^{AB}(\alpha_1,\alpha_2)O^B(\kappa_1',\kappa_2'), \\
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int Dw\,(\kappa_-)^{d_B-d_A}\,\widetilde{K}^{AB}(w_1,w_2)O^B(\kappa_1',\kappa_2') \\
&= \int_0^1 dw_2 \int_{-1+w_2}^{1-w_2} dw_1\,(\kappa_-)^{d_B-d_A}\,\widetilde{K}^{AB}_{\rm sym}(w_1,w_2) \\
&\quad \times O^B(\kappa_1',\kappa_2'),
\end{aligned}
$$

$$
\widetilde{K}^{AB}_{\rm sym}(w_1,w_2) = \frac{1}{2}\left[\widetilde{K}_0^{AB}(w_1,w_2)+(-1)^{d_B}\widetilde{K}_0^{AB}(w_1,-w_2)\right].
$$

Spin-Towers : Two-fold Moment Expansion

$$\mu^2 \frac{d}{d\mu^2} O^A_{n_1 n_2} = \sum_{n_1', n_2'} \gamma^{AB}_{n_1, n_2; n_1', n_2'} O^B_{n_1' n_2'} ,$$

$$\gamma^{AB}_{n_1, n_2; n_1', n_2'} = \frac{\partial^{n_1}}{\partial \kappa_1^{n_1}} \frac{\partial^{n_2}}{\partial \kappa_2^{n_2}} \int_{\kappa_2}^{\kappa_1} d\kappa_1' \int_{\kappa_2}^{\kappa_1} d\kappa_2' \frac{(\kappa_1')^{n_1'}}{n_1'!} \frac{(\kappa_2')^{n_2'}}{n_2'!} \times \gamma^{AB}(\kappa_1, \kappa_2; \kappa_1', \kappa_2')_{\kappa_1 = \kappa_2 = 0} . \quad (2)$$

$$\begin{aligned}
\gamma^{qq}_{nn'} &= \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qq}(w_1, w_2) \right\}, \\
\gamma^{qG}_{nn'} &= n \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qG}(w_1, w_2) \right\} \\
\gamma^{Gq}_{nn'} &= \frac{1}{n} \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{Gq}(w_1, w_2) \right\}, \\
\gamma^{GG}_{nn'} &= \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{GG}(w_1, w_2) \right\},
\end{aligned}$$

with

$$\begin{aligned}
\sigma^{(\pm)}_{n\,n'} &= \frac{1}{4} \left(1 + (-1)^{n-n'} \right) \left(1 \pm (-1)^{n'-2d_B} \right) \\
&= \frac{1}{4} (1 \pm (-1)^n)(1 \pm (-1)^{n'})
\end{aligned}$$

1) Unpolarized anomalous dimensions

$$
\begin{aligned}
\widehat{K}_0^{qq}(\alpha_1,\alpha_2) &= C_F\left\{1-\delta(\alpha_1)-\delta(\alpha_2)+\delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+\right.\\
&\quad\left.+\delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+ +\frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\right\},\\
\widehat{K}_0^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\left\{1-\alpha_1-\alpha_2+4\alpha_1\alpha_2\right\},\\
\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) &= -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)+2\right\},\\
\widehat{K}_0^{GG}(\alpha_1,\alpha_2) &= C_A\left\{4(1-\alpha_1-\alpha_2)+12\alpha_1\alpha_2\right.\\
&\quad+\delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ -2+\alpha_2\right)\\
&\quad\left.+\delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ -2+\alpha_1\right)\right\}+\frac{1}{2}\beta_0\,\delta(\alpha_1)\delta(\alpha_2),
\end{aligned}
$$

where $C_F=(N_c^2-1)/2N_c\equiv 4/3$, $T_R=1/2$, $C_A=N_c\equiv 3$, and the β–function in leading order, $\beta_0=(11C_A-4T_RN_f)/3$.

$$
\int_0^1 dx\,[f(x,y)]_+\,\varphi(x)=\int_0^1 dx\,f(x,y)\,[\varphi(x)-\varphi(y)],
$$

if the singularity of f is of the type $\sim 1/(x-y)$. 2) Polarized anomalous dimensions

$$
\begin{aligned}
\Delta\widehat{K}_0^{qq}(\alpha_1,\alpha_2) &= \widehat{K}_0^{qq}(\alpha_1,\alpha_2),\\
\Delta\widehat{K}_0^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\left\{1-\alpha_1-\alpha_2\right\},\\
\Delta\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) &= -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)-2\right\},\\
\Delta\widehat{K}_0^{GG}(\alpha_1,\alpha_2) &= \widehat{K}_0^{GG}(\alpha_1,\alpha_2)-12C_A\alpha_1\alpha_2
\end{aligned}
$$

1) Unpolarized anomalous dimensions:

$$\gamma^{qq}_{nn'} = C_F\left\{\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1}\frac{1}{j}\right]\delta_{nn'} - \left[\frac{1}{(n+1)(n+2)} + \frac{2}{n-n'}\frac{n'+1}{n+1}\right]\theta_{nn'}\right\}$$

$$\gamma^{qG}_{nn'} = -N_f T\frac{1}{(n+1)(n+2)(n+3)}\left[(n^2+3n+4) - (n-n')(n+1)\right],$$

$$\gamma^{Gq}_{nn'} = -C_F\frac{1}{n(n+1)(n+2)}\left[(n^2+3n+4)\delta_{nn'} + 2\theta_{nn'}\right],$$

$$\gamma^{GG}_{nn'} = C_A\left\{\left[\frac{1}{6} - \frac{2}{n(n+1)} - \frac{2}{(n+2)(n+3)} + 2\sum_{j=2}^{n+1}\frac{1}{j} + \frac{2N_fT}{3C_A}\right]\delta_{nn'} + \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'}\right) -(n-n'+2)\left(\frac{1}{n(n+1)} + \frac{1}{(n+2)(n+3)}\right)\right]\theta_{nn'}\right\},$$

with the following notation

$$\sigma^{(\pm)}_{n\,n'} = \frac{1}{4}(1\pm(-1)^n)(1\pm(-1)^{n'})$$

$$\theta_{n\,n'} = \begin{cases} 1 & \text{for } n' < n, \\ 0 & \text{otherwise}. \end{cases}$$

2) Polarized local anomalous dimensions:

$$
\begin{aligned}
\Delta\gamma_{nn'}^{qq} &= \gamma_{nn'}^{qq}, \\
\Delta\gamma_{nn'}^{qG} &= -N_f T \frac{n'}{(n+1)(n+2)}, \\
\Delta\gamma_{nn'}^{Gq} &= \frac{1}{(n+1)(n+2)}\left[(n+3)\delta_{nn'} - \frac{2}{n}\theta_{nn'}\right], \\
\Delta\gamma_{nn'}^{GG} &= C_A\Biggl\{\left[\frac{1}{6} - \frac{4}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1}\frac{1}{j} + \frac{2N_f T}{3C_A}\right]\delta_{nn'} \\
&\quad + \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'}\right)\right. \\
&\quad \left. -(n-n'+2)\frac{2}{(n+1)(n+2)}\right]\theta_{nn'}\Biggr\}.
\end{aligned}
$$

4 Scalar and Vector GDP's

unpol.

$$
\begin{aligned}
e^2\left\langle p_2, S_2 \middle| O^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1 \right\rangle
&= i\int Dz e^{-i\tilde{x}p_z/2} \underline{F(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma^\mu u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma\tilde{x} u(p_1, S_1)\right] \\
&+ i\int Dz e^{-i\tilde{x}p_z/2} \underline{G(z_1, z_2)} \left[\overline{u}(p_2, S_2)\sigma^{\mu\nu} p_{-\nu} u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\sigma^{\alpha\beta}\tilde{x}_\alpha p_{-\beta} u(p_1, S_1)\right]
\end{aligned}
$$

∞ $p_- \to 0$
FORWARD.

$p_z = p_1 z_1 + p_2 z_2$.

pol.

$$
\begin{aligned}
e^2\left\langle p_2, S_2 \middle| O_5^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1 \right\rangle
&= i\int Dz e^{-i\tilde{x}p_z/2} \underline{F_5(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma_5\gamma^\mu u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma_5\gamma\tilde{x} u(p_1, S_1)\right] \\
&+ i\int Dz e^{-i\tilde{x}p_z/2} \underline{G_5(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma_5\sigma^{\mu\nu} p_{-\nu} u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma_5\sigma^{\alpha\beta}\tilde{x}_\alpha p_{-\beta} u(p_1, S_1)\right],
\end{aligned}
$$

SCALAR

$$\boxed{H_{(5)}(z_1, z_2) = \int_0^1 \frac{d\lambda}{\lambda^2} h_{(5)}\left(\frac{z_1}{\lambda}, \frac{z_2}{\lambda}\right)}$$

ARGUMENTS: MOMENTUM FRACTIONS

SYMMETRY PROP:

$$\begin{aligned} H(z_1, z_2) &= -H(-z_1, -z_2) \\ H_5(z_1, z_2) &= +H_5(-z_1, -z_2) \end{aligned}$$

$$\begin{aligned} T_{\mu\nu}(p_+, p_-, q) &= i \int d^4x\, e^{iqx} \langle p_2, S_2 | T(J_\mu(x/2) J_\nu(-x/2)) | p_1, S_1 \rangle \\ &= \int d^4x\, e^{iqx} \left\{ -\frac{\tilde{x}^\lambda}{i\pi^2 (x^2 - i\varepsilon)^2} \left[S_{\alpha\mu\lambda\nu} \left\langle p_2 \middle| O^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1 \right\rangle \right.\right. \\ &\qquad \left.\left. + i\varepsilon_{\mu\lambda\nu\alpha} \left\langle p_2 \middle| O_5^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1 \right\rangle \right] \right\}. \end{aligned}$$

PERT. NON. PERT.

$$\begin{aligned} T_{\mu\nu}(q, p_+, p_-) &= -2 \int Dz \frac{1}{Q^2 + i\varepsilon} \Big\{ \overline{u}(p_2, S_2) \Gamma^F_{\mu\nu}(q, p_+, p_-) u(p_1, S_1)\ F(z_+, z_-) \\ &\quad + \overline{u}(p_2, S_2) \Gamma^{F5}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1)\ F_5(z_+, z_-) \\ &\quad + \overline{u}(p_2, S_2) \Gamma^{G}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1)\ G(z_+, z_-) \\ &\quad + \overline{u}(p_2, S_2) \Gamma^{G5}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1)\ G_5(z_+, z_-) \Big\} \end{aligned}$$

DIRAC - TYPE

PAULI - TYPE

$$z_\pm = \tfrac{1}{2}(z_2 \pm z_1)$$

$$\begin{aligned} Dz = dz_+\, dz_-\, &\theta(1 + z_+ + z_-)\, \theta(1 + z_+ - z_-) \\ &\cdot \theta(1 - z_+ + z_-)\, \theta(1 - z_+ - z_-) \end{aligned}$$

STRUCTURE OF THE $T_{\mu\nu}$ TENSORS:

$$
\begin{aligned}
\Gamma^F_{\mu\nu}(q,p_z) &= \Big[Q_\mu\gamma_\nu + Q_\nu\gamma_\mu - g_{\mu\nu}\gamma_\alpha Q^\alpha\Big] \\
&\quad -\frac{1}{2}\Big[p_{z\mu}\gamma_\nu + p_{z\nu}\gamma_\mu - g_{\mu\nu}\gamma_\alpha p_z^\alpha\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\gamma_\alpha Q^\alpha\Big[Q_\nu p_{z\mu} + Q_\mu p_{z\nu} - g_{\mu\nu} Q.p_z\Big] \\
&\simeq \Big[q_\mu\gamma_\nu + q_\nu\gamma_\mu - g_{\mu\nu}\gamma_\alpha q^\alpha\Big] - \Big[p_{z\mu}\gamma_\nu + p_{z\nu}\gamma_\mu\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\gamma_\alpha q^\alpha\Big[-p_{z\nu}p_{z\mu} + q_\nu p_{z\mu} + q_\mu p_{z\nu} - g_{\mu\nu} q.p_z\Big] \\
\Gamma^{F5}_{\mu\nu}(q,p_z) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[Q^\lambda\gamma^\sigma - \frac{1}{2}p_z^\sigma\gamma^\lambda + \frac{1}{Q^2+i\varepsilon}Q^\lambda p_z^\sigma\gamma_\alpha Q^\alpha\right] \\
&\simeq i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\gamma^\sigma + \frac{1}{Q^2+i\varepsilon}q^\lambda p_z^\sigma\gamma_\alpha q^\alpha\right] \\
\Gamma^G_{\mu\nu}(q,p_z) &= \Big[Q_\mu\sigma_{\nu\alpha}p_-^\alpha + Q_\nu\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\alpha\beta}p_-^\beta Q^\alpha\Big] \\
&\quad -\frac{1}{2}\Big[p_{z\mu}\sigma_{\nu\alpha}p_-^\alpha + p_{z\nu}\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha p_z^\beta\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\sigma_{\beta\alpha}p_-^\alpha Q^\beta\Big[Q_\nu p_{z\mu} + Q_\mu p_{z\nu} - g_{\mu\nu} Q.p_z\Big] \\
&\simeq \Big[q_\mu\sigma_{\nu\alpha}p_-^\alpha + q_\nu\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha q^\beta\Big] - \Big[p_{z\mu}\sigma_{\nu\alpha}p_-^\alpha + p_{z\nu}\sigma_{\mu\alpha}p_-^\alpha\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\sigma_{\beta\alpha}p_-^\alpha q^\beta\Big[-p_{z\mu}p_{z\nu} + q_\nu p_{z\mu} + q_\mu p_{z\nu} - g_{\mu\nu} q.p_z\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{G5}_{\mu\nu}(q,p_z) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[Q^\lambda\sigma^{\sigma\alpha}p_{-\alpha} - \frac{1}{2}p_z^\sigma\sigma^{\lambda\alpha}p_{-\alpha} + \frac{1}{Q^2+i\varepsilon}Q^\lambda p_z^\sigma\sigma^{\alpha\beta}Q_\alpha p_{-\beta}\right] \\
&\simeq i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\sigma^{\sigma\alpha}p_{-\alpha} + \frac{1}{Q^2+i\varepsilon}q^\lambda p_z^\sigma\sigma^{\alpha\beta}q_\alpha p_{-\beta}\right],
\end{aligned}
$$

$$Q = q - \frac{p_z}{2}\,.$$

$\simeq$: TERMS WHICH REMAIN BETWEEN Bi-SPINORS (i.e. $\neq 0$)

2.3 Lorentz Structure

$$
\begin{aligned}
T_{\mu\nu}(q,p_+,p_-) &= -2\overline{u}(p_2,S_2)\Big[\Gamma^F_{\mu\nu}(q,p_+,p_-)+\Gamma^{F5}_{\mu\nu}(q,p_+,p_-) \\
&\qquad +\Gamma^G_{\mu\nu}(q,p_+,p_-)+\Gamma^{G5}_{\mu\nu}(q,p_+,p_-)\Big]u(p_1,S_1)\ ,
\end{aligned}
$$

REORGANIZE: REMOVE THE z+- INTEGRALS FROM THE TENSORS

$$
\begin{aligned}
\Gamma^F_{\mu\nu}(q,p_+,p_-) &= \Big[q_\mu\gamma_\nu+q_\nu\gamma_\mu-g_{\mu\nu}\gamma_\alpha q^\alpha\Big]\underline{F_1(\xi,\eta)} \\
&\quad -\gamma_\mu F_{1,\nu}(\xi,\eta)-\gamma_\nu F_{1,\mu}(\xi,\eta)+\gamma_\alpha q^\alpha \underline{F_{2,\mu\nu}(\xi,\eta)} \\
\Gamma^{F5}_{\mu\nu}(q,p_+,p_-) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\Big[q^\lambda\gamma^\sigma \underline{F_1^5(\xi,\eta)}+q^\lambda\gamma_\alpha q^\alpha \underline{F_2^{\sigma,5}(\xi,\eta)}\Big] \\
\Gamma^G_{\mu\nu}(q,p_+,p_-) &= \Big[q_\mu\sigma_{\nu\alpha}p_-^\alpha+q_\nu\sigma_{\mu\alpha}p_-^\alpha-g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha q^\beta\Big]\underline{G_1(\xi,\eta)} \\
&\quad -\sigma_{\mu\alpha}p_-^\alpha \underline{G_{1,\nu}(\xi,\eta)}-\sigma_{\nu\alpha}p_-^\alpha \underline{G_{1,\mu}}(\xi,\eta)p_-^\alpha+\sigma_{\beta\alpha}p_-^\alpha q^\beta \underline{G_{2,\mu\nu}}(\xi,\eta) \\
\Gamma^{G5}_{\mu\nu}(q,p_+,p_-) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\Big[q^\lambda\sigma^{\sigma\alpha}p_{-\alpha}\underline{G_1^5(\xi,\eta)}+q^\lambda\sigma^{\alpha\beta}q_\alpha p_{-\beta}\underline{G_2^{\sigma,5}(\xi,\eta)}\Big]\ .
\end{aligned}
$$

$$Q = q - \frac{p_z}{2}.$$

$$
\begin{aligned}
H_1(\xi,\eta) &= \int Dz\frac{1}{Q^2+i\varepsilon}H(z_+,z_-) \\
H_k^\sigma(\xi,\eta) &= \int Dz\frac{p_+^\sigma z_+ + p_-^\sigma z_-}{(Q^2+i\varepsilon)^k}H(z_+,z_-) = \int Dz\frac{p_+^\sigma t+\pi_\sigma z_-}{(Q^2+i\varepsilon)^k}H(z_+,z_-) \\
H_{2,\mu\nu}(\xi,\eta) &= \int Dz\frac{1}{(Q^2+i\varepsilon)^2}\left[-p_{z\mu}p_{z\nu}+q_\nu p_{z\mu}+q_\mu p_{z\nu}-g_{\mu\nu}q.p_z\right]H(z_+,z_-) \\
&= \int Dz\frac{1}{(Q^2+i\varepsilon)^2}\Big[-p_{+\mu}p_{+\nu}t^2+(q_\nu p_{+\mu}+q_\mu p_{+\nu})t-g_{\mu\nu}q.p_z \\
&\qquad -\pi_\mu\pi_\nu z_-^2+(q_\nu\pi_\mu+q_\mu\pi_\nu)z_-+(p_{+\nu}\pi_\mu+p_{+\mu}\pi_\nu)tz_-\Big] \\
&\qquad \times H(z_+,z_-)\ ,
\end{aligned}
$$

OUTER VARIABLES OF THE PROCESS

$$
\begin{aligned}
\boxed{t = z_+ + \eta z_-} \\
\pi_\sigma = p_{-\sigma} - \eta p_{+\sigma}
\end{aligned}
$$

5 Kinematic Relations

KINEMATIC FRAME: BREIT FRAME

$$\begin{aligned} p_+ &= p_1 + p_2 = (2E_p;\ \vec{0}) \\ -p_- &= p_1 - p_2 = (0;\ 2\vec{p}) = (0;\ 0, 0, 2p_3) \\ q &= \frac{1}{2}(q_1 + q_2) = (q_0;\ q_1, 0, q_3)\ . \end{aligned}$$

$\nu \equiv q p_+$ (LARGE)

$$\begin{aligned} q_1 \cdot q_1 &= -\nu(\xi - \eta) \\ q_2 \cdot q_2 &= -\nu(\xi + \eta) \\ q \cdot p_+ &= \nu \\ q \cdot p_- &= \eta\nu \\ q \cdot q &= -\xi\nu \\ q \cdot p_z &= q^2 - Q^2 = (z_+ + z_- \eta)\nu \equiv t\ \nu \\ p_+^2 &\approx p_-^2 \approx p_+ p_- \approx 0\ . \end{aligned}$$

STUDY: HELICITY PROJECTIONS: γ_1^*, γ_2^*.

$$\mathrm{T}_{kl} = \varepsilon^{\mu}_{2,k} T_{\mu\nu} \varepsilon^{\nu}_{1,l}, \quad k, l \in \{0, 1, 2, 3\}$$

$$\begin{aligned} n_0 &= (1;\ 0, 0, 0) \\ n_2 &= (0;\ 0, 1, 0)\ . \end{aligned}$$

$\propto p_+$

NEW.

HELICITY-VECTORS:

$$
\begin{aligned}
\varepsilon^{(1)}_{0\mu} &= \frac{q_{1\mu}}{\sqrt{|q_1^2|}} = \frac{q_{1\mu}}{\nu^{1/2}}\frac{1}{\sqrt{|\xi-\eta|}} \\
\varepsilon^{(2)}_{0\mu} &= \frac{q_{2\mu}}{\sqrt{|q_2^2|}} = \frac{q_{2\mu}}{\nu^{1/2}}\frac{1}{\sqrt{|\xi+\eta|}} \\
\varepsilon^{(i)}_{1\mu} &= n_{2\mu} \\
\varepsilon^{(i)}_{2\mu} &= \frac{1}{N_{2i}}\varepsilon_{\mu\alpha\beta\gamma}n_0^\alpha n_2^\beta q_i^\gamma \\
\varepsilon^{(i)}_{3\mu} &= \frac{1}{N_{3i}}\left[q_{i\mu}q_i.n_0 - n_{0\mu}q_i.q_i\right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
N_{21} &= \frac{\nu}{\mu}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi-\eta)\right|} \\
N_{22} &= \frac{\nu}{\mu}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi+\eta)\right|}
\end{aligned}
$$

$$
\begin{aligned}
N_{31} &= \frac{\nu^{3/2}}{\mu}\sqrt{|\xi-\eta|}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi-\eta)\right|} \\
N_{32} &= \frac{\nu^{3/2}}{\mu}\sqrt{|\xi+\eta|}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi+\eta)\right|}
\end{aligned}
$$

$\mu^2 = p_T^2$.

NORMALIZATION:

$$\varepsilon^{(i)}_{k\mu}.\varepsilon^{(i)\mu}_{l} = s_k\delta_{kl} \ ,$$

$s_k = -1 \qquad k = 0, 1, 2$

$s_k = +1 \qquad k = 3$

CONSIDER: $\mu^2 \ll \nu$:

$$
\begin{aligned}
\varepsilon^{(1(2))}_{0\rho} &= \frac{1}{\sqrt{|\xi|}}\left[\frac{q_\rho}{\nu^{1/2}} \pm \frac{p_{-\rho}}{2\nu^{1/2}}\right]\frac{1}{\sqrt{|1 \mp \eta/\xi|}} \\
\varepsilon^{(1(2))}_{1\rho} &= n_{2\rho} \\
\varepsilon^{(1(2))}_{2\rho} &= \frac{\mu}{\nu}\left|1 - \frac{\mu^2}{2\nu}(\xi \mp \eta)\right| \varepsilon_{\rho\alpha\beta\gamma} n_0^\alpha n_2^\beta \left(q^\gamma \pm \frac{1}{2}p_-^\gamma\right) \\
\varepsilon^{(1(2))}_{3\rho} &= \frac{1}{\nu^{1/2}}\frac{1}{\sqrt{|\xi|}}\left|1 - \frac{\mu^2}{2\nu}(\xi \mp \eta)\right| \left[q_\rho \pm \frac{1}{2}p_{-\rho} + \mu n_{0\rho}(\xi \mp \eta)\right]\frac{1}{\sqrt{|1 \mp \eta/\xi|}}
\end{aligned}
$$

$\varepsilon^{(1,2)}_{3\rho} \rightarrow \varepsilon^{(1,2)}_{0\rho}$. : EFFECTIVELY 2 HELIC. VECTORS.

q_2 : LIGHTLIKE.

$$
\begin{aligned}
\varepsilon^{(2)}_{0\mu} &= \frac{1}{\sqrt{2}q_0^{(2)}} q_\mu = \frac{1}{\sqrt{2}q_0^{(2)}}(q_0,\ \vec{q}_2) \\
\varepsilon^{(2)}_{1\mu} &= n_{2\mu} \\
\varepsilon^{(2)}_{2\mu} &= \frac{1}{q_0^{(2)}} \varepsilon_{\mu\alpha\beta\gamma} n_0^\alpha n_2^\beta q_2^\gamma\ ,
\end{aligned}
$$

$$
q_0^{(2)} = \frac{\nu}{\mu}\ .
$$

$$
\tilde{\varepsilon}^{(2)}_{0\mu} = \frac{1}{\sqrt{2}q_0^{(2)}}(q_0,\ -\vec{q}_2)
$$

$$
\tilde{\varepsilon}^{(2)}_{0\mu} + \varepsilon^{(2)}_{0\mu} = \sqrt{2} n_{0\mu}\ .
$$

6 Current Conservation

$$\partial_\mu^x J^\mu(x) = 0$$

$$\begin{aligned} T_{\mu\nu}(p_+, p_-, q) &= i \int d_x^4 e^{-iq_2 x} \langle p_2, S_2 \| RT(J_\mu(0) J_\nu(x) \| p_1, S_1 \rangle \\ &= i \int d_x^4 e^{-iq_1 x} \langle p_2, S_2 \| RT(J_\mu(-x) J_\nu(0) \| p_1, S_1 \rangle \end{aligned}$$

$$q_2^\mu T_{\mu\nu} = T_{\mu\nu} q_1^\nu = 0 \; .$$

$$\begin{aligned} \overline{u}(p_2, S_2)\gamma_\mu q^\mu u(p_1, S_1) &\propto \nu \\ \varepsilon_{\alpha,\beta,\gamma,\delta}\, p_\pm^\gamma q^\delta &\propto \nu \end{aligned}$$

$$\Longrightarrow \int Dz\, H(z_+, z_-) = \int_{-1}^{+1} dz_+ \int_{-1+|z_+|}^{+1-|z_+|} H(z_+, z_-) = 0$$ CRUCIAL.

EXTRACT LOWEST TWIST TERMS.

CURRENT CONSERVATION TWIST BY TWIST.

HELICITY PROJ. WITH ONE "0":

$$\mathrm{T}^F_{00} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{\sqrt{|\xi^2-\eta^2|}}\int Dz\, F(z_+,z_-) \tag{4.4}$$

$$\mathrm{T}^F_{01},\mathrm{T}^F_{10},\mathrm{T}^F_{02},\mathrm{T}^F_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.5}$$

$$\mathrm{T}^F_{03} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{|\xi-\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\, F(z_+,z_-) \tag{4.6}$$

$$\mathrm{T}^F_{30} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{|\xi+\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\, F(z_+,z_-) \tag{4.7}$$

$$\mathrm{T}^{F5}_{00} = 0 \tag{4.8}$$

$$\mathrm{T}^{F5}_{01},\mathrm{T}^{F5}_{10},\mathrm{T}^{F5}_{02},\mathrm{T}^{F5}_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.9}$$

$$\mathrm{T}^{F5}_{03},\mathrm{T}^{F5}_{30} = O\left(\frac{1}{\nu}\right) \tag{4.10}$$

$$\mathrm{T}^G_{00} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{\sqrt{|\xi^2-\eta^2|}}\int Dz\, G(z_+,z_-) \tag{4.11}$$

$$\mathrm{T}^G_{01},\mathrm{T}^G_{10},\mathrm{T}^G_{02},\mathrm{T}^G_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.12}$$

$$\mathrm{T}^G_{03} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\sigma_{\beta\alpha}p_-^\alpha q^\beta u(p_1,S_1)\frac{1}{|\xi-\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\, G(z_+,z_-) \tag{4.13}$$

$$\mathrm{T}^G_{30} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\sigma_{\beta\alpha}p_-^\alpha q^\beta u(p_1,S_1)\frac{1}{|\xi+\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\, G(z_+,z_-) \tag{4.14}$$

$$\mathrm{T}^{G5}_{00} = 0 \tag{4.15}$$

$$\mathrm{T}^{G5}_{01},\mathrm{T}^{G5}_{10},\mathrm{T}^{G5}_{02},\mathrm{T}^{G5}_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.16}$$

$$\mathrm{T}^{G5}_{03},\mathrm{T}^{G5}_{30} = O\left(\frac{1}{\nu}\right). \tag{4.17}$$

LEADING ORDER: ν^0

ALL PROJECTIONS ARE EITHER $\frac{1}{\nu^{\frac{1}{2}+k}}$, $k \geq 0$

OR VANISH EXACTLY.

$\longrightarrow$ PROBLEM MOVED TO HIGHER TWIST OPERATORS! TO BE STUDIED THERE.

7 The Helicity Projections of the Compton Amplitude

UNPOLARIZED: DIRAC

$$
\begin{aligned}
\mathsf{T}^F_{11} &= 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta)+\varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right]\\
\mathsf{T}^F_{22} &= 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta)+\varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right]\\
\mathsf{T}^F_{kl} &\propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections} \quad k,l\,\epsilon\,\{1,2,3\} \quad \text{and} \quad n\geq 0\,,
\end{aligned}
$$

POLARIZED: DIRAC

$$
\mathsf{T}^{F5}_{12} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon^{(2)}_{1\mu}\varepsilon^{(1)}_{2\nu}\int Dz\frac{q_\lambda}{Q^2+i\varepsilon}\left[S_{21,\sigma}+\frac{q.S_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right]F_5(z_+,z_-)\,.
$$

$$
\mathsf{T}^{F5}_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections} \quad k,l\,\epsilon\,\{1,2,3\} \quad \text{and} \quad n\geq 0\,.
$$

$$
\varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2\mu\nu}(\xi,\eta)=\varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2\mu\nu}(\xi,\eta)=\int Dz\frac{q.p_z}{(Q^2+i\varepsilon)^2}F(z_+,z_-)
$$

$$
\boxed{\begin{aligned}\mathsf{T}^F_{11} &= \mathsf{T}^F_{22}\,.\\ \mathsf{T}^{F5}_{12} &= -\mathsf{T}^{F5}_{21}\end{aligned}}
$$

NON-FWD.
SPIN VECTOR

$$
S^\sigma_{21} := -\frac{1}{2}\overline{u}(p_2,S_2)\gamma_5\gamma^\sigma u(p_1,S_1)\,.
$$

STRUCTURE IN FORW. CASE:

$$\propto q_\lambda S_{\sigma,21} \quad \rightarrow \quad g_1(x_B) + g_2(x_B)$$
$$\propto q_\lambda p_{z\sigma} \quad \rightarrow \quad g_2(x_B)\ .$$

UNPOLARIZED: PAULI

$$\mathsf{T}^G_{11} = 2\overline{u}(p_2, S_2)\sigma_{\alpha\beta}q^\alpha p_-^\beta u(p_1, S_1)\left[G_1(\xi,\eta) + \varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}G_{2,\mu\nu}(\xi,\eta)\right]$$
$$\mathsf{T}^G_{22} = 2\overline{u}(p_2, S_2)\sigma_{\alpha\beta}q^\alpha p_-^\beta u(p_1, S_1)\left[G_1(\xi,\eta) + \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}G_{2,\mu\nu}(\xi,\eta)\right]$$
$$\mathsf{T}^G_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } k,l \in \{1,2,3\} \text{ and } n \geq 0\ ,$$

POLARIZED: PAULI

$$\mathsf{T}^{G5}_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } k,l \in \{1,2,3\} \text{ and } n \geq 0$$

$$\mathsf{T}^{G5}_{12} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon_1^{(2)\mu}\varepsilon_2^{(1)\nu}\int Dz\frac{q_\lambda}{Q^2+i\varepsilon}\left[\Sigma_{21,\sigma} + \frac{q.\Sigma_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right]G_5(z_+,z_-)\ ,$$

$$\Sigma^\sigma_{21} := -\frac{1}{2}\overline{u}(p_2,S_2)\gamma_5\sigma^{\sigma\alpha}p_{-\alpha}u(p_1,S_1)$$

'TENSORIAL'
NON-FORW.
SPIN VECTOR

$$\boxed{\begin{gathered}\mathsf{T}^G_{11} = \mathsf{T}^G_{22}\ . \\ \mathsf{T}^{G5}_{12} = -\mathsf{T}^{G5}_{21}\end{gathered}}$$

8. Integral Relations

8.1 Unpolarized Contributions

$$\mathsf{T}_{11}^{F,G} = \mathsf{T}_{22}^{F,G}$$

$$F_2(x_B) = 2xF_1(x_B) \equiv \sum_q e_q^2 x \left[q(x_B) + \overline{q}(x_B)\right]$$

FORWARD

PARTON DENSITIES

BUT:

$$\begin{aligned}
\mathsf{H}_1(\xi,\eta) &= \int Dz \frac{\nu}{Q^2 + i\varepsilon} H(z_+, z_-) = -\int Dz \frac{H(z_+, z_-)}{\xi + t - i\varepsilon} \\
\mathsf{H}_2(\xi,\eta) &= \int Dz \frac{\nu\ q.p_z}{(Q^2 + i\varepsilon)^2} H(z_+, z_-) = \int Dz \frac{tH(z_+, z_-)}{(\xi + t - i\varepsilon)^2} \,.
\end{aligned}$$

$t = t(z_+, z_-)$

$$\widehat{H}(t,\eta) = \int_{z_-^{\min}}^{z_-^{\max}} dz_- H(t-\eta z_-, z_-) = \int_0^1 \frac{d\lambda}{\lambda^2} \int_{z_-^{\min}}^{z_-^{\max}} dz_- h\left(\frac{t}{\lambda} - \eta\frac{z_-}{\lambda}, \frac{z_-}{\lambda}\right)$$
$$= \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta) \ ,$$

TERMS OF THIS KIND SHOULD CANCEL.

$$z_-^{\min,\max} = \frac{t \pm 1}{\eta \pm 1} \ ,$$

$$\hat{h}(z,\eta) = \int_{\rho_{\min}}^{\rho_{\max}} d\rho \, h(z - \eta\rho, \rho) \ ,$$

$H_2(\xi,\eta) =$

$$\int_{-1}^{+1} dt \frac{t}{(\xi + t - i\varepsilon)^2} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta) = \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta) - \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \hat{h}(t,\eta) \ ,$$

$$\mathsf{H}_2(\xi,\eta) = -\mathsf{H}_1(\xi,\eta) - \int_{-1}^{+1} dt \frac{\hat{h}(t,\eta)}{\xi + t - i\varepsilon} \ .$$

$$\mathsf{T}^H_{11(22)}(\xi,\eta) \propto - \int_{-1}^{+1} dt \frac{\hat{h}(t,\eta)}{\xi + t - i\varepsilon} = -\mathsf{P} \int_{-1}^{+1} dt \frac{\hat{h}(t,\eta)}{\xi + t} - i\pi \hat{h}(\xi,\eta)$$

PARTONIC INTERPRET.

TENSOR: UNPOL.

$$A^{\mu\nu} = -2\frac{q.P_{21}}{\nu}\left[g^{\mu\nu} - \frac{q^\mu p_+^\nu + q^\nu p_+^\mu}{q.p_+}\right]\int_{-1}^{+1} dt \frac{\mathsf{F}_1(t,\eta)}{\xi + t - i\varepsilon}$$

$$+\frac{2}{\nu}\left[q^\mu\left(P_{21}^\nu - p_+^\nu \frac{q.P_{21}}{\nu}\right) + q^\nu\left(P_{21}^\mu - p_+^\mu\frac{q.P_{21}}{\nu}\right)\right]\int_{-1}^{+1} dt \frac{\hat{\mathsf{H}}(t,\eta)}{\xi + t - i\varepsilon},$$

$$-\frac{q.P_{21}}{\nu^2}p_+^\mu p_+^\nu \int_{-1}^{+1} dt \frac{\mathsf{F}_2(t,\eta)}{\xi + t - i\varepsilon}$$

$$-\frac{2}{\nu}\left[p_+^\mu\left(p_{21}^\nu - p_+^\nu\frac{q.P_{21}}{\nu}\right) + p_+^\nu\left(p_{21}^\mu - p_+^\mu\frac{q.P_{21}}{\nu}\right)\right]\int_{-1}^{+1} dt \frac{t\,\hat{\mathsf{H}}(t,\eta)}{\xi + t - i\varepsilon}$$

$$\mathsf{F}_1(t,\eta) = \hat{h}(t,\eta)$$
$$\mathsf{F}_2(t,\eta) = 2t\,\hat{h}(t,\eta)\,.$$

Def.

$$\hat{\mathsf{H}}(t,\eta) = \int_t^{\mathrm{sign}(t)} \frac{dz}{z}\hat{h}(z,\eta)$$
$$\tilde{\mathsf{H}}_k(t,\eta) = \int_t^{\mathrm{sign}(t)} \frac{dz}{z}\tilde{h}_k(z,t,\eta)\,,$$

$$\tilde{h}_k(z,t,\eta) = \left(\frac{t}{z}\right)^k \int_{\rho_{\min}}^{\rho_{\max}} d\rho\, \rho^k\; h(z-\eta\rho,\rho)\,.$$

$$P_{21}^\sigma := \overline{u}(p_2,S_2)\gamma^\sigma u(p_1,S_1),$$

$$\Pi^\mu = P_{21}^\mu - p_+^\mu \frac{q.P_{21}}{\nu}$$

$$q_\nu \Pi^\nu,\; p_{\pm\nu}\Pi^\nu,\; n_{2\nu}\Pi^\nu = O(\mu^2)$$

$$\boxed{\mathsf{F}_2(t,\eta) = 2t\,\mathsf{F}_1(t,\eta)}$$

NON FORW. CALLEN-GROSS REL.

8.2 Polarized Contributions

$$\mathsf{T}_{12}^{H5} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon_{1\mu}^{(2)}\varepsilon_{2\nu}^{(1)}B_{\lambda\sigma}$$

$$B_{\lambda\sigma} = \int Dz \frac{q_\lambda}{Q^2 + i\varepsilon}\left[S_{21,\sigma}^{H} + \frac{q.S_{21}^{H}}{Q^2 + i\varepsilon}p_{z\sigma}\right] H_5(z_+, z_-),$$

$S_{21}^{H} = S_{21}(\Sigma_{21})$ for $H = F(G)$. It may be rewritten as

$$B_{\lambda\sigma} = -\frac{1}{\nu}\int Dz \frac{q_\lambda}{\xi + t - i\varepsilon}\left[S_{21,\sigma}^{H} - \frac{1}{\nu}\frac{t\; q.S_{21}^{H}}{\xi + t - i\varepsilon}p_{+\sigma} + \frac{1}{\nu}\frac{q.S_{21}^{H}}{\xi + t - i\varepsilon}z_-\pi_\sigma\right] H_5(z_+, z_-$$

$$\begin{aligned}
B_{\lambda\sigma} \quad = \quad & -\frac{1}{\nu}q_\lambda S_{21,\sigma}^{H}\int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon}\int_t^{\mathrm{sign}(t)} \frac{dz}{z}\hat{h}_5(z,\eta) \\
& -\frac{1}{\nu^2}q_\lambda p_{+\sigma}\; q.S_{21}^{H}\int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon}\left[\hat{h}_5(t,\eta) - \int_t^{\mathrm{sign}(t)} \frac{dz}{z}\hat{h}_5(z,\eta)\right] \\
& -\frac{1}{\nu^2}q_\lambda \pi_\sigma\; q.S_{21}^{H}\int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon}\int_t^{\mathrm{sign}(t)} \frac{dz}{z}\widetilde{h}_5(z,t,\eta)\ .
\end{aligned}$$

$$\widetilde{h}_5(z,t,\eta) = \left(\frac{t}{z}\right)\int_{\rho_{\min}}^{\rho_{\max}} d\rho\; \rho h\left(z - \eta\rho, \rho\right)$$

$q_\mu \pi^\mu$, $p_{\pm\mu}\pi^\mu$, $n_{2\mu}\pi^\mu \propto O(\mu^2)$.

$$B_{\lambda\sigma} = -\frac{1}{\nu} q_\lambda \int_{-1}^{+1} \frac{dt}{\xi + t - i\varepsilon}$$
$$\times \left\{ S^H_{21,\sigma} \left[\mathsf{G}_1(t,\eta) + \mathsf{G}_2(t,\eta) \right] + \frac{1}{\nu} p_{+\sigma} \; q.S^H_{21} \mathsf{G}_2(t,\eta) + \frac{1}{\nu} \pi_\sigma \; q.S^H_{21} \mathsf{G}_3(t,\eta) \right\}$$

$$\boxed{\begin{aligned} \mathsf{G}_1(t,\eta) &:= \hat{h}_5(t,\eta) \\ \mathsf{G}_2(t,\eta) &= -\mathsf{G}_1(t,\eta) + \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \mathsf{G}_1(z,\eta) \end{aligned}}$$
$$\mathsf{G}_3(t,\eta) := \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \tilde{h}_5(z,t,\eta) \, .$$

NON - FORW.
WANDZURA -
WILCZEK
REL.

THE NON- FORWARD (INTEGRAL) RELATIONS HOLD ALREADY AT THE AMPLITUDE LEVEL !

6.3 Forward Scattering

$$W_{\mu\nu} = \frac{1}{2\pi} \mathrm{Im}\, T_{\mu\nu} \ .$$

PARTON CONTENT:

$$\begin{aligned} \mathsf{F}_1(t,0) &= \sum_q e_q^2 \left[q(t)\theta(t) - \overline{q}(-t)\theta(-t) \right] \\ \mathsf{G}_1(t,0) &= \sum_q e_q^2 \left[\Delta q(t)\theta(t) + \Delta\overline{q}(-t)\theta(-t) \right] \ . \end{aligned}$$

$$q_\mu A^{\mu\nu} = p^\nu \left[\int_{-1}^{+1} dt\, \frac{2\xi \mathsf{F}_1(t,0) - \mathsf{F}_2(t,0)}{\xi - t - i\varepsilon} - \int_{-1}^{+1} dt\, \frac{2\xi \mathsf{F}_1(t,0) + \mathsf{F}_2(t,0)}{\xi + t - i\varepsilon} \right] = 0 \ .$$

$$\pm 2\xi \mathsf{F}_1(\pm\xi, 0) = \mathsf{F}_2(\pm\xi, 0) \ .$$

$$\begin{aligned} F_1(x_B) &= \frac{1}{2}\left[\mathsf{F}_1(\xi,0) - \mathsf{F}_1(-\xi,0)\right] = \frac{1}{2}\sum_q e_q^2 \left[q(x_B) + \overline{q}(x_B)\right] \\ F_2(x_B) &= \mathsf{F}_2(\xi,0) + \mathsf{F}_2(-\xi,0) \ , \end{aligned}$$

CALLAN-GROSS:

$$F_2(x_B) = 2x_B F_1(x_B) \ .$$

$$
\begin{aligned}
B_{\lambda\sigma} &= -\frac{1}{2\nu} q_\lambda S^H_{21,\sigma} \int_{-1}^{+1} dt \left[\frac{\mathrm{G}_1(t,0) + \mathrm{G}_2(t,0)}{\xi + t - i\varepsilon} + \frac{\mathrm{G}_1(t,0) + \mathrm{G}_2(t,0)}{\xi - t - i\varepsilon} \right] \\
&\quad - \frac{1}{2\nu^2} q_\lambda p_{+\sigma}\ q.S^H_{21} \int_{-1}^{+1} dt \left[\frac{\mathrm{G}_2(t,0)}{\xi + t - i\varepsilon} + \frac{\mathrm{G}_2(t,0)}{\xi - t - i\varepsilon} \right] .
\end{aligned}
$$

$$
\begin{aligned}
g_1(x_B) &= \frac{1}{2} \left[\mathrm{G}_1(\xi,0) + \mathrm{G}_1(-\xi,0) \right] = \frac{1}{2} \sum_q e_q^2 \left[\Delta q(x_B) + \Delta \overline{q}(x_B) \right] \\
g_2(x_B) &= -g_1(x_B) + \int_{x_B}^{1} \frac{dz}{z} g_1(z) .
\end{aligned}
$$

WANDZURA - WILCZEK.

9. Diffractive Scattering: Operator Approach

Diffractice Scattering: Important Process at HERA

1. Why is the Q^2 Dependence of $F_2^{\rm DIS}(x,Q^2)$ and $F_2^{\rm Diffr}(x,Q^2)$ about the same?
2. Why is their Ratio about $1/8 ... 1/10$?

- Question 1 should be studied within Perturbative QCD.
- One should formulate the problem such, that Question 2 can later be studied within Lattice Gauge Theory.
- Can one formulate the problem in a way that Higher Twists find their place?

$\rightarrow$ Operator Approach !

ABRAMOWICZ, DAINTON '96

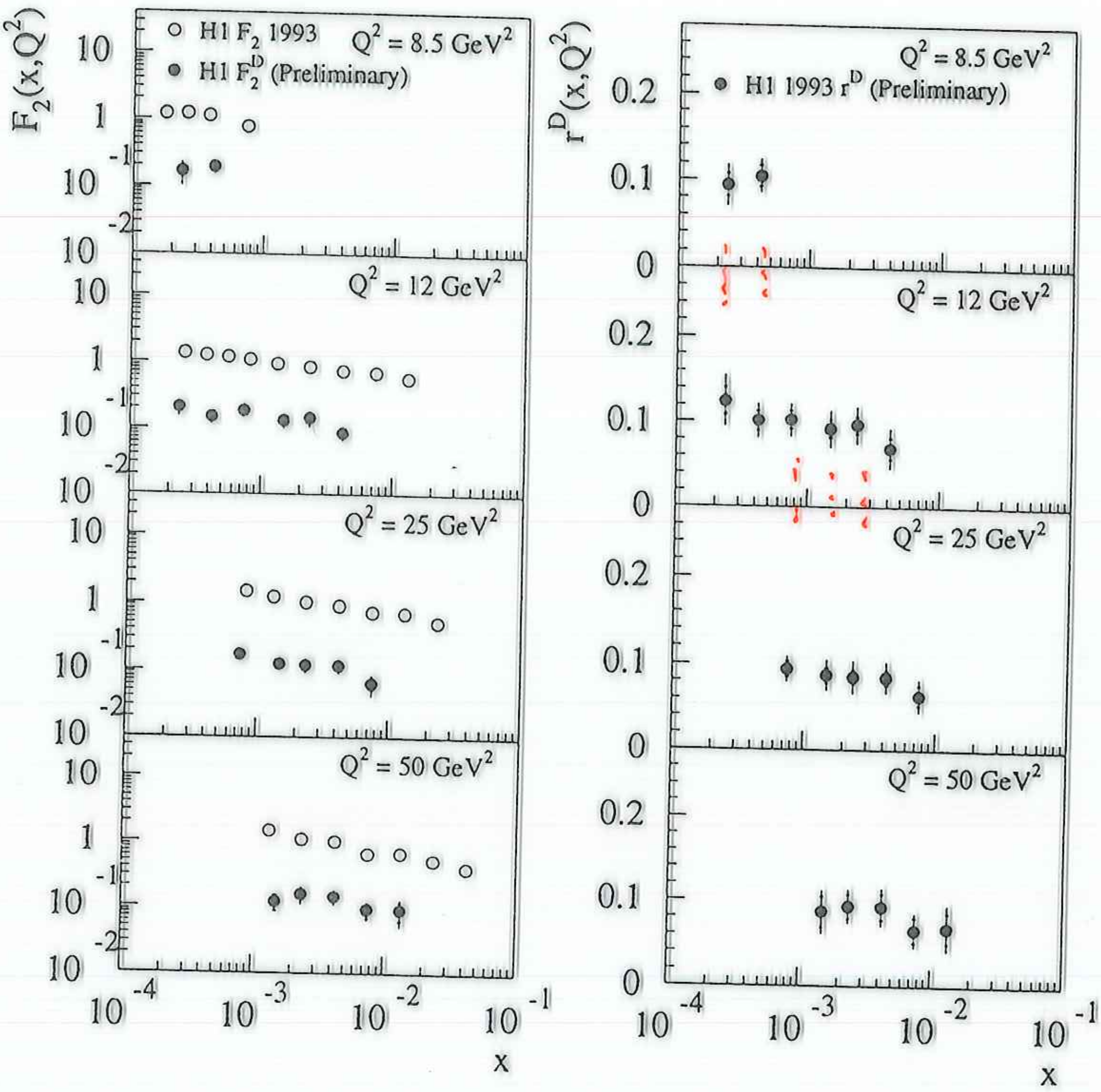
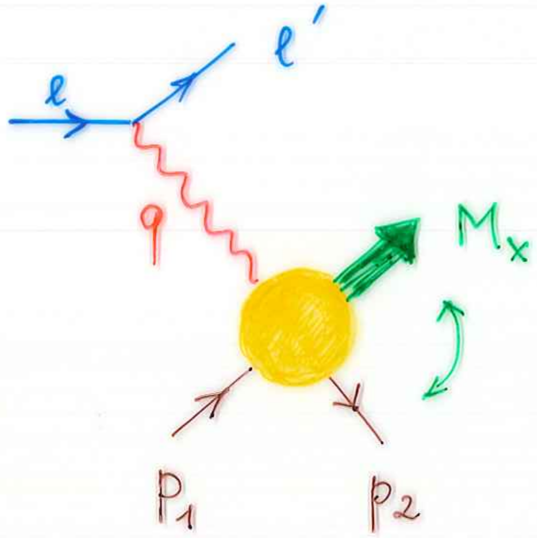


Figure 3. The structure function $F_2^D(x, Q^2)$ and the ratio $r^D = F_2^D(x, Q^2)/F_2(x, Q^2)$ for $x_{\mathbb{P}} < 0.05$. The result is for deep-inelastic diffraction in which the proton does not dissociate. Approximately one third of deep-inelastic diffractive interactions are consistent with proton dissociation.

2. Lorentz Structure

$$d^5\sigma_{\rm DIFFR} = \frac{1}{2(S-M^2)}\,\frac{1}{4}\,dPS^{(3)} \sum_{\rm spins} \frac{e^4}{Q^4}\, L_{\mu\nu} W^{\mu\nu}$$



$$\Delta\eta_{\mathbb{P}} \sim \frac{1}{x_{\mathbb{P}}}$$

$$x = \frac{Q^2}{W^2+Q^2-M^2}\,, \quad t = +(p_1-p_2)^2\,, \quad M_x^2 = (q+p_1-p_2)^2$$

$$W^2 = (q+p_1)^2$$

$$x_{\mathbb{P}} = -\frac{2\eta}{1-\eta} \geq x$$

$$\eta = \frac{q(p_2-p_1)}{q(p_2+p_1)} \in \left[-1, \frac{-x}{2-x}\right].$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \hat{P}_{1\mu}\hat{P}_{1\nu}\frac{W_3}{M^2} + \hat{P}_{2\mu}\hat{P}_{2\nu}\frac{W_4}{M^2}$$

$$+ \left[\hat{P}_{1\mu}\hat{P}_{2\nu} + \hat{P}_{1\nu}\hat{P}_{2\mu}\right]\frac{W_5}{M^2}$$

$$\hat{P}_{i\mu} = P_{i\mu} - q_\mu \frac{p_i . q}{q^2}$$

3. The Compton Amplitude

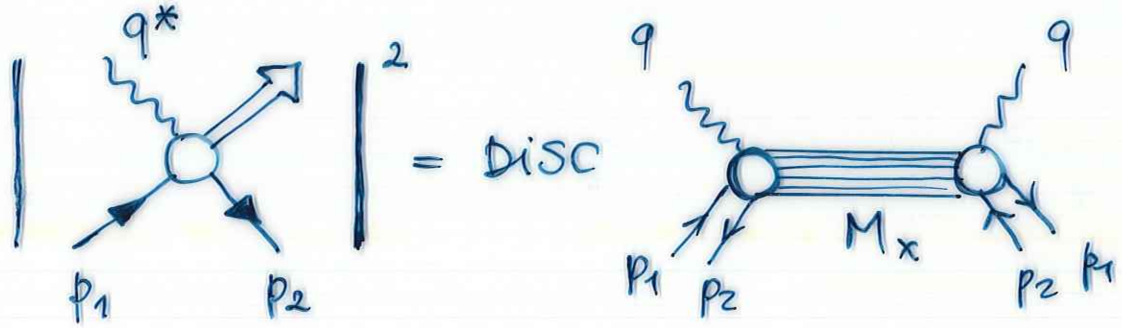
OPERATOR:

$$\hat{T}_{\mu\nu} = i RT \left[J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right) S \right]$$

$$\approx -e^2 \frac{\tilde{x}^\lambda}{i\pi^2 (x^2 - i\varepsilon)} \times$$

$$\left[S_{\alpha\mu\lambda\nu} O^\alpha\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) + i\varepsilon_{\mu\lambda\nu\sigma} O_5^\sigma\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \right]$$

COMPTON AMPLITUDE & DIFFRACTIVE SCATTERING ?



AH MUELLER'S THEOREM:

TURN THE FINAL STATE PROTON INTO AN INITIAL STATE ANTIPROTON.

⟹ DIS OFF A 2-PARTICLE INITIAL STATE.

$$W_i = W_i(x, Q^2, x_{I\!P}, t)$$

APPROXIMATION

$$t, M^2 \sim 0$$

$$p_2 \to z\, p_1$$

$$z = 1 - x_{I\!P} = \frac{1+\eta}{1-\eta}$$

$$W_{\mu\nu} \stackrel{!}{=} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \hat{P}_{1\mu} \hat{P}_{2\nu} \frac{W_2}{M^2}$$

$$W_2 = W_3 + (1-x_{I\!P}) W_5 + (1-x_{I\!P})^2 W_4$$

GENERALIZED BJORKEN LIMIT:

$$2p_1 q,\ 2p_2 q \to \infty \quad , \quad Q^2 \to \infty$$

$$x,\ x_{I\!P} = \text{FIXED}.$$

RAPIDITY GAP: $\Delta\eta_R \sim \log \frac{1}{x_{I\!P}}$.

WHAT ARE THE OPERATOR MATRIX ELEMENTS?

$$\langle p_1, -p_2 | O^{A\rho}_{(5)}(\kappa_+ \tilde{x}, \kappa_- \check{x}) | p_1, -p_2 \rangle$$

$$= \int_0^1 d\lambda \; \partial_x^\rho \langle p_1, -p_2 | O^A_{(5)}(\lambda \kappa_+ x, \lambda \kappa_- x) | p_1, -p_2 \rangle$$

$$A = q, G.$$

NON-PERTURBATIVE REPRESENTATION:

$$\langle p_1, -p_2 | O^q | p_1, -p_2 \rangle = x p_- \int Dz \, e^{-i\kappa_- x p_z} f_q(z_+, z_-)$$
$$+ x \pi_- \int Dz \, e^{-i\kappa_- x p_z} f_\pi(z_+, z_-)$$

$$p_z = p_- z_- + p_+ z_+ \equiv p_- \vartheta + \pi_- z_+$$

$$\vartheta = z_- + \frac{1}{\eta} z_+$$

$$\pi_- = p_+ - \frac{1}{\eta} p_-$$

$t \to 0 \quad \curvearrowright \quad \pi \to 0 \; ; \quad$ ALWAYS: $\quad q \cdot \pi_- = 0.$

$$T_{\mu\nu}(p_1, p_2, q) = -2 \, S_{\varrho\mu\sigma\nu} \int Dz \; F(z_+, z_-)$$
$$\times \left[\frac{p_-^\varrho Q_z^\sigma}{(Q_z^2 + i\varepsilon)} - \frac{1}{2} \frac{p_z^\varrho p_-^\sigma}{Q_z^2 + i\varepsilon} + \frac{Q_z \cdot p_-}{(Q_z^2 + i\varepsilon)^2} p_z^\varrho Q_z^\sigma \right]$$

$$F(z_+, z_-) = \int_0^1 \frac{d\lambda}{\lambda^2} f\left(\frac{z_+}{\lambda}, \frac{z_-}{\lambda}\right) \theta(\lambda - |z_+|) \, \theta(\lambda - |z_-|).$$

PROPAGATORS:

$$\frac{1}{Q_z^2 + i\varepsilon} = \frac{1}{qp_-} \frac{1}{\vartheta - 2\beta + i\varepsilon}$$

$$\beta = \frac{x}{x_{\mathbb{P}}} = \frac{q^2}{2qp_-}$$

$T_{\mu\nu}$ IS GAUGE INVARIANT.

$$q_\mu T^{\mu\nu} = T^{\mu\nu} q_\nu = 0.$$

DUE TO:

$$\int Dz \; F(z_+, z_-) = 0.$$

z_+, z_- ARE INTERNAL MOMENTUM FRACTIONS.

$\longrightarrow$ COME TO OBSERVABLE KINEM. VARIABLES.

$$\hat{F}(\vartheta, \eta) = \int Dz \; F(z_+, z_-) \, \delta\left(\vartheta - z_- - \frac{1}{\eta} z_+\right)$$

$$= \int_{\vartheta}^{-\mathrm{sign}(\vartheta/\eta)} \frac{dz}{z} \; \hat{f}(z, \eta)$$

$$\hat{f}(z, \eta) = \int_{\eta(1+z)}^{\eta(1-z)} d\varrho \; \theta(1-\varrho)\, \theta(1+\varrho)\, f(\varrho, z - \varrho/\eta).$$

PARTIAL INTEGRATIONS :

$$T_{\mu\nu} = 2\int_{1/\eta}^{-1/\eta} d\vartheta \left[-g_{\mu\nu} + \frac{P_{1\mu}q_\nu + P_{1\nu}q_\mu}{P_1\cdot q} + \vartheta x_{I\!P} \frac{P_{1\mu}P_{1\nu}}{qP_1}\right] \cdot \frac{\hat{f}(\vartheta,\eta)}{\vartheta - 2\beta + i\varepsilon}$$

NEARLY THE FORM OF $T_{\mu\nu}$ WE SEEK.

$$\Rightarrow$$

$$\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(\beta,\eta,Q^2) + \frac{1}{qP_1}\hat{P}_{1\mu}\hat{P}_{1\nu} F_2(\beta,\eta,Q^2)$$

$$\boxed{2\underline{x} F_1(\beta,\eta,Q^2) = F_2(\beta,\eta,Q^2)}$$

$\uparrow$ not β !

DIFFRACTIVE CALLAN - GROSS RELATION.

$$\overset{(-)}{f}{}^D(\beta,Q^2,x_{I\!P}) = -\int_{-\frac{x_{I\!P}+2x}{2-x_{I\!P}}}^{-\frac{x_{I\!P}-2x}{2-x_{I\!P}}} d\varrho\, f(\varrho, (\pm), 2\beta + \varrho(2-x_{I\!P})/x_{I\!P}, Q^2).$$

$$F_1(\beta,\eta,Q^2) = \sum_q e_q^2 \left[f^D(\beta,Q^2,x_{I\!P}) + \bar{f}^D(\beta,Q^2,x_{I\!P})\right].$$

4. Evolution Equations

HOW DO DIFFRACTIVE PARTON DENSITIES EVOLVE ?

START WITH GENERAL NON-FORWARD FORMALISM, TRACE η DEPENDENCE.

$$\frac{d}{d\log\mu^2} O^A(\kappa_+\tilde{x}, \kappa_-\tilde{x}; \mu^2) = \int DK' \gamma^{AB}(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-; \mu^2) \times O_B(\kappa'_+\tilde{x}, \kappa'_-\tilde{x})$$

THE OME's $\langle p_1, -p_2 | O | p_1, -p_2 \rangle$ ARE INTRODUCED.

$$f^A(\vartheta; \eta) = \int \frac{d\kappa_- \tilde{x} p_-}{2\pi} e^{i\kappa_- \tilde{x} p_- \vartheta} \langle p_1, -p_2 | O^A | p_1, -p_2 \rangle \cdot (\tilde{x} p_-)^{1-d_A}$$

$d_A = 1 : q$
$d_A = 2 : G$

$\longrightarrow$ NO κ_+ DEPENDENCE FOR THIS PROJECTION

$$\gamma^{AB}(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-) \rightarrow \gamma^{AB}(0, \kappa_-, \kappa'_+, \kappa'_-).$$

ALL - ORDER RESCALING PROPERTY:

$$\gamma^{AB}(k_+, k_-, k'_+, k'_-) = \sigma^{d_{AB}} \gamma^{AB}(\sigma k_+, \sigma k_-, \sigma k'_+, \sigma k'_-)$$

$$d_{AB} = 2 + d_A - d_B .$$

$$\int Dk' \; k_-^{d_B - d_A} \; \gamma^{AB}\left(0, 1, \frac{k'_+}{k_-}, \frac{k'_-}{k_-} ; \mu^2\right)$$

$$= \int D\alpha \; k_-^{d_B - d_A} \; \hat{K}^{AB}(\alpha_1, \alpha_2 ; \mu^2)$$

FURTHER CONVERSION:

$$\mu^2 \frac{d}{d\mu^2} f^A(\vartheta; \eta, \mu^2) = \int_0^1 du \int_{\vartheta}^{-\mathrm{sign}(\vartheta/\eta)} d\vartheta' \; \tilde{O}^{AB}(u\vartheta' - \vartheta)$$

$$\cdot \hat{\hat{K}}^{AB}(u, \mu^2) \; f_B(\vartheta'; \eta, \mu^2)$$

$$\tilde{O}^{AB}(u\vartheta' - \vartheta) = \begin{cases} \delta(u\vartheta' - \vartheta) & A = B \\ \partial_u \delta(u\vartheta' - \vartheta) & A = q, \; B = G \\ \Theta(u\vartheta' - \vartheta)/\vartheta & A = G, \; B = q \end{cases}$$

$\longrightarrow$ EVOLUTION EQUATION.

EVOLUTION EQUATION IN:

$$\vartheta = z_- + \frac{1}{\eta} z_+$$

$$\mu^2 \frac{d}{d\mu^2} f^A(\vartheta,\eta,\mu^2) = \int_{\vartheta}^{-\mathrm{sign}(\vartheta/\eta)} \frac{d\vartheta'}{\vartheta'} P^{AB}\left(\frac{\vartheta}{\vartheta'},\mu^2\right) f_B(\vartheta',\eta,\mu^2)$$

YET NOT IN THE RANGE: $\vartheta \in [0,1]$!

HOWEVER:

ACTION OF THE ABSORPTION CONDITION
$\delta(\vartheta - 2\beta)$.

$$\boxed{\mu^2 \frac{d}{d\mu^2} f_A^D(\beta,x_P,\mu^2) = \int_{\beta}^{1} \frac{d\beta'}{\beta'} P^{AB}\left(\frac{\beta}{\beta'},\mu^2\right) f_B(\beta',x_P,\mu^2)}$$

η OR x_P ARE BARE PARAMETERS
AND DO NOT INTERFERE WITH THE EVOLUTION,
WHICH IS IN β.

10. Conclusions

1. The virtual Compton Amplitude for deep-inelastic Nonforward Scattering was studied in the Generalized Bjorken Region for the Twist–2 contributions.

2. There exist several equivalent methods to derive the Nonforward Evolution Kernels and anomalous dimensions, which yield the same results. The problem of Spin Towers can be solved in terms of integral representations. This is likely the solution of the Spin Tower problem arising for Higher Twist Operators, in generalized form, for Forward Scattering too.

3. The Nonforward Compton Amplitude consists of unpolarized and polarized DIRAC and PAULI –type contributions at leading twist. For the Operator-Expectaion Values an expansion in $1/\nu$ has to be performed to find the Leading Twist terms, in the spirit of the Bjorken Limit.

4. For the unpolarized terms only the Amplitude matrix elements T_{11} and T_{22} and the polarized terms the projections T_{12} and T_{21} contribute in this order.

5. In this order the Light-Cone expansion conserves the electromagnetic current. This property has to be studied twist by twist for the remaining contributions.

6. Generalizations of the CALLAN-GROSS and WANDZURA-WILCZEK relations known in the forward case for the matrix element square level for the Nonforward Case were derived at the Matrix Element Level.

7. Diffractive Scattering can be described in the Light-Cone Expansion, applying A.H. Mueller's Optical Theorem. One obtains a slightly modified Callan-Gross Relation. The Evolution is forward. x_P behaves as plain parameter. This method applies to All Twists.