$\mathcal{O}(\alpha^2 L^2)$ Radiative Corrections to Deep Inelastic ep Scattering for Different Kinematical Variables

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1. The Different Variables
2. The Corrections up to $\mathcal{O}(\alpha^2 L^2)$
3. Numerical Results
4. Conclusions

1. The Different Variables

Goal:

Measurement of a Born Cross section: $2 \rightarrow 2$ Reaction

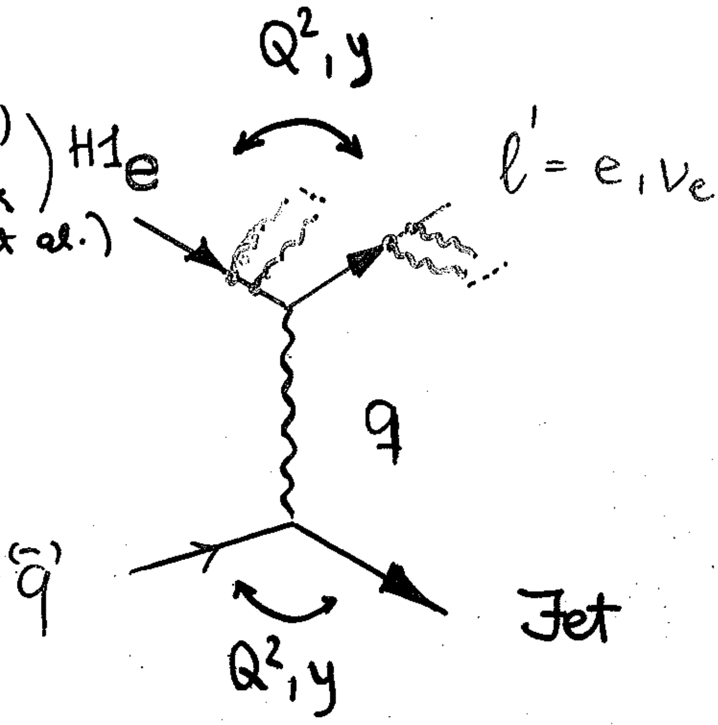
$\hookrightarrow$ Integrating over the DOF of the radiated Photon(s).

$\hookrightarrow$ Different Correction Functions for Different Variables are obtained !

$$\boxed{\delta^{NC,CC}_{(x,y)}}$$

- Double Angle Method $\quad \Theta_e, \Theta_J$ } ZEUS
- θ_e & y_J
- Jet Measurement: NC
- Jet Measurement: CC
- Mixed Variables (Q^2_e, y_J)) H1 e
- (Lepton Measurement)

(KRIPFGANZ et al.)



	$\hat{s}$	$\hat{Q}^2$	$\hat{y}$	$\mathcal{J}(x,y,z)$
lepton measurement	zs	$Q^2 z$	$(z+y-1)/z$	$y/(z+y-1)$
jet measurement	zs	$Q^2(1-y)/(1-y/z)$	y/z	$(1-y)/(z-y)$
mixed variables	zs	$Q^2 z$	y/z	1
double angle method	zs	$Q^2 z^2$	y	z
y_{JB} and θ_e	zs	$Q^2 z(z-y)/(1-y)$	y/z	$(z-y)/(1-y)$

Table 1: The shifted variables for different types of cross section measurement

- z_0 :

LEPTON MEASUREMENT
JET MEASUREMENT $\hat{x}(z_0) = 1$

MIXED VARIABLES
θ_e, y_J $z_0 = y$

DOUBLE ANGEL : $z_0 \equiv 0.$

$\hat{\mathcal{J}}(\hat{x} \to 0, \hat{Q}^2 \to 0)$!
for $z \to z_0$!

BUT:

$$2E_e = E'_e(1-\cos\theta_e) + E_J(1-\cos\theta_J) \geq \mathcal{A} \quad ! \tag{3}$$

FORTUNATELY: $z_0 = \dfrac{\mathcal{A}}{2E_e}.$

$\Longrightarrow$ ZEUS: THIS HELPS ONLY IN THE CASE OF THE DOUBLE ANGEL METHOD !

2. The Corrections up to $\mathcal{O}(\alpha^2)$

Contributions:

1. Bremsstrahlung: Diagrams a,b
2. Electron Pair Production: Diagram c
3. Fermion Pair Production: Diagram d , $f = e, \mu, \tau, u, d, s, c, b$

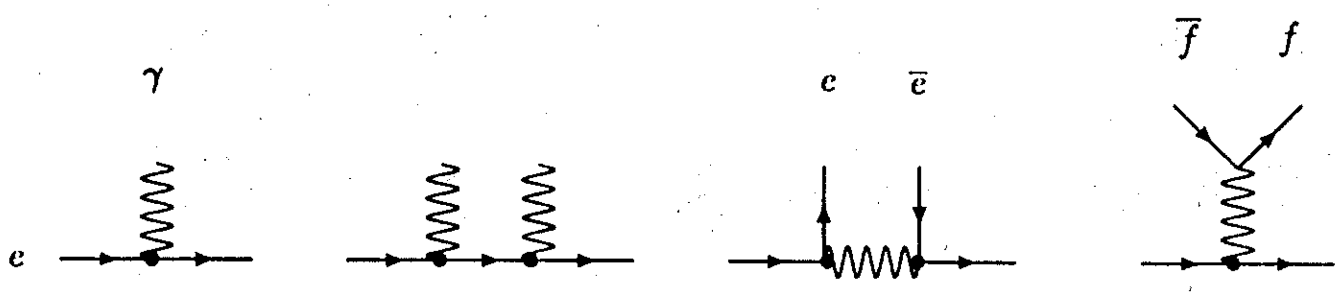
The Radiator-Method is applied.

Meaning of the bullet: Collinear Bremsstrahlung contribution

including

soft & virtual corrections.

An individual consideration of initial and final state bremsstrahlung is possible.



(a) (b) (c) (d)

BORN $\quad\quad\quad\quad\quad\quad\quad\quad$ $O(\alpha)$

$$
\begin{aligned}
\frac{d^2\sigma^{(2)}}{dxdy} &= \frac{d^2\sigma^{(0)}}{dxdy} + \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\int_0^1 P_{ee}^{(1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\} \\
&+ \frac{1}{2}\left[\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\right]^2\int_0^1 P_{ee}^{(2,1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\} \\
&+ \left(\frac{\alpha}{2\pi}\right)^2\int_{z_0}^1\left\{\ln^2\left(\frac{Q^2}{m_e^2}\right)P_{ee}^{(2,2)}(z) + \sum_{f=l,q}\ln^2\left(\frac{Q^2}{m_f^2}\right)P_{ee,f}^{(2,3)}(z)\right\}\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}}
\end{aligned}
\Bigg\} O(\alpha^2)
$$

$$
\mathcal{J}(x,y,z) = \begin{vmatrix} \partial\hat{x}/\partial x & \partial\hat{y}/\partial x \\ \partial\hat{x}/\partial y & \partial\hat{y}/\partial y \end{vmatrix}. \tag{2}
$$

$O(\alpha)$

$$
P_{ee}^{(1)}(z) = \frac{1+z^2}{1-z} \tag{4}
$$

$O(\alpha^2L^2)$

$$
\begin{aligned}
P_{ee}^{(2,1)}(z) &= \frac{1}{2}\left[P_{ee}^{(1)} \otimes P_{ee}^{(1)}\right](z) \\
&= \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
P_{ee}^{(2,2)}(z) &= \frac{1}{2}\left[P_{e\gamma}^{(1)} \otimes P_{\gamma e}^{(1)}\right](z) \\
&\equiv (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3)
\end{aligned} \tag{6}
$$

$$
P_{ee,f}^{(2,3)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(1)}(z)\theta\left(1-z-\frac{2m_f}{E_e}\right) \tag{7}
$$

$$
\times\left[1-\exp(-A^2\hat{Q}^2)\right] \quad \text{with} \quad A^2 = 3.37\,\text{GeV}^{-2}. \tag{12}
$$

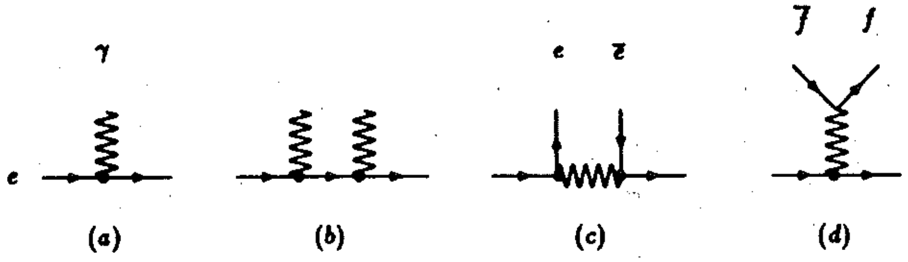


Figure 1: Diagrams contributing to the radiative corrections up to $\mathcal{O}(\alpha^2 L^2)$.

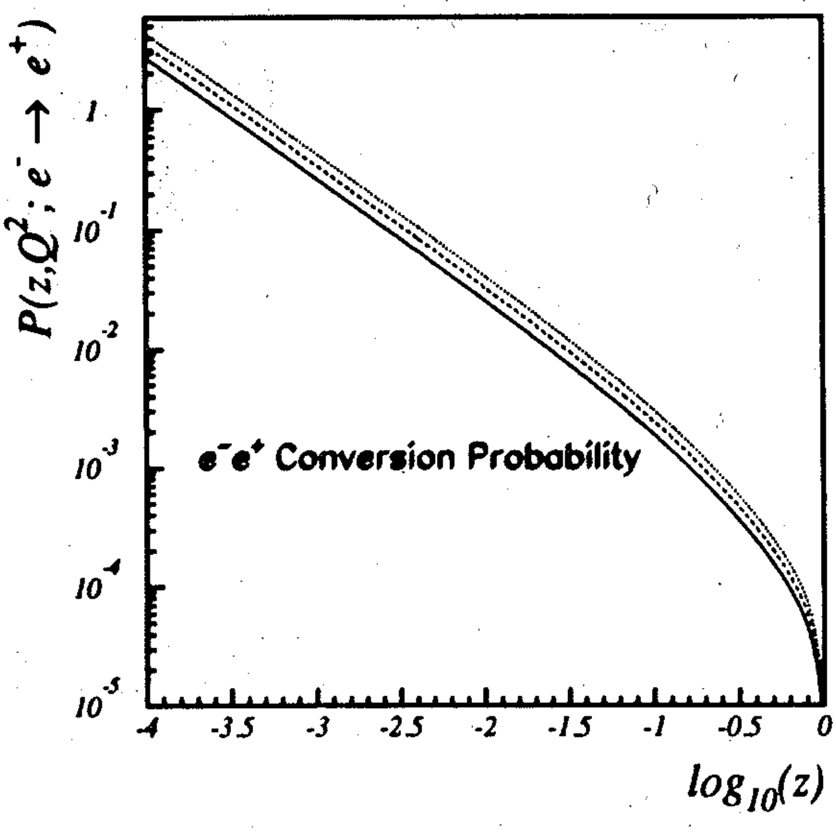


Figure 2: $e^- \to e^+$ transition probability for different values of Q^2. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 100\,\mathrm{GeV}^2$, and dotted line: $Q^2 = 1000\,\mathrm{GeV}^2$.

SOFT EXPONENTIATION :

SOLVE : LO - GRIBOV LIPATOV eq. (NS) FOR $z \to 1$

$$D_{NS}(z,Q^2) = \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2}-2\gamma_E\right)\right]}{\Gamma(1+\zeta)} \tag{8}$$

with

$$\zeta = -3\ln\left[1-(\alpha/3\pi)\ln(Q^2/m_e^2)\right] \tag{9}$$

(RUNNING α_{QED} !)

THESE TERMS WERE TAKEN INTO ACC. ALREADY !

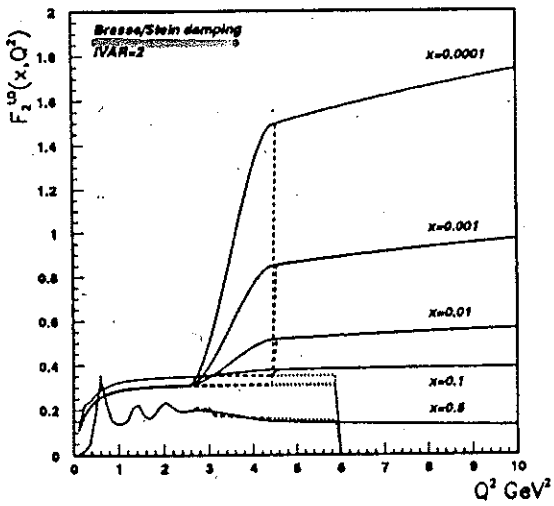
$$P_{ee}^{>2,\ soft}(z,Q^2) = D_{NS}(z,Q^2) - \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\frac{2}{1-z}\left\{1+\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\left[\frac{11}{6}+2\ln(1-z)\right]\right\} \tag{10}$$

and[6]

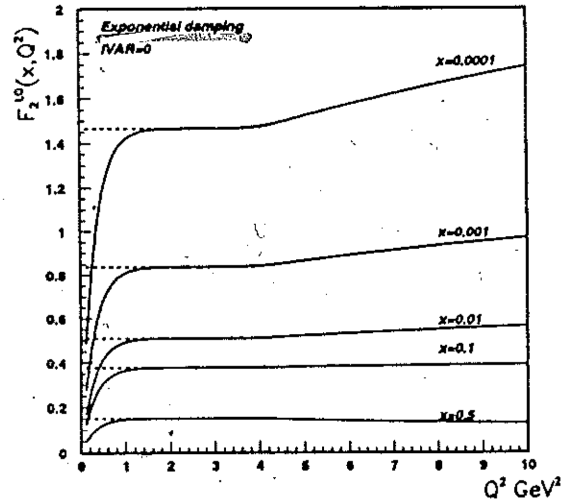
$$\frac{d^2\sigma^{(>2,soft)}}{dxdy} = \int_0^1 dz P_{ee}^{(>2)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\} \tag{11}$$

$\longrightarrow$ NOTE: NO 'UNIQUE' EXPONENTIATION EXISTS !

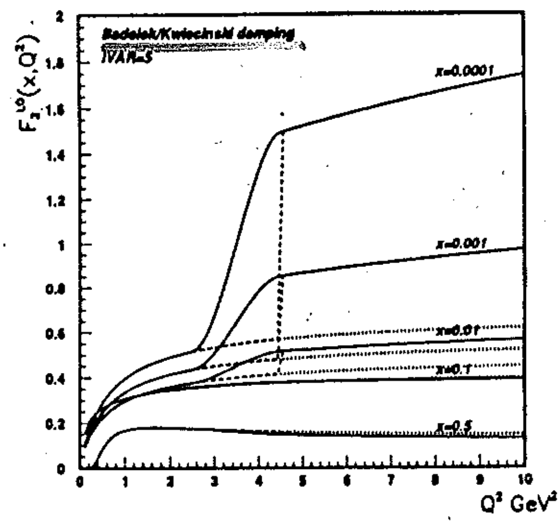
F₂ AT LOW Q^2



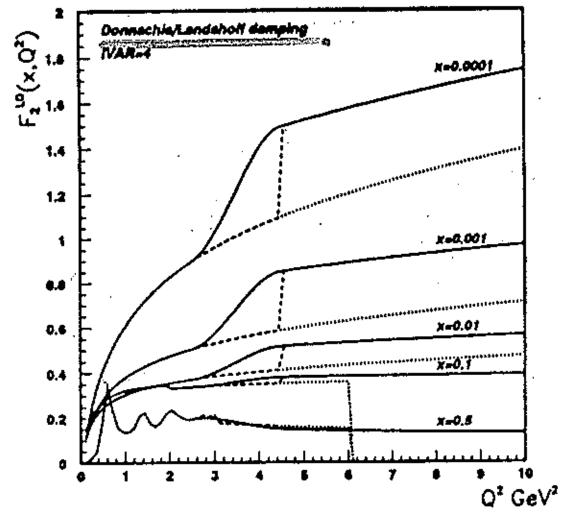
Brasse/Stein damping
IVAR=2
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2



Exponential damping
IVAR=0
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2



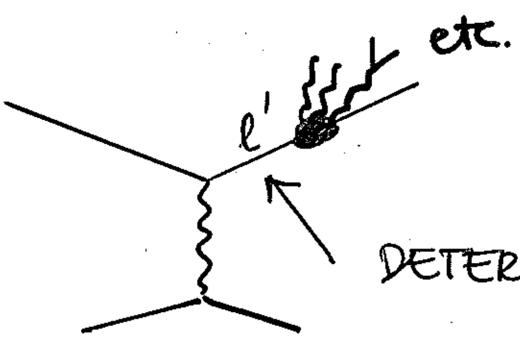
Badelek/Kwiecinski damping
IVAR=5
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2



Donnachie/Landshoff damping
IVAR=4
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2

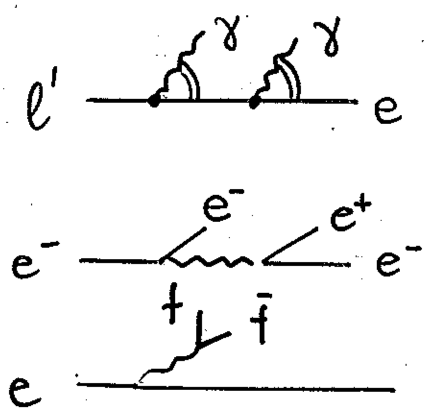
REMARK :

FBR : $\mathcal{O}(\alpha^2)$ FROM LEPTONS.



CALORIMETRIC MEASUREMENT

DETERMINE ℓ' KINEMATICS!



$$E_{\ell'} = E_e + E_{\gamma_i} + E_{e^+e^-} + \sum_i E_{f_i\bar{f}_i}$$

COLLECT ALL RADIATED ENERGY IN THE ANGULAR VICINITY OF e'!

3. Numerical Results

- UPDATE : $\mathcal{O}(\alpha)$
- STATUS : COMPARISON LLA & FULL CALCULATION IN $\mathcal{O}(\alpha)$
- $\mathcal{O}(\alpha^2 L^2)$ RESULTS IN ALL VARIABLES

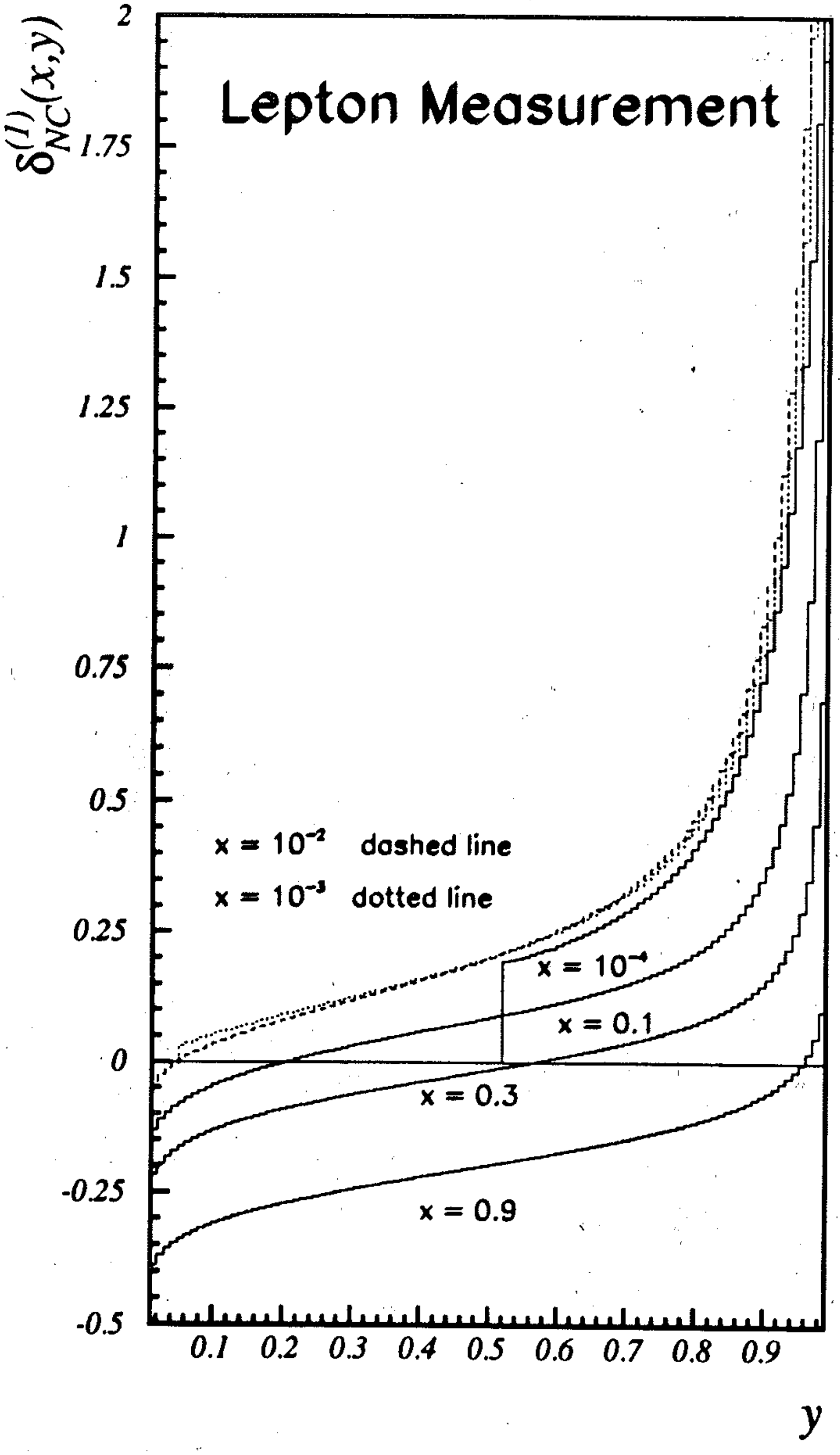
→ ISR LEPTON

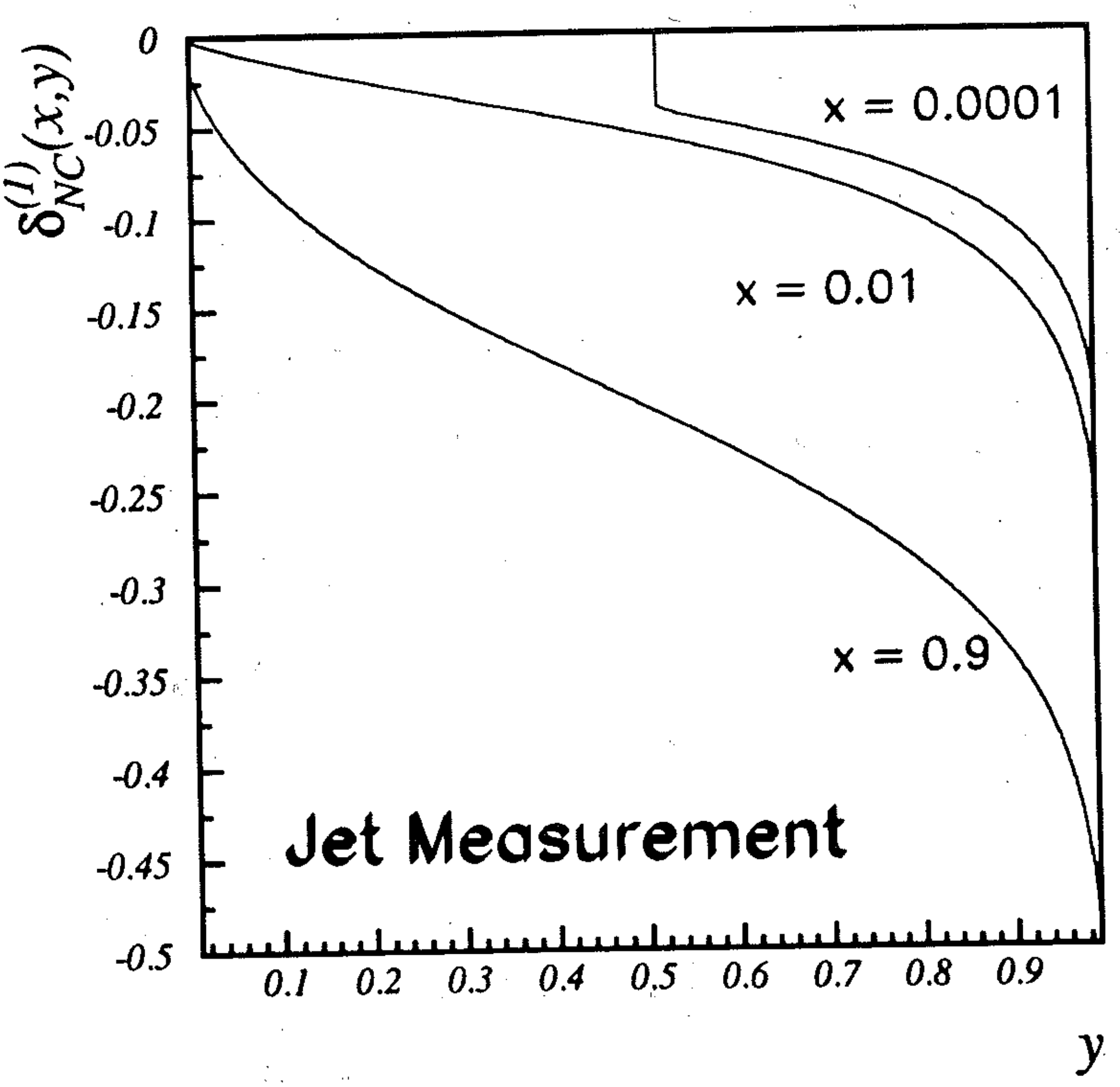
- FSR LEPTON → KLN
- ISR / FSR QUARK → SCAL. VIOL $\sim 1\%$
- COMPTON → EXCL. SIGNATURE

#'s NOT COUNTED TO THE DIS SAMPLE.

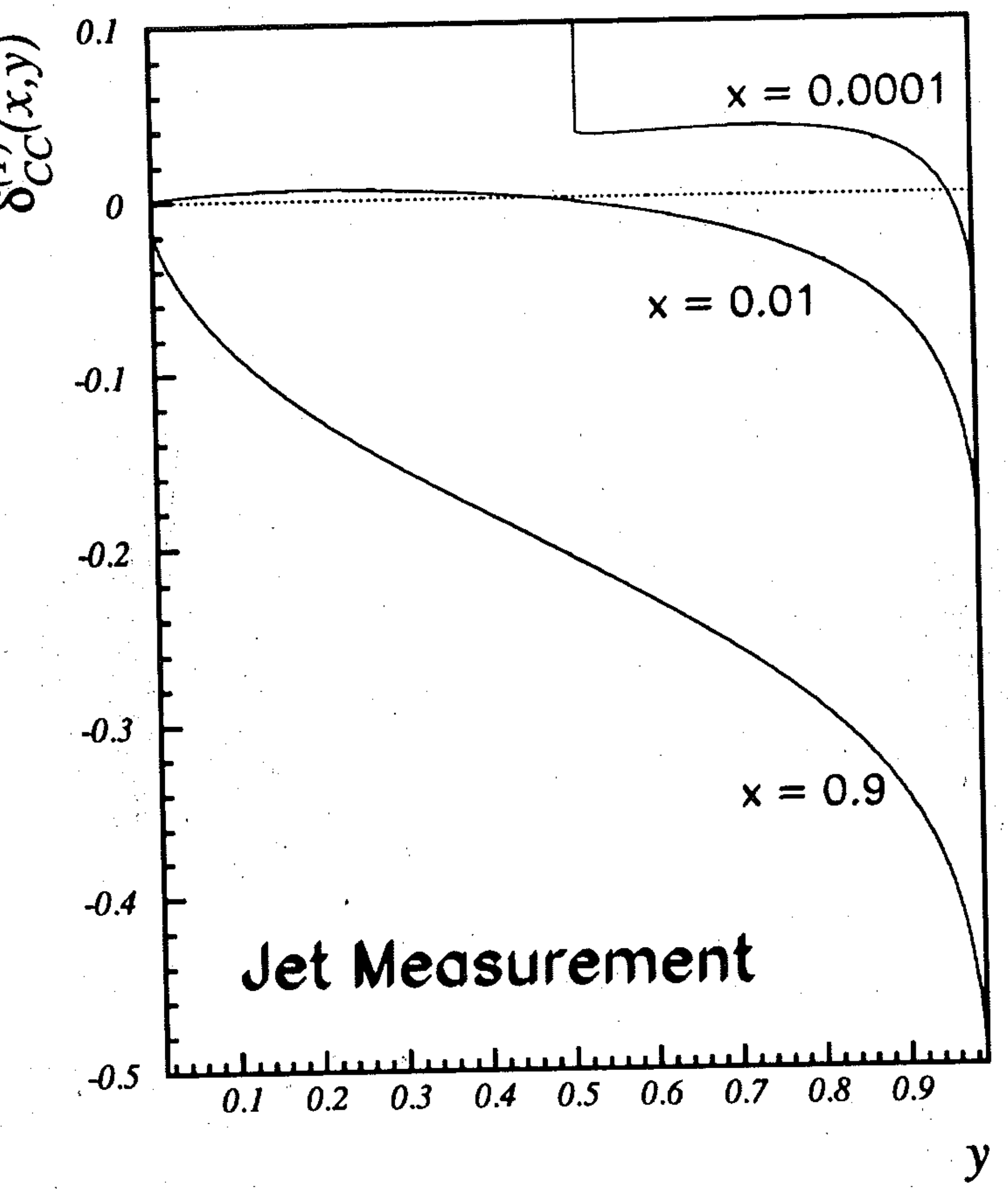
➡ MRS D_-', SIMILAR RES. HMRS CETQ 2

$O(\alpha)$

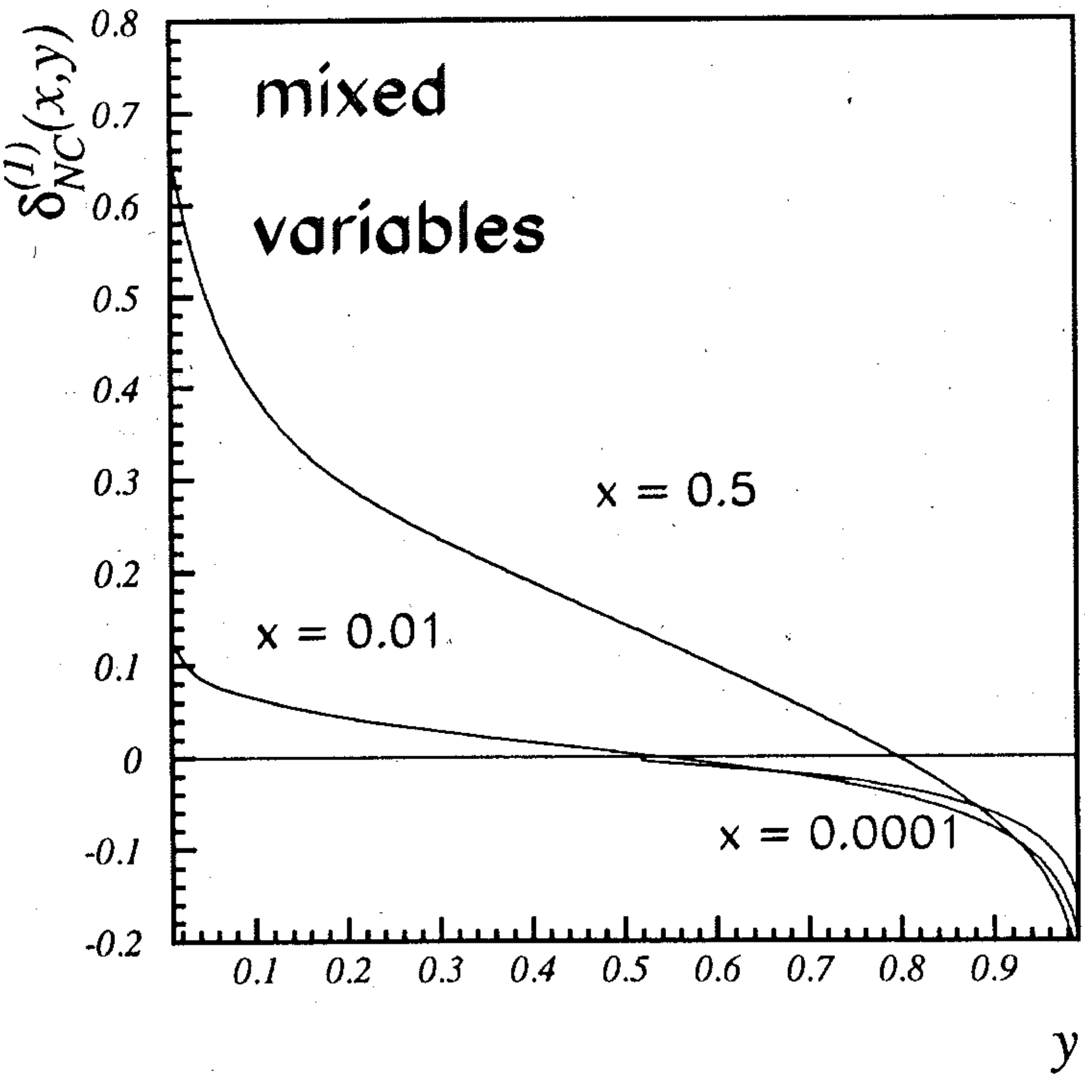

Lepton Measurement
$\delta_{NC}^{(1)}(x,y)$
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
-0.25
-0.5
x = 10⁻² dashed line
x = 10⁻³ dotted line
x = 10⁻⁴
x = 0.1
x = 0.3
x = 0.9
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y



$\delta_{NC}^{(1)}(x,y)$
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement



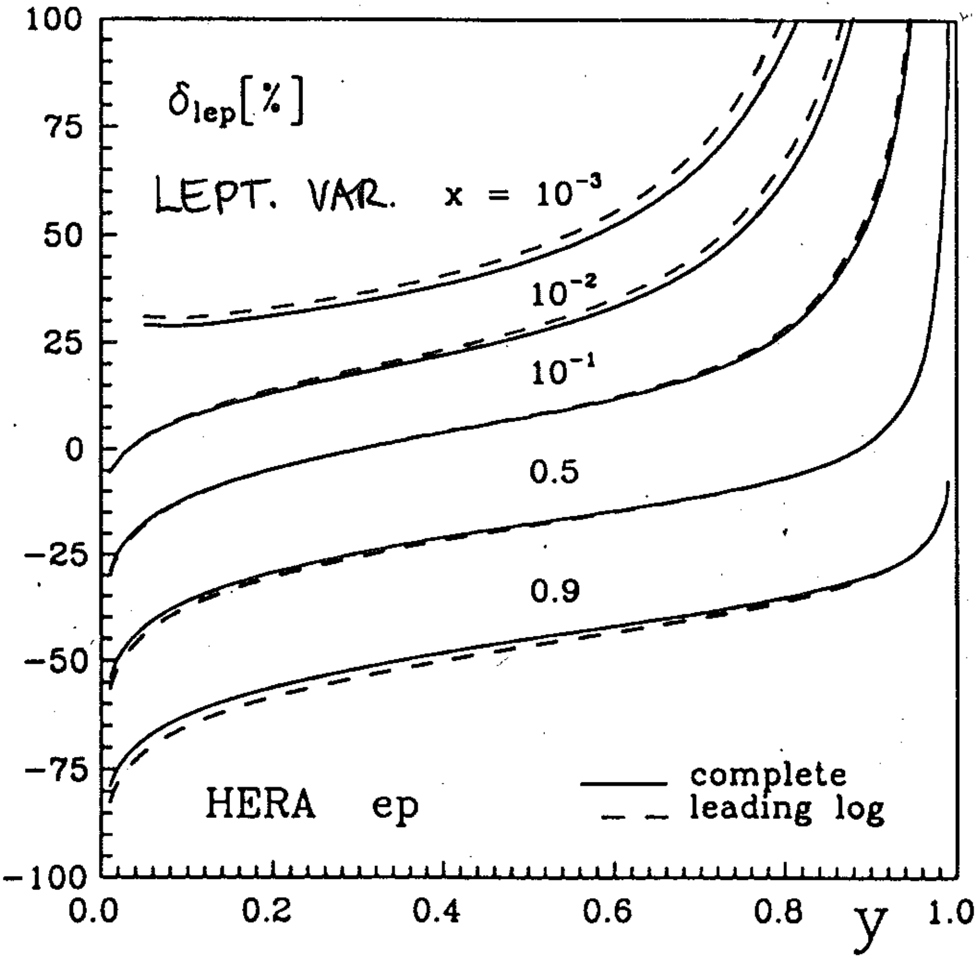
$\delta^{(1)}_{CC}(x,y)$
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

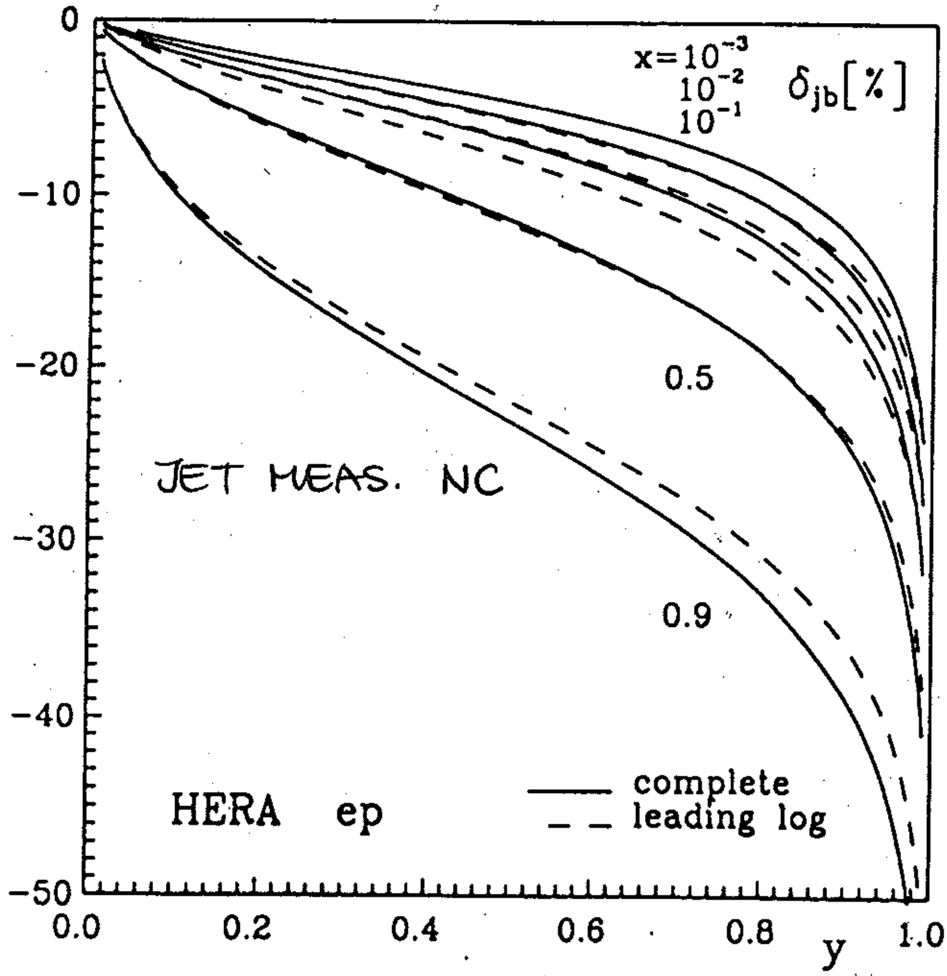


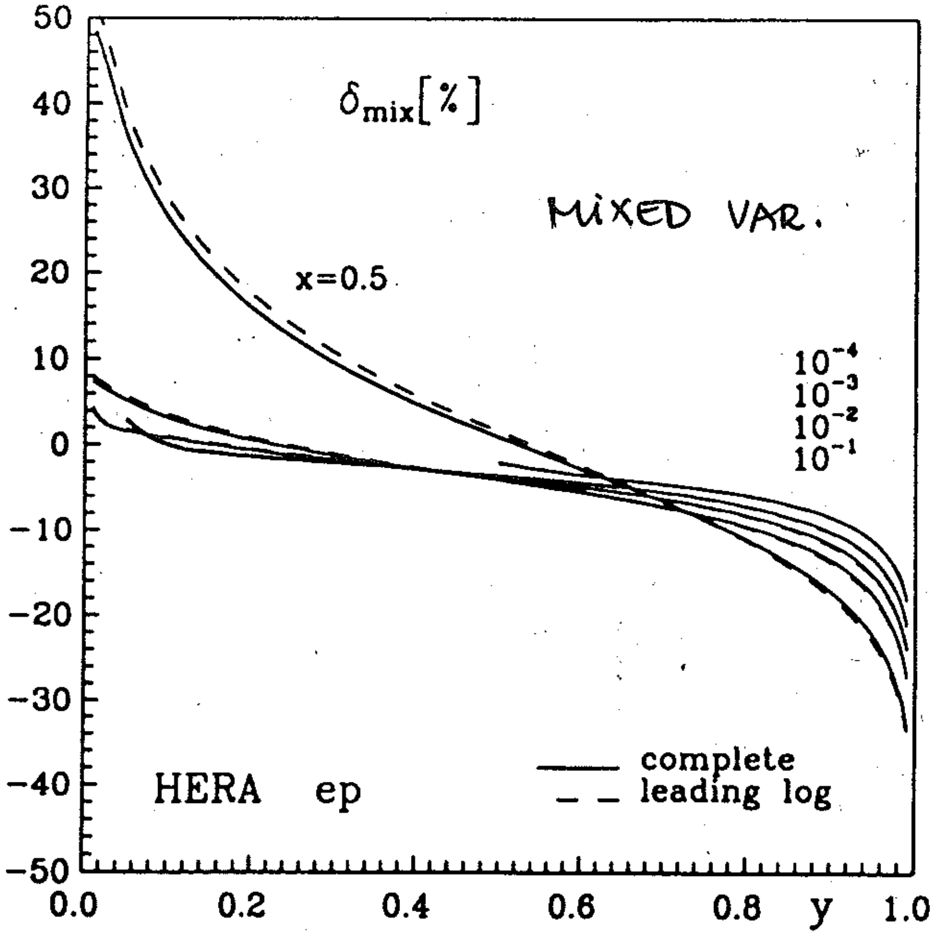
$\delta_{NC}^{(1)}(x,y)$
mixed
variables
x = 0.5
x = 0.01
x = 0.0001
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

Comparison with a Full $\mathcal{O}(\alpha)$ Calculation

TERAD, D.Y. Bardin et al.

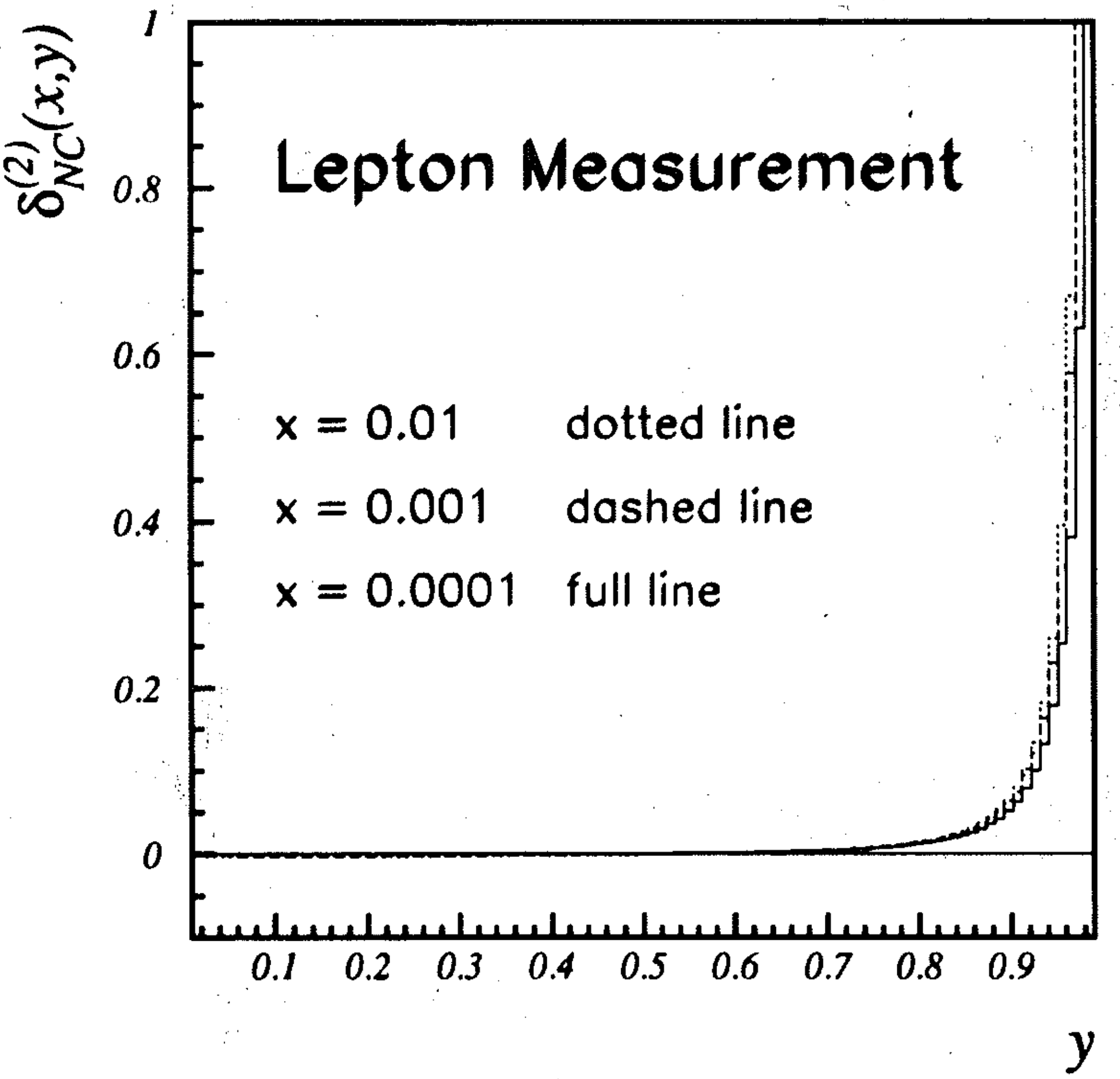




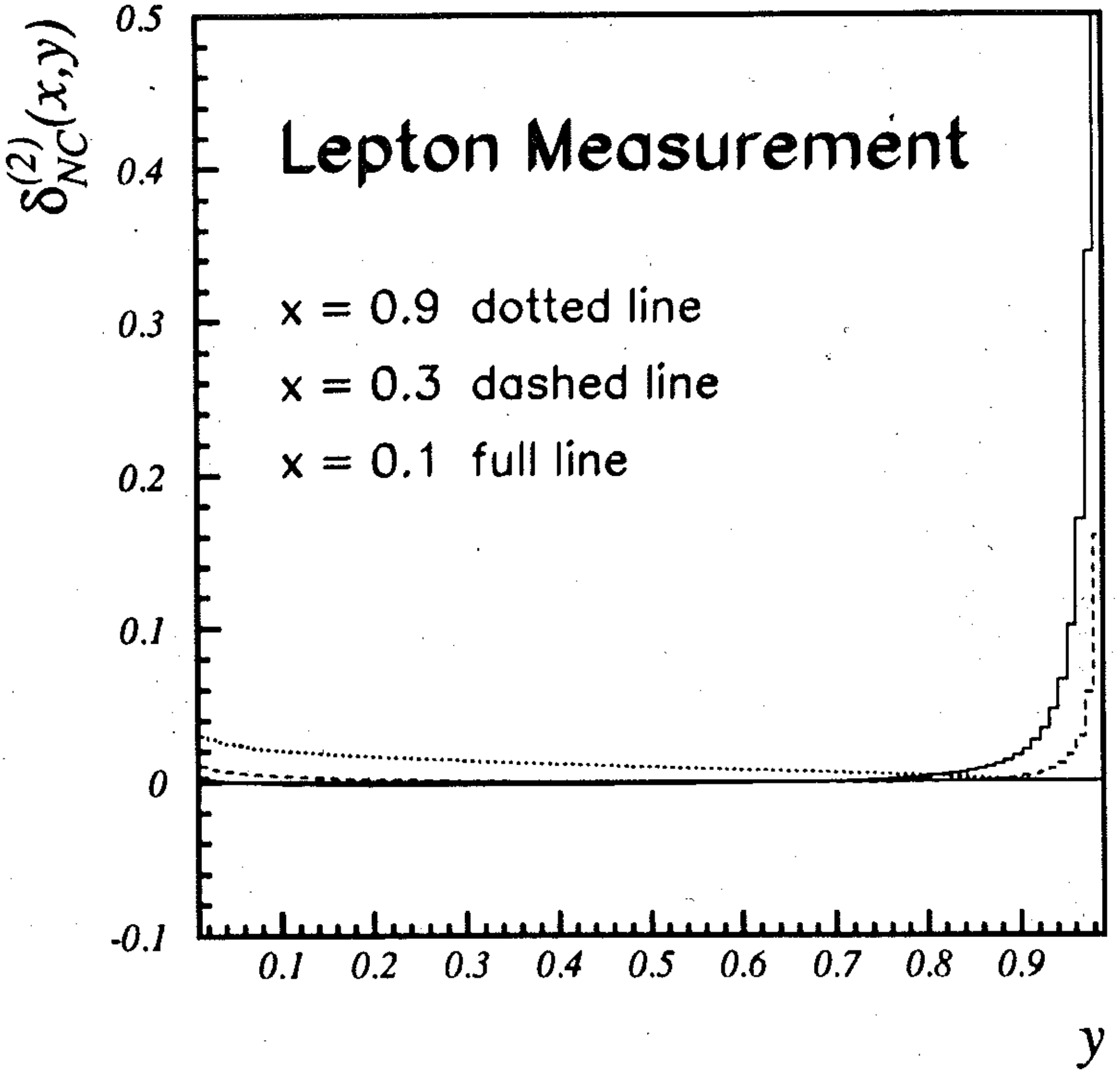


$\delta_{mix}[\%]$
MIXED VAR.
x=0.5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
HERA ep
complete
leading log
y
50
40
30
20
10
0
−10
−20
−30
−40
−50
0.0
0.2
0.4
0.6
0.8
1.0

$O(\alpha^2 L^2)$



Lepton Measurement
x = 0.01 dotted line
x = 0.001 dashed line
x = 0.0001 full line
$\delta_{NC}^{(2)}(x,y)$
1
0.8
0.6
0.4
0.2
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y



$\delta_{NC}^{(2)}(x,y)$
Lepton Measurement
x = 0.9 dotted line
x = 0.3 dashed line
x = 0.1 full line
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

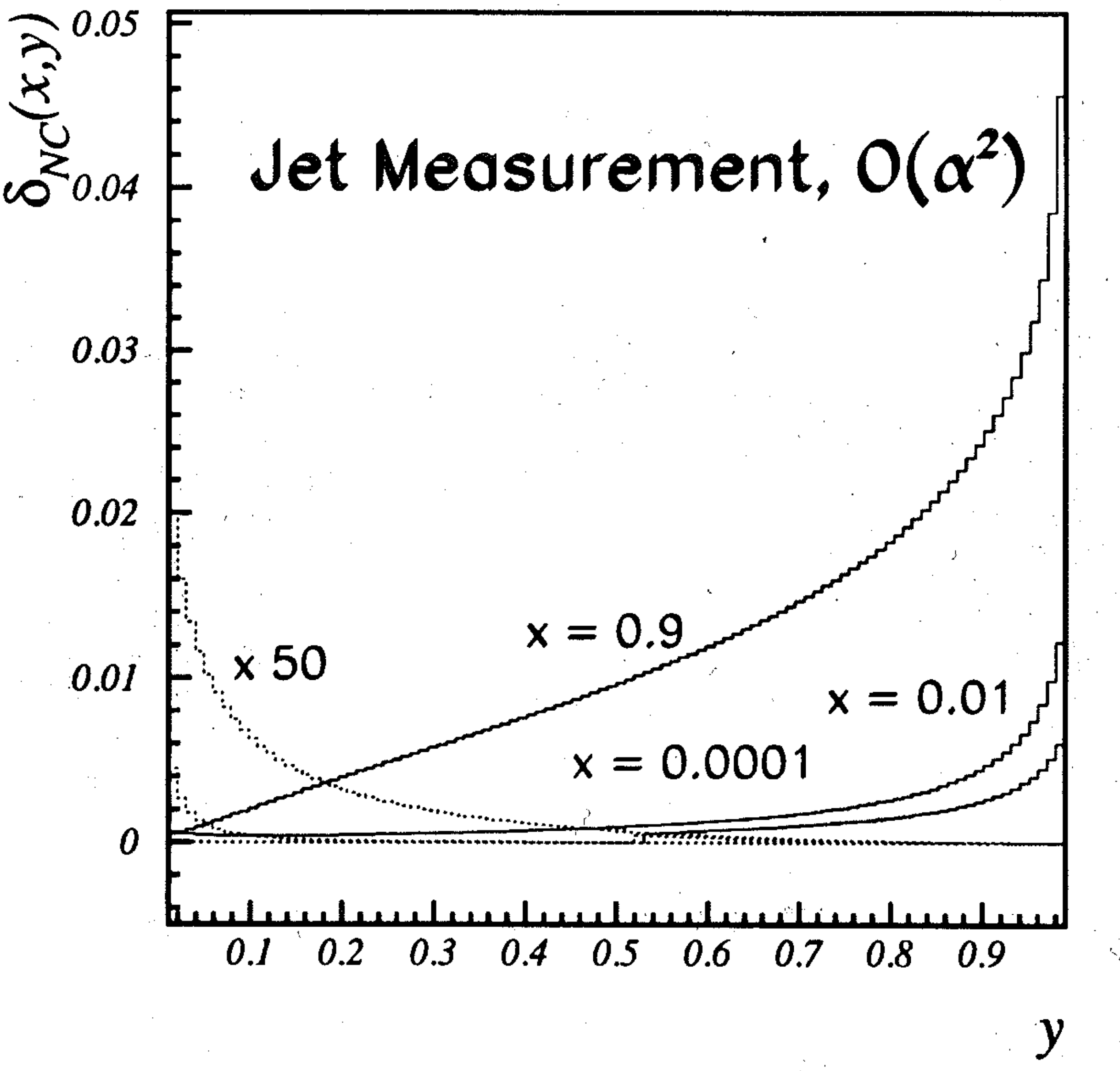


Figure 3: Leptonic initial state radiative corrections $\delta_{NC}(x,y) = (d\sigma^{(2+>2,soft)}/dxdy)/(d\sigma^0/dxdy)$ in LLA for e^-p deep inelastic scattering in the case of jet measurement for $\sqrt{s} = 314$ GeV, $\mathcal{A} = 0$, and $Q^2 \geq 5\,\mathrm{GeV}^2$. Full lines: $\mathcal{O}(\alpha^2)$ corrections; dotted lines: contributions due to $e^- \to e^+$ conversion eq. (13), $\delta_{NC}^{e^-\to e^+}(x,y) = (d\sigma^{(2,e^-\to e^+)}/dxdy)/(d\sigma^0/dxdy)$ scaled by $\times 50$; upper line: $x = 0.01$, middle line: $x = 0.0001$, lower line $x = 0.9$.

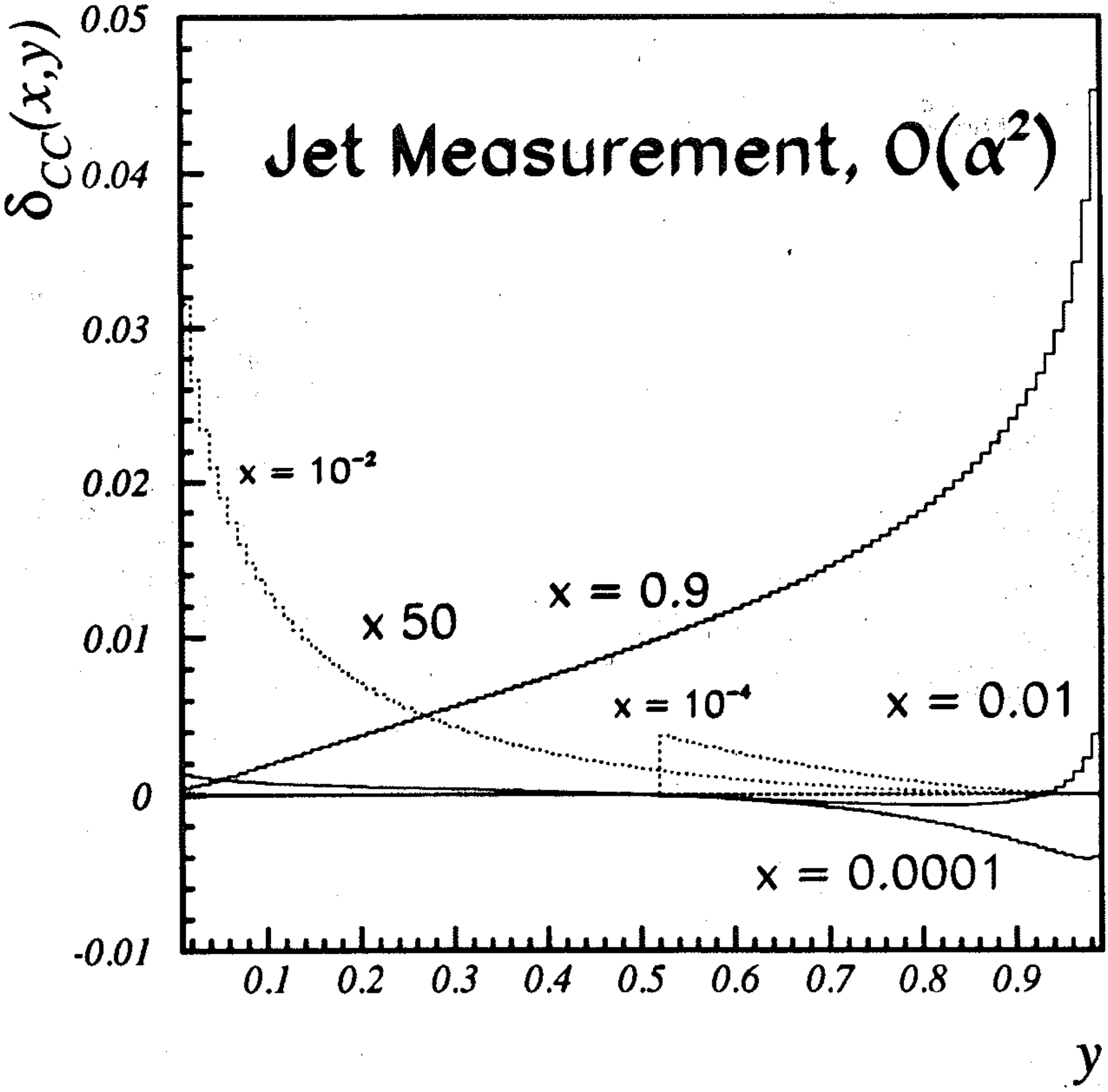


Figure 4: $\delta_{CC}(x,y) = (d\sigma_{CC}^{(2+>2,soft)}/dxdy)/(d\sigma_{CC}^{0}/dxdy)$ for deep inelastic e^-p scattering in the case of jet measurement. Dotted lines: $\delta_{CC}^{e^-\to e^+}(x,y)$. The other parameters are the same as in figure 3.

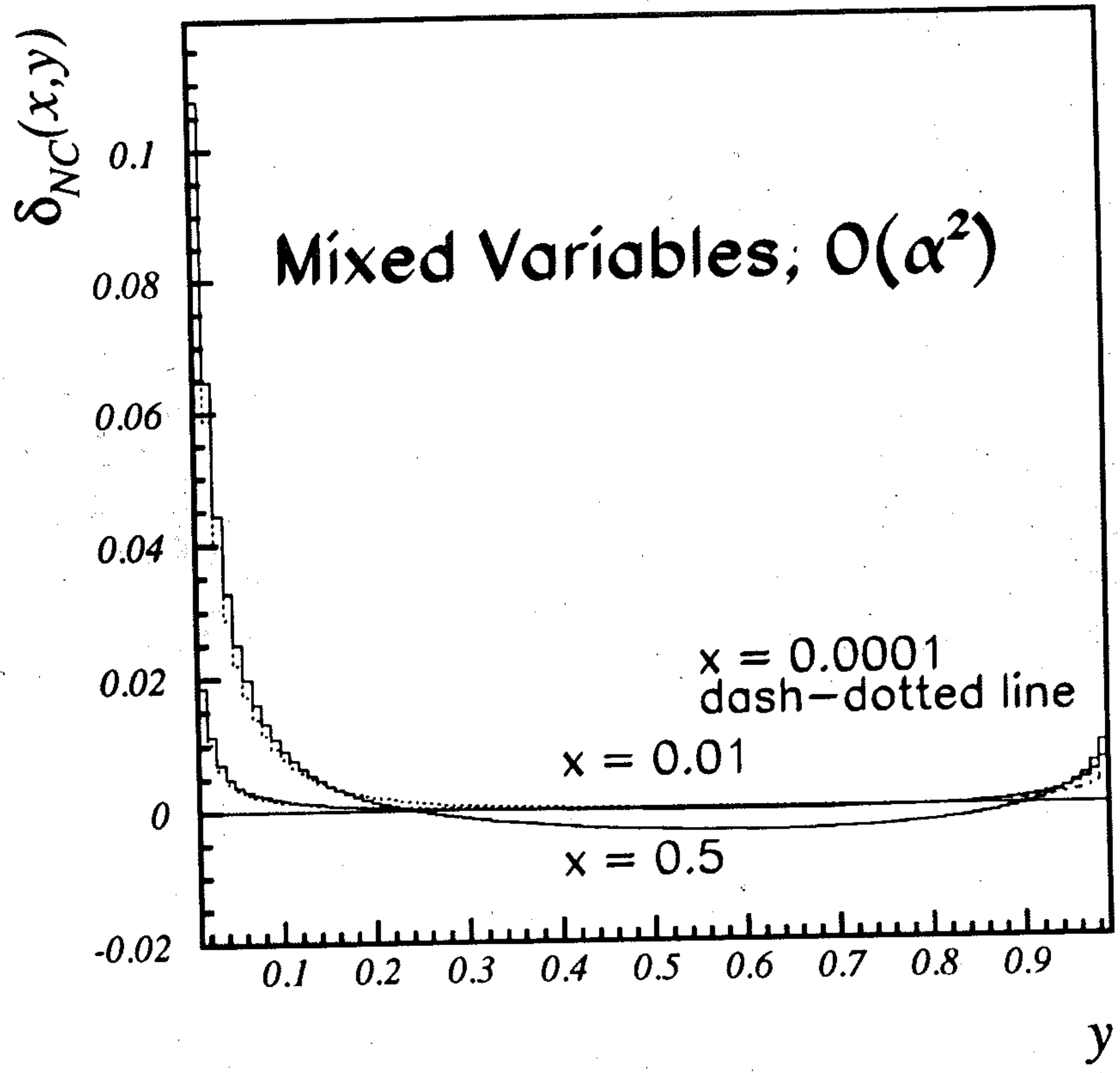


Figure 5: $\delta_{NC}(x,y)$ **for the case of mixed variables. Dotted lines:** $\delta_{NC}^{e^-\to e^+}(x,y)$**; upper line:** $x = 0.5$**, lower line** $x = 0.01$**. The other parameters are the same as in figure 3.**

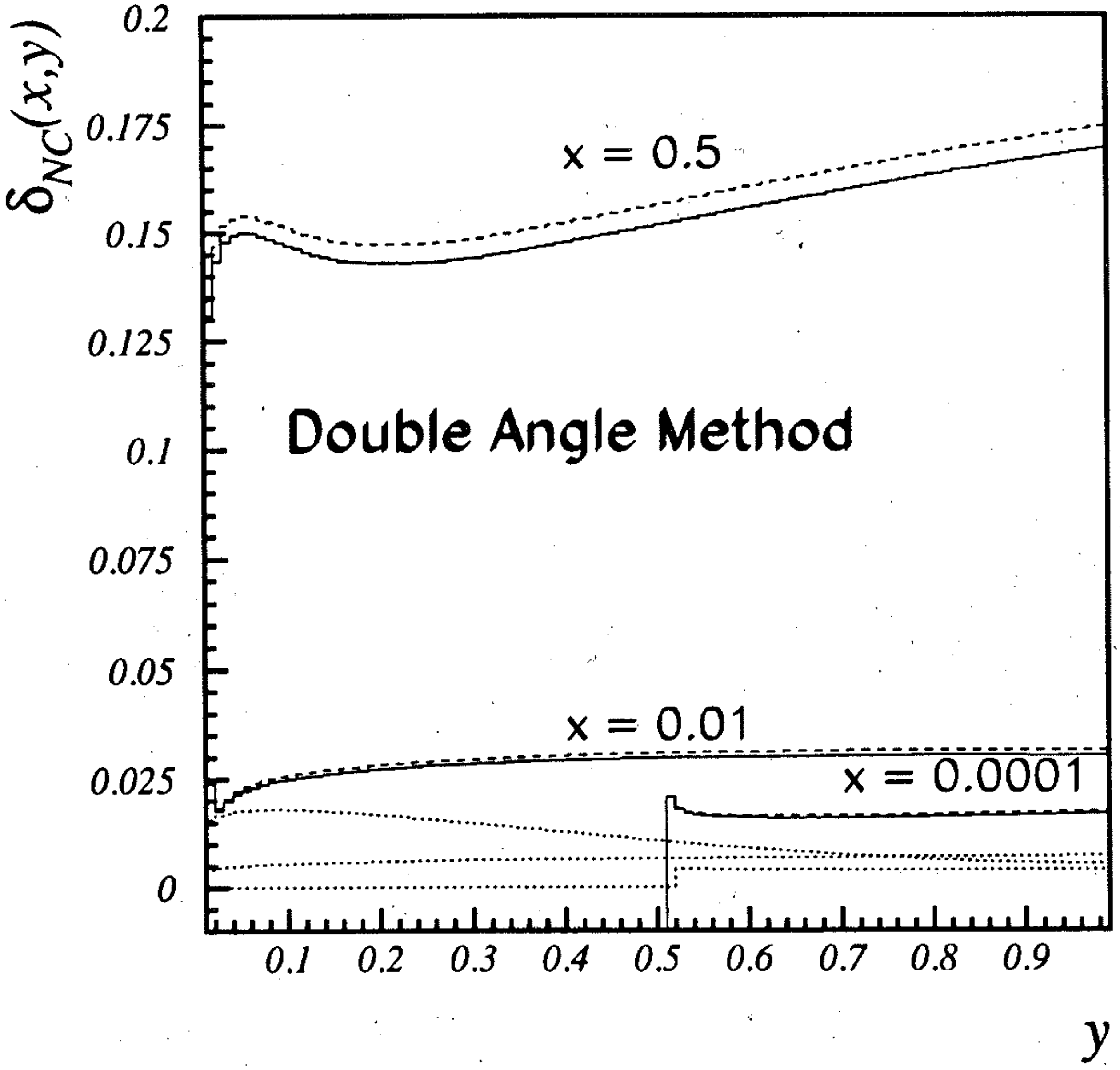


Figure 6: $\delta_{NC}(x,y)$ for the case of the double angle method for $A = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dashed lines: $\delta_{NC}^{(1)}(x,y)$. Dotted lines: $\delta_{NC}^{e^-\to e^+}(x,y)$ scaled by $\times 100$; upper line: $x = 0.5$, middle line: $x = 0.01$, lower line: $x = 0.0001$. The other parameters are the same as in figure 3.

A DANGEROUS CASE:

θ_e & y_J

RESCALING: ISR

$$\hat{Q}^2 = Q^2 \, z \, \frac{z-y}{1-y}$$

$$\hat{x} = x \, \frac{z\,(z-y)}{1-y}$$

$$\boxed{z_0 = y}$$

ZEUS: $\boxed{z_0 = \max\left\{\frac{35\ \mathrm{GeV}}{2E_e}, y\right\}}$

$\delta_{NC}(x,y)$ JUMPS! AT $y \gtrsim \frac{\mathcal{A}}{2E_e}$, $\mathcal{A} = 35$ GeV.

$$\frac{\sigma'(Q^2 \& x \to 0)}{\sigma(Q^2, x)} \quad !$$

NO CONTROL ON INPUT AT ALL !

$\boxed{\longrightarrow \text{UNFORTUNATE CHOICE OF VARIABLES.}}$

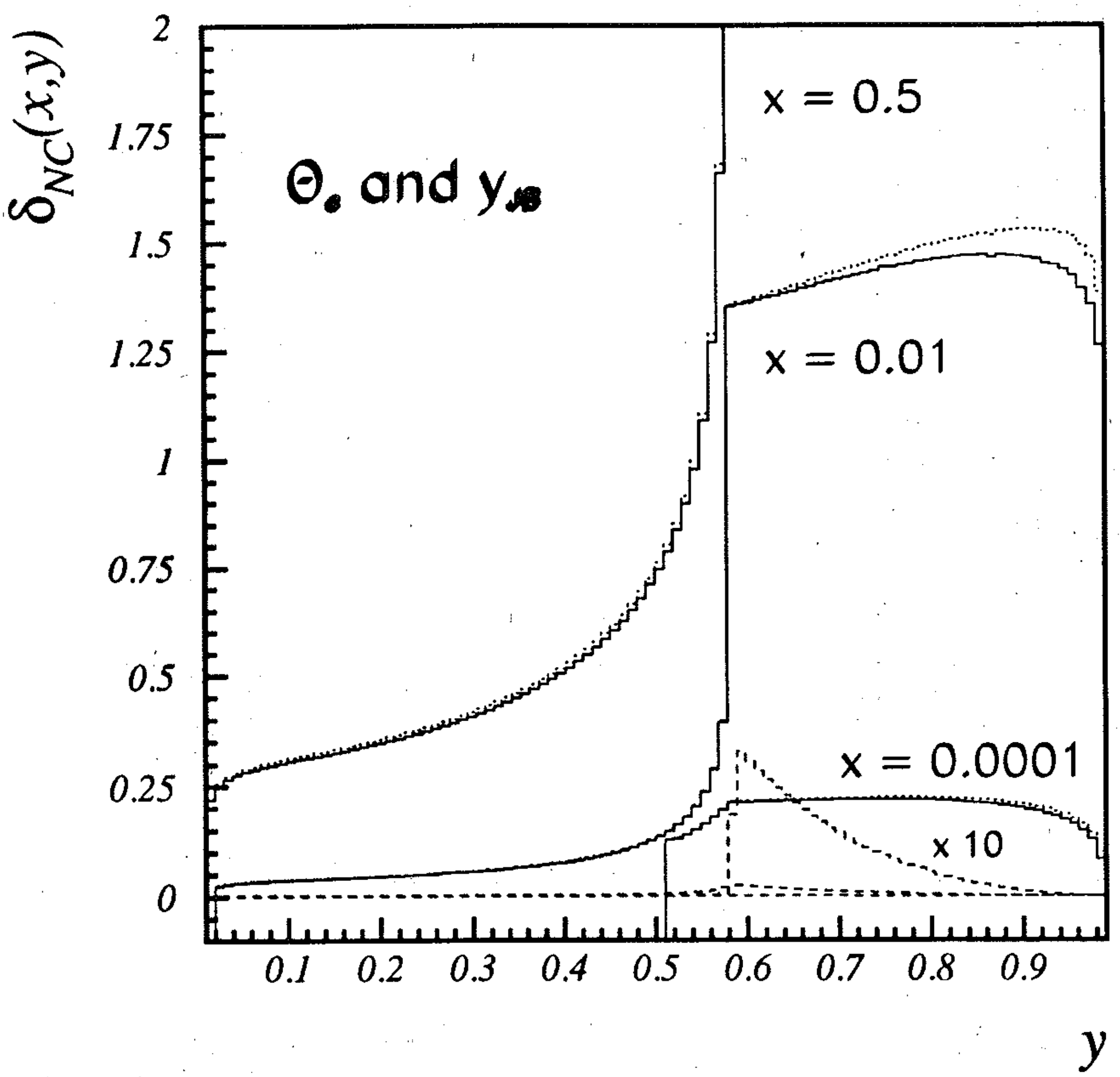


Figure 7: $\delta_{NC}(x,y)$ for the measurement based on θ_e and y_{JB} for $\mathcal{A} = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dotted lines: $\delta_{NC}^{(1)}(x,y)$. Dashed lines: $\delta_{NC}^{e^-\to e^+}(x,y)$; upper line: $x = 0.5$, middle line: $x = 10^{-2}$, lower line: $x = 10^{-4}$. The other parameters are the same as in figure 3.

$\boxed{\text{HECTOR}}$ – a program to calculate QED and electroweak corrections to ep and $l^\pm N$ deep inelastic NC and CC scattering

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ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of NC and CC deep-inelastic charged lepton–proton (or –deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for the calculations at HERA or other ep–colliders, but may be also used for similar processes like muon–proton scattering in fixed–target experiments. The QED corrections may be calculated in several different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading-logarithmic approximation up to order $\mathcal{O}(\alpha^2)$, the exact $\mathcal{O}(\alpha)$corrections and soft-photon exponentiation are taken into account. The photoproduction region is also covered.

† Supported by the Heisenberg-Landau fund.

HADRON
ELECTRON
LEAD-
ING
ORDER
CORRECTIONS

$\oplus$

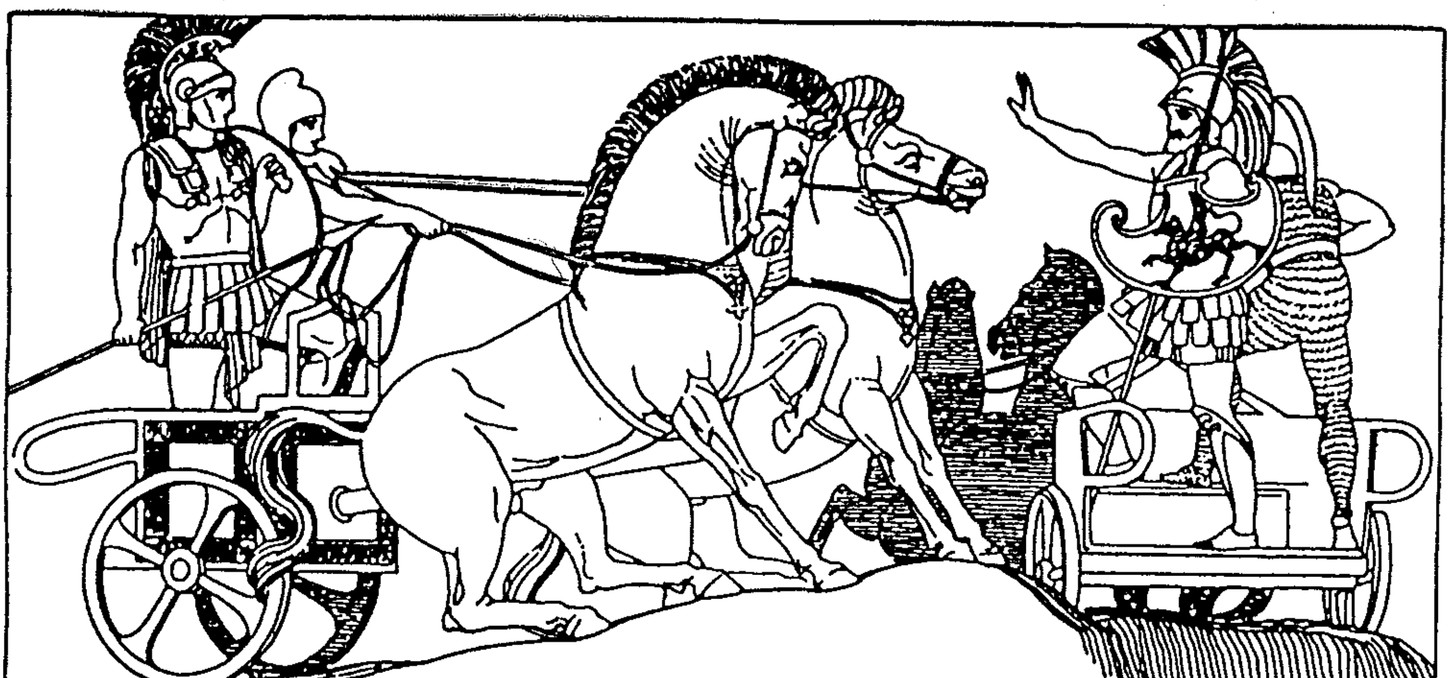
TERAD 91
BARDIN
RIEMANN
AKHUDOV
CHRISTOVA
KALINOVSKAYA

HELIOS
JB.

$\oplus$ UPGRADES QED
QCD
new variables ...

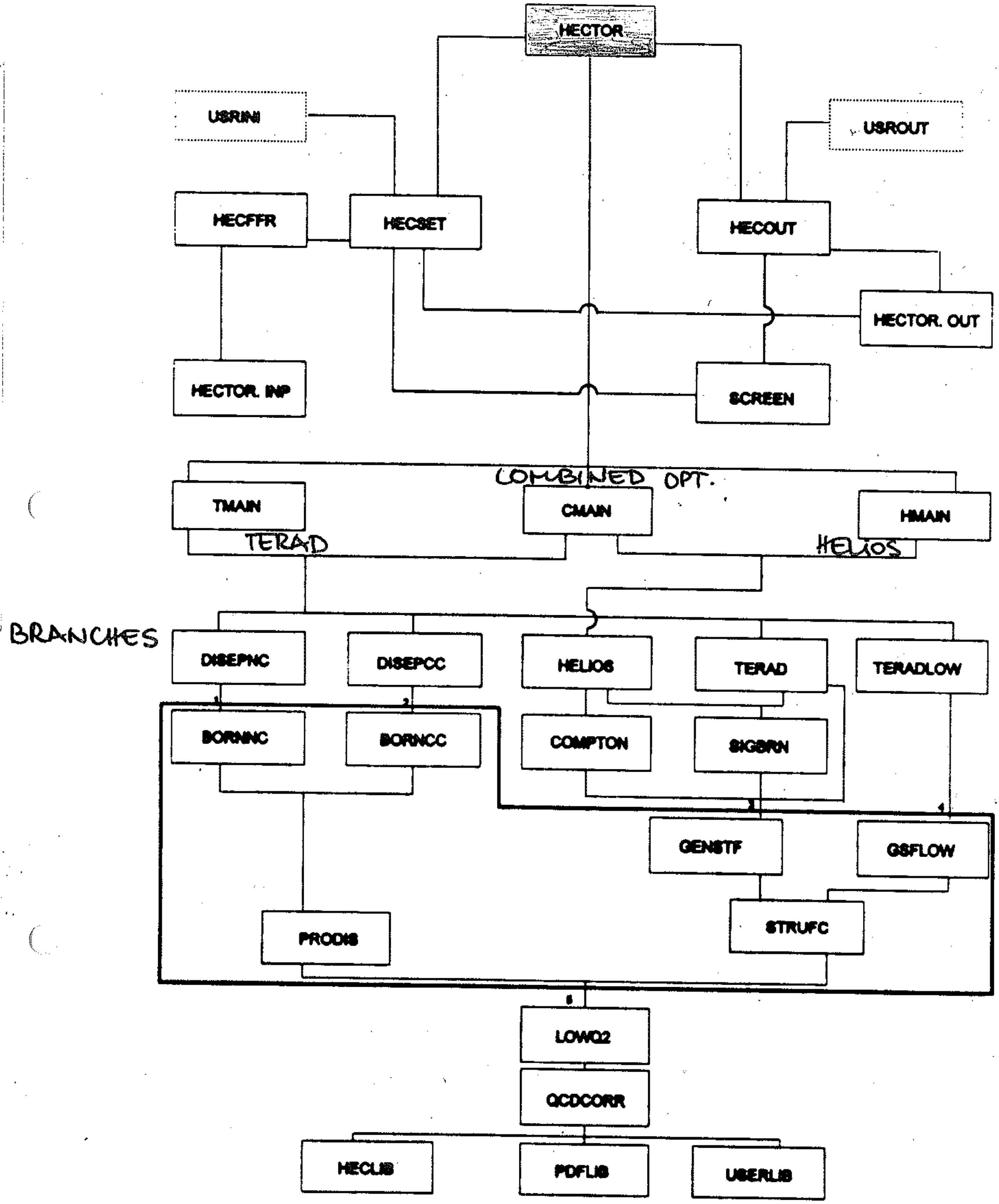
HECTOR

HADRON
ELECTRON
CODE
TO CALCULATE 1st & HIGHER
ORDER
RADIATIVE CORRECTIONS



POLYDAMAS ADVICES HECTOR TO MAKE THE ASSAULT ONTO THE CAMP OF THE GREEKS ON FOOT.

engraving by J. FLAXMAN 1780ies.


HECTOR
USRINI
USROUT
HECFFR
HECSET
HECOUT
HECTOR. OUT
HECTOR. INP
SCREEN
COMBINED OPT.
TMAIN
CMAIN
HMAIN
TERAD
HELIOS
BRANCHES
DISEPNC
DISEPCC
HELIOS
TERAD
TERADLOW
1
2
BORNNC
BORNCC
COMPTON
SIGBRN
3
4
GENSTF
GSFLOW
STRUFC
PRODIS
5
LOWQ2
QCDCORR
HECLIB
PDFLIB
USERLIB

4. Conclusions

1. THE $\mathcal{O}(\alpha L)$ & $\mathcal{O}(\alpha^2 L^2)$ RC'S TO DIS HAVE BEEN CALCULATED FOR:

H1
- LEPTONIC VARIABLES
- JET MEASUREMENT NC & CC
- MIXED VARIABLES

ZEUS
- THE DOUBLE ANGLE METHOD
- VARIABLES BASED ON θ_e & y_J.

$\Big\}$ 1^{ST} CALC.

2. IN $\mathcal{O}(\alpha)$ THE DOMINANCE OF LLA IS ESTABLISHED, GOOD TO EXCELLENT AGREEMENT WITH FULL $\mathcal{O}(\alpha)$ RESULTS IS FOUND.

3. $\delta_{NS}^{(1)}$: DOUBLE ANGLE METHOD $\lesssim^{+} 18\%$ $x \leq 0.5$

$\sim^{+} 2\%$ $x \sim 10^{-4}$.

$\delta_{NS}^{(2)}$

$-5‰$ $x \sim .5$

$-1‰$ $x \sim 10^{-4}$

FLAT BEHAVIOUR! IN y, $x =$ CONST.

4. PROBLEMATIC CASE: θ_e & y_J.

UNSTABLE RC's FOR $y > \frac{Q}{2E_e}$!

ONE SHOULD NOT USE IT FOR THE F_2 MEASUREMENT IN THE DIS RANGE!

5. PERHAPS POSSIBLE : $\dfrac{F_2(x \to 0, Q^2 \to 0)}{}$!
UNFOLDABLE FROM $\delta_{NC}^{(x,y)}$ & $F_2(x, Q^2)$

$\uparrow$ MEAS. $\qquad\qquad$ $\uparrow$ 'KNOWN' BY A DIFFERENT MEASUREMENT

6. JET MEASUREMENT:

$\delta^{(2)}_{NC,CC}$: $\quad x = 10^{-4} \qquad < 5‰$

$\qquad\qquad\quad x = .9 \qquad < 5\%$

7. MIXED VARIABLES

$\delta^{(2)}_{NC}$: $\quad x = 0.5 \qquad \sim 10\% \quad y \to 0$

$\qquad\qquad\qquad\qquad\quad \sim 1\% \quad y \to 1$

8. LEPTON MEASUREMENT : $\delta^{(2)}_{NC} > 10\%$

$y > 0.9$

SLIGHT BULK $\sim 5\%$ $\quad x \sim 0.9 \quad y \lesssim 0.6.$

9. $\mathcal{O}(\alpha^2)$ CORRECTIONS ARE NEEDED FOR A PRECISION ANALYSIS IN ALL VARIABLES.

$(\delta_{\text{LIM.}} < 1\%\ ,\ 0.5\%)$

SOON, $O(\alpha^2 L)$ CORRECTIONS WILL BE AVAILABLE TOO.