3-Loop Non-Singlet Heavy Flavor Wilson Corrections to Deep-Inelastic Scattering and Sum Rules

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in collaboration with :

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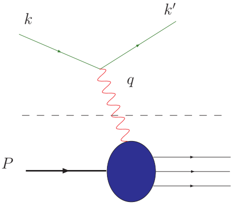
Introduction

Unpolarized Deep–Inelastic Scattering (DIS):



$\longrightarrow L_{\mu\nu}$ $\qquad Q^2 := -q^2, \quad x := \frac{Q^2}{2P.q}$ Bjorken–x

$\longrightarrow W_{\mu\nu}$ $\qquad \frac{d\sigma}{dQ^2\, dx} \sim W_{\mu\nu} L^{\mu\nu}$

$$W_{\mu\nu}(q, P, s) = \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P, s\rangle =$$

$$\frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L(x, Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2(x, Q^2)\ .$$

Structure Functions: $F_{2,L}$ contain light and heavy quark contributions.
Polarized case: $g_{1,2}$

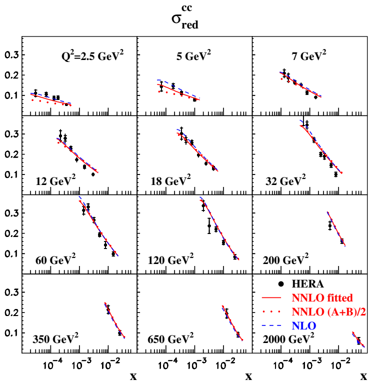
$\alpha_s(M_Z^2)$ from NNLO DIS(+) analyses

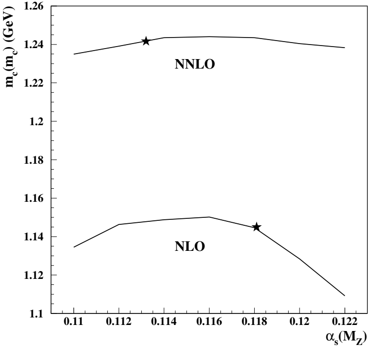
	$\alpha_s(M_Z^2)$	
BBG	$0.1134\ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
GRS	0.112	valence analysis, NNLO
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$
JR	0.1128 ± 0.0010	dynamical approach
JR	0.1162 ± 0.0006	including NLO-jets
MSTW	0.1171 ± 0.0014	
Thorne	0.1136	[DIS+DY+HT*] (2014)
ABM11$_J$	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.
ABM13	0.1133 ± 0.0011	
ABM13	0.1132 ± 0.0011	(without jets)
CTEQ	0.1159..0.1162	
CTEQ	0.1140	(without jets)
NN21	$0.1174 \pm 0.0006 \pm 0.0001$	
Gehrmann et al.	$0.1131\ ^{+\ 0.0028}_{-\ 0.0022}$	e^+e^- thrust
Abbate et al.	0.1140 ± 0.0015	e^+e^- thrust
BBG	$0.1141\ ^{+0.0020}_{-0.0022}$	valence analysis, $\mathrm{N^3LO}$

$$\Delta_{\rm TH}\alpha_s = \alpha_s(\mathrm{N^3LO}) - \alpha_s(\mathrm{NNLO}) + \Delta_{\rm HQ} = +0.0009 \pm 0.0006_{\rm HQ}$$

NNLO accuracy is needed to analyze the world data. $\Longrightarrow$ NNLO HQ corrections needed.

Deep–Inelastic Scattering (DIS):





NNLO:

S. Alekhin, J. Blümlein, K. Daum, K. Lipka, Phys.Lett. B720 (2013) 172 [1212.2355]

$$m_c(m_c) = 1.24 \pm 0.03(exp) \begin{array}{c} +0.03 \\ -0.02 \end{array} (scale) \begin{array}{c} +0.00 \\ -0.07 \end{array} (thy),$$

$$m_b(m_b) = 3.97 \pm 0.14(exp) \begin{array}{c} +0.00 \\ -0.11 \end{array} (thy) \text{ (preliminary)},$$

Yet approximate NNLO treatment [Kawamura et al. [1205.5227]].

Factorization of the Structure Functions

At leading twist the structure functions factorize in terms of a Mellin convolution

$$F_{(2,L)}(x, Q^2) = \sum_j \underbrace{\mathbb{C}_{j,(2,L)}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right)}_{perturbative} \otimes \underbrace{f_j(x, \mu^2)}_{nonpert.}$$

into (pert.) Wilson coefficients and (nonpert.) parton distribution functions (PDFs).

$\otimes$ denotes the Mellin convolution

$$f(x) \otimes g(x) \equiv \int_0^1 dy \int_0^1 dz \, \delta(x - yz) f(y) g(z) \, .$$

The subsequent calculations are performed in Mellin space, where $\otimes$ reduces to a multiplication, due to the Mellin transformation

$$\hat{f}(N) = \int_0^1 dx \, x^{N-1} f(x) \, .$$

Wilson coefficients:

$$\mathbb{C}_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)=C_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2}\right)+H_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)\ .$$

At $Q^2 \gg m^2$ the heavy flavor part

$$H_{j,(2,L)}\left(N,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)=\sum_i C_{i,(2,L)}\left(N,\frac{Q^2}{\mu^2}\right)A_{ij}\left(\frac{m^2}{\mu^2},N\right)$$

[Buza, Matiounine, Smith, van Neerven 1996 Nucl.Phys.B]

factorizes into the light flavor Wilson coefficients C and the massive operator matrix elements (OMEs) of local operators O_i between partonic states j

$$A_{ij}\left(\frac{m^2}{\mu^2},N\right)=\langle j \mid O_i \mid j\rangle\ .$$

$\rightarrow$ additional Feynman rules with local operator insertions for partonic matrix elements.

The unpolarized light flavor Wilson coefficients are known up to NNLO

[Moch, Vermaseren, Vogt, 2005 Nucl.Phys.B].

For $F_2(x,Q^2)$: at $Q^2 \gtrsim 10m^2$ the asymptotic representation holds at the 1% level.

The Non-Singlet Wilson Coefficients for Photon Exchange

We consider different unpolarized and polarized flavor non-singlet combinations of structure function.

- $F_2^{ep,\mathrm{NS}}(x, Q^2)$
- $g_1^{ep,\mathrm{NS}}(x, Q^2)$
- $xF_3^{\overline{\nu}N} + xF_3^{\nu N}$

Associated sum rules:

- Adler and unpolarized Bjorken sum rule
- polarized Bjorken sum rule
- Gross-Llewellyn Smith sumrule.

The calculation of the respective OMEs delivers the contributions to the 3-loop anomalous dimensions $\propto T_F$ as a by-product.

The Wilson Coefficients at large Q^2

2014 $L_{q,(2,L)}^{\mathsf{NS}}(N_F+1) = a_s^2\Big[A_{qq,Q}^{(2),\mathsf{NS}}(N_F+1)\ \delta_2 + \hat{C}_{q,(2,L)}^{(2),\mathsf{NS}}(N_F)\Big] + a_s^3\Big[A_{qq,Q}^{(3),\mathsf{NS}}(N_F+1)\ \delta_2 + A_{qq,Q}^{(2),\mathsf{NS}}(N_F+1)C_{q,(2,L)}^{(1),\mathsf{NS}}(N_F+1) + \hat{C}_{q,(2,L)}^{(3),\mathsf{NS}}(N_F)\Big]$

2010 $L_{q,(2,L)}^{\mathsf{PS}}(N_F+1) = a_s^3\Big[\ A_{qq,Q}^{(3),\mathsf{PS}}(N_F+1)\ \delta_2 + A_{gq,Q}^{(2)}(N_F)\ \ N_F\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + N_F\hat{\tilde{C}}_{q,(2,L)}^{(3),\mathsf{PS}}(N_F$

2010 $L_{g,(2,L)}^{\mathsf{S}}(N_F+1) = a_s^2A_{gg,Q}^{(1)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + a_s^3\Big[\ A_{qg,Q}^{(3)}(N_F+1)\ \delta_2 + A_{gg,Q}^{(1)}(N_F+1)\ \ N_F\tilde{C}_{g,(2,L)}^{(2)}(N_F+1) + A_{gg,Q}^{(2)}(N_F+1)\ \ N_F\tilde{C}_{g,(2,L)}^{(1)}(N_F + {} + A_{Qg}^{(1)}(N_F+1)\ \ N_F\tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(N_F+1) + N_F\hat{\tilde{C}}_{g,(2,L)}^{(3)}(N_F)\Big]\ ,$

2014 $H_{q,(2,L)}^{\mathsf{PS}}(N_F+1) = a_s^2\Big[\ A_{Qq}^{(2),\mathsf{PS}}(N_F+1)\ \delta_2 + \ \tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(N_F+1)\Big] + a_s^3\Big[\ A_{Qq}^{(3),\mathsf{PS}}(N_F+1)\ \delta_2 + \tilde{C}_{q,(2,L)}^{(3),\mathsf{PS}}(N_F+1) + A_{gq,Q}^{(2)}(N_F+1)\ \tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + A_{Qq}^{(2),\mathsf{PS}}(N_F+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(N_F+1)\Big]\ ,$

$$H_{g,(2,L)}^{\mathsf{S}}(N_F+1) = a_s\Big[\ A_{Qg}^{(1)}(N_F+1)\ \delta_2 + \ \tilde{C}_{g,(2,L)}^{(1)}(N_F+1)\Big] + a_s^2\Big[\ A_{Qg}^{(2)}(N_F+1)\ \delta_2 + A_{Qg}^{(1)}(N_F+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(N_F+1) + \ A_{gg,Q}^{(1)}(N_F+1)\ \tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \tilde{C}_{g,(2,L)}^{(2)}(N_F+1)\Big] + \ a_s^3\Big[\ A_{Qg}^{(3)}(N_F+1)\ \delta_2 + \ A_{Qg}^{(2)}(N_F+1)\ C_{q,(2,L)}^{(1),\mathsf{NS}}(N_F + {} + A_{gg,Q}^{(2)}(N_F+1)\ \tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \ A_{Qg}^{(1)}(N_F+1)\Big\{C_{q,(2,L)}^{(2),\mathsf{NS}}(N_F+1) + \tilde{C}_{q,(2,L)}^{(2),\mathsf{PS}}(N_F+1)\Big\} + \ A_{gg,Q}^{(1)}(N_F+1)\ \tilde{C}_{g,(2,L)}^{(2)}(N_F+1) + \ \tilde{C}_{g,(2,L)}^{(3)}(N_F+1$$

[Ablinger et al. 2010, Ablinger et al., 2014a, Ablinger et al., 2014b]

Variable Flavor Number Scheme

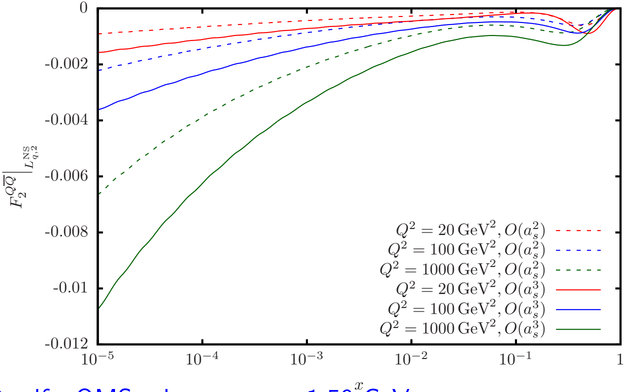
$$
\begin{aligned}
f_k(n_f+1,\mu^2) + f_{\bar{k}}(n_f+1,\mu^2) &= A_{qq,Q}^{\rm NS}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes \left[f_k(n_f,\mu^2) + f_{\bar{k}}(n_f,\mu^2)\right] \\
&\quad + \tilde{A}_{qq,Q}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes \Sigma(n_f,\mu^2) + \tilde{A}_{qg,Q}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes G(n_f,\mu^2) \\
f_{Q+\bar{Q}}(n_f+1,\mu^2) &= \tilde{A}_{Qq}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes \Sigma(n_f,\mu^2) + \tilde{A}_{Qg}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes G(n_f,\mu^2)\ . \\
G(n_f+1,\mu^2) &= A_{gq,Q}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes \Sigma(n_f,\mu^2) + A_{gg,Q}^{S}\left(n_f,\frac{\mu^2}{m^2}\right) \otimes G(n_f,\mu^2)\ . \\
\Sigma(n_f+1,\mu^2) &= \sum_{k=1}^{n_f+1}\left[f_k(n_f+1,\mu^2) + f_{\bar{k}}(n_f+1,\mu^2)\right] \\
&= \left[A_{qq,Q}^{\rm NS}\left(n_f,\frac{\mu^2}{m^2}\right) + n_f\,\tilde{A}_{qq,Q}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right) + \tilde{A}_{Qq}^{\rm PS}\left(n_f,\frac{\mu^2}{m^2}\right)\right] \\
&\quad \otimes \Sigma(n_f,\mu^2) \\
&\quad + \left[n_f\,\tilde{A}_{qg,Q}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right) + \tilde{A}_{Qg}^{\rm S}\left(n_f,\frac{\mu^2}{m^2}\right)\right] \otimes G(n_f,\mu^2)
\end{aligned}
$$

All master integrals for $A_{gg}^{(3)}$ have just been completed (June 2015).

The Non-Singlet Heavy Flavor Contributions to $F_2(x, Q^2)$

J. Ablinger et al., [Nucl.Phys. B886 (2014) 733.]

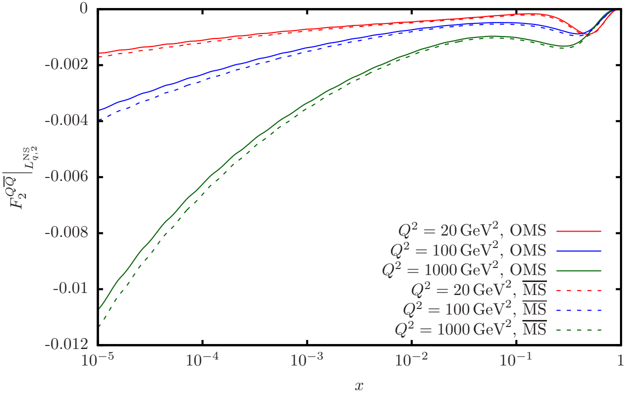
- Starting from 3-loop order, only the inclusive case has a clear definition. The separation into 'tagged' and 'remainder heavy flavor' in the NS case is only possible up to 2-loop order.
- Consider first the inclusive contributions at $O(a_s^2)$ and $O(a_s^3)$:



ABM12 pdfs, OMS scheme, $m_c = 1.59$ GeV.

The Non-Singlet Heavy Flavor Contributions to $F_2(x, Q^2)$

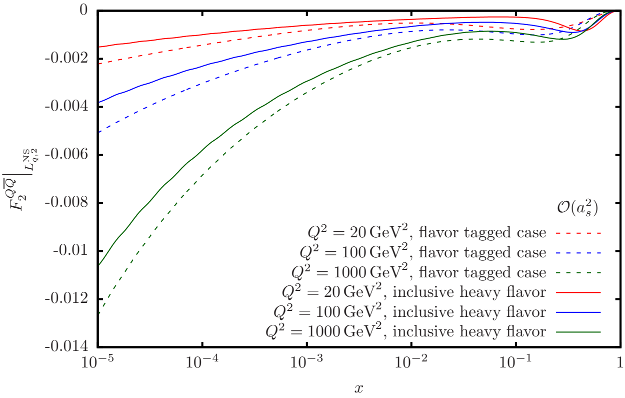
The choice of the renormalization scheme for m_c: OMS vs $\overline{\mathrm{MS}}$:



ABM12 pdfs, OMS scheme, $m_c = 1.59$ GeV.

The Non-Singlet Heavy Flavor Contributions to $F_2(x, Q^2)$

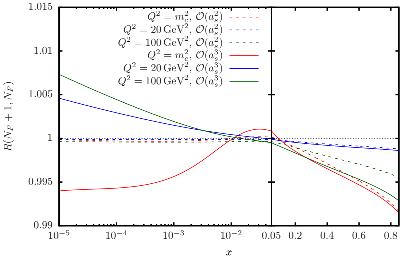
$O(a_s^2)$: The difference between the inclusive and the tagged case.



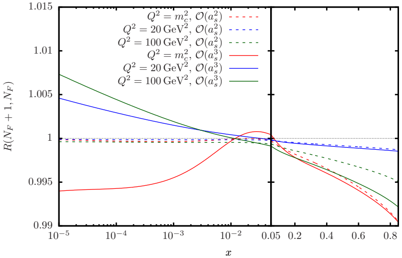
ABM12 pdfs, OMS scheme, $m_c = 1.59$ GeV.

The Non-Singlet Heavy Flavor Contributions to $F_2(x, Q^2)$

Variable flavor scheme matching at $O(a_s^2)$ and $O(a_s^3)$:



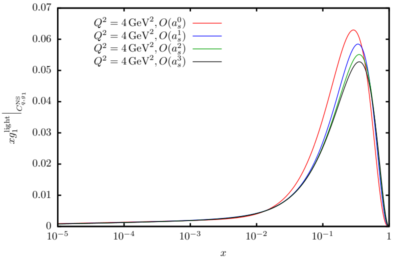
$u + \overline{u}$



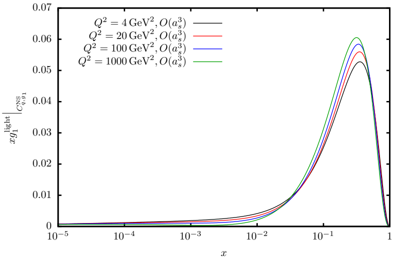
$d + \overline{d}$

ABM12 pdfs, OMS scheme, $m_c = 1.59$ GeV.

A. Behring et al. [Nucl.Phys. B897 (2015) 612]
The massless and asympotic massive 3-Loop corrections. BB10 pdfs (NLO) are used.
Massless contributions:



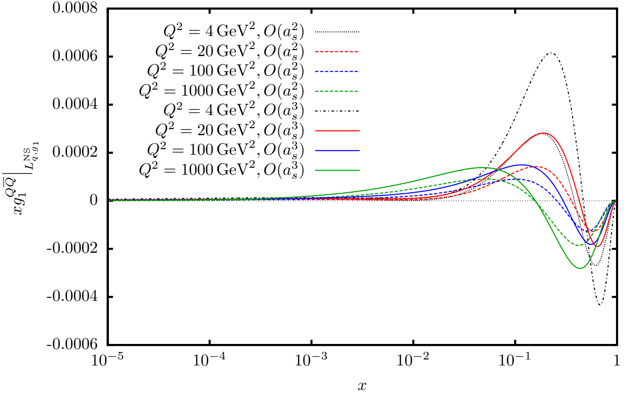
different orders



evolution in Q^2

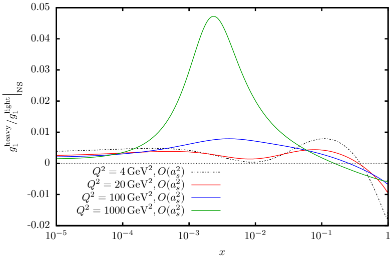
$g_1(x, Q^2)$

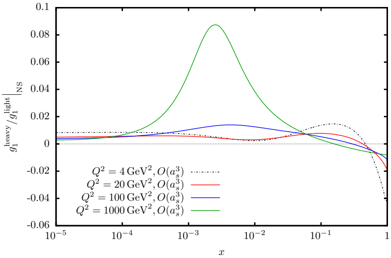
The heavy quark (charm) contribution.



$g_1(x, Q^2)$

The ratio of the inclusive charm contribution to those of the light partons.

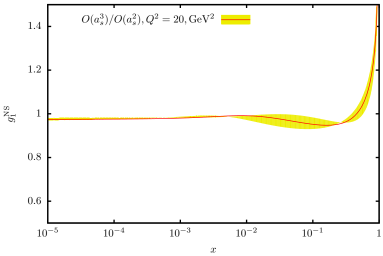


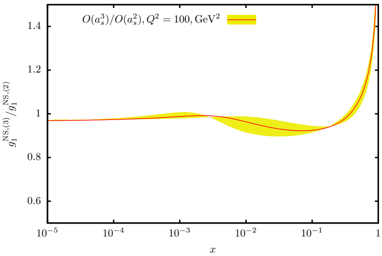


a_s^2 a_s^3

$g_1(x, Q^2)$

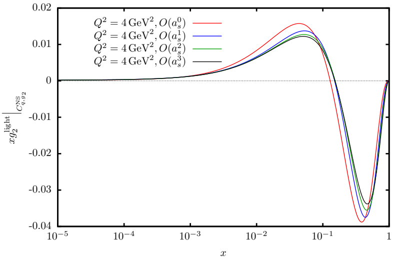
Scale dependence. $Q^2/4 < \mu^2_{R,F} < 4Q^2$:



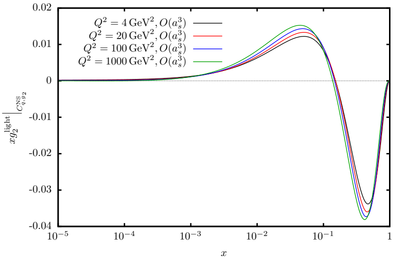


$g_2(x, Q^2)$

A. Behring et al. [Nucl.Phys. B897 (2015) 612]
The massless and asymptic massive 3-Loop corrections. BB10 pdfs (NLO) are used.
Massless contributions:



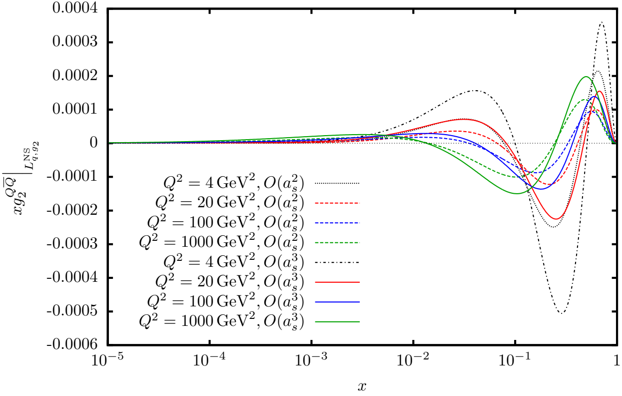
different orders



evolution in Q^2

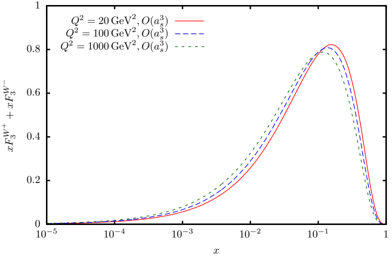
$g_2(x, Q^2)$

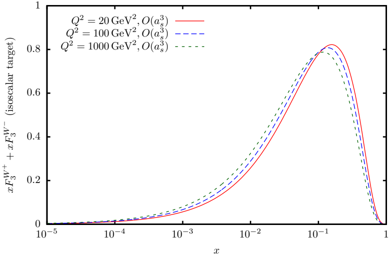
A. Behring et al. [Nucl.Phys. B897 (2015) 612]
The massive contribution for charm.



$xF_3(x, Q^2)$

A. Behring et al., arXiv:1508.01449.
The massless and asymptotic massive contributions for charm. ABM13 pdfs are used.



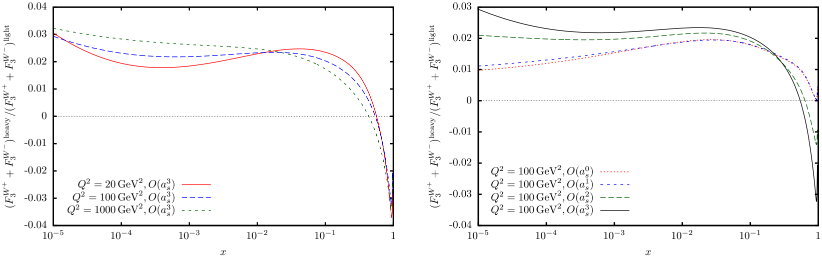


proton target　　isoscalar target

$\Longrightarrow$ The combination is nearly isoscalar from the combination of currents.

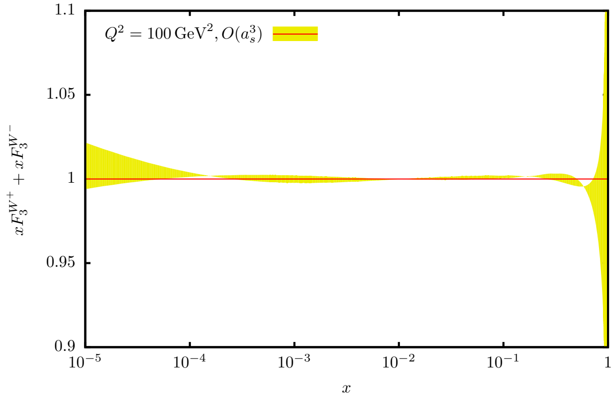
$xF_3(x, Q^2)$

The scale evolution of the heavy vs the light contributions.



$xF_3(x, Q^2)$

The scale variations $Q^2/4 < \mu^2_{R,F} < 4Q^2$



Sum Rules

The first moments of the Wilson coefficients form named sum rules. $[N_F = 3]$

Adler sum rule

$$\int_0^1 \frac{dx}{x} \left[F_2^{\overline{\nu}p} - F_2^{\nu p}\right] = 2(1 + s_\theta^2)$$

It receives neither QCD nor mass corrections.

Unpolarized Bjorken sum rule

$$\int_0^1 dx \left[F_1^{\overline{\nu}N} - F_1^{\nu N}\right] = (1 + s_\theta^2) C_{\mathrm{uBj}}$$

Polarized Bjorken sum rule

$$\int_0^1 dx \left[g_1^{ep} - g_1^{en}\right] = \frac{1}{6} \left|\frac{g_A}{g_V}\right| C_{\mathrm{pBj}}$$

Gross-Llewellyn Smith sum rule

$$\int_0^1 dx \left[F_3^{\overline{\nu}p} + F_3^{\nu p}\right] = 3(1 - s_\theta^2) C_{\mathrm{GLS}}$$

All other sum rules receive QCD and mass corrections.

Sum Rules

In the limit $Q^2 \gg m^2$ the sum rules are just modified by setting $N_F \to N_F + 1$ and adjusting the CKM matrix elements accordingly. There are no logarithmic, but just power corrections, yet computable only in the region, where Q^2 is a deep-inelastic scale. $[\hat{a}_s = \alpha_s/\pi]$.

Unpolarized Bjorken sum rule [Chetyrkin et al. (2014)]

$$
\begin{aligned}
C_{\rm uBj} &= 1 - \hat{a}_s 0.6667 - \hat{a}_s^2(3.833 - 2.963 N_F) - \hat{a}_s^3(36.15 - 6.331 N_F + 0.1595 N_F^2) \\
&\quad + \hat{a}_s^4(-436.8 + 111.9 N_F - 7.115 N_F^2 + 0.1017 N_F^3)
\end{aligned}
$$

Polarized Bjorken sum rule [Baikov et al. (2012)]

$$
\begin{aligned}
C_{\rm pBj} &= 1 - \hat{a}_s + \hat{a}_s^2(-4.58333 + 0.33333 N_F) + \hat{a}_s^3(-41.4399 + 7.60729 N_F \\
&\quad - 0.17747 N_F^2) + \hat{a}_s^4(-479.448 + 123.472 N_F - 7.69747 N_F^2 + 0.10374 N_F^3)
\end{aligned}
$$

Gross-Llewellyn Smith sum rule [Baikov et al. (2010)]

$$
\begin{aligned}
C_{\rm GLS} &= 1 - \hat{a}_s + \hat{a}_s^2(-4.58333 + 0.33333 N_F) + \hat{a}_s^3(-41.4399 + 7.74370 N_F \\
&\quad - 0.17747 N_F^2) + \hat{a}_s^4(-479.448 + 140.796 N_F - 8.39702 N_F^2 + 0.10374 N_F^3)
\end{aligned}
$$

Power corrections: [J.B. et al. (2015)].

2-mass case

$$
\begin{aligned}
\tilde{a}_{qq,Q}^{(3),\mathrm{NS}} \quad = \quad & C_F T_F^2 \Bigg\{ \left(\frac{32}{27} S_1 - \frac{8\left(3N^2+3N+2\right)}{27N(N+1)} \right) \ln^3(\eta) + \Bigg[-\frac{R_1}{18N^2(N+1)^2\eta} \\
& + \Bigg[\frac{\left(3N^2+3N+2\right)(\eta+1)\left(5\eta^2+22\eta+5\right)}{36N(N+1)\eta^{3/2}} - \frac{(\eta+1)\left(5\eta^2+22\eta+5\right)}{9\eta^{3/2}} S_1 \Bigg] \ln\left(\frac{1+\eta_1}{1-\eta_1}\right) \\
& + \frac{2\left(5\eta^2+2\eta+5\right)}{9\eta} S_1 + \ln(1-\eta)\left(\frac{16\left(3N^2+3N+2\right)}{9N(N+1)} - \frac{64}{9} S_1 \right) + \frac{32}{9} S_2 \Bigg] \ln^2(\eta) \\
& + \Bigg[\frac{40(\eta-1)(\eta+1)}{9\eta} S_1 - \frac{10\left(3N^2+3N+2\right)(\eta-1)(\eta+1)}{9N(N+1)\eta} + \frac{(\eta+1)\left(5\eta^2+22\eta+5\right)}{9\eta^{3/2}} \\
& \times \Bigg[8S_1 - \frac{2\left(3N^2+3N+2\right)}{N(N+1)} \Bigg] \mathrm{Li}_2(\eta_1) + \frac{(\eta_1+1)^2\left(-10\eta^{3/2}+5\eta^2+42\eta-10\eta_1+5\right)}{9\eta^{3/2}} \\
& \times \left[\frac{\left(3N^2+3N+2\right)}{2N(N+1)} - 2S_1 \right] \mathrm{Li}_2(\eta) \Bigg] \ln(\eta) + \frac{16\left(3N^4+6N^3+47N^2+20N-12\right)\zeta_2}{27N^2(N+1)^2} \\
& + \frac{(\eta+1)\left(5\eta^2+22\eta+5\right)}{9\eta^{3/2}} \Bigg[\frac{4\left(3N^2+3N+2\right)}{N(N+1)} - 16S_1 \Bigg] \mathrm{Li}_3(\eta_1) - \frac{1280}{81} S_3 + \frac{256}{27} S_4 \\
& + \frac{(\eta_1+1)^2\left(-10\eta^{3/2}+5\eta^2+42\eta-10\eta_1+5\right)}{9\eta^{3/2}} \Bigg[2S_1 - \frac{\left(3N^2+3N+2\right)}{2N(N+1)} \Bigg] \mathrm{Li}_3(\eta) + \left[\frac{128\zeta_2}{9} + \frac{3712}{81} \right] S_2 \\
& + \left[\frac{16\left(405\eta^2-3238\eta+405\right)}{729\eta} + \frac{256\zeta_3}{27} - \frac{640\zeta_2}{27} \right] S_1 - \frac{64\left(3N^2+3N+2\right)\zeta_3}{27N(N+1)} - \frac{4R_2}{729N^4(N+1)^4\eta} \Bigg\} .
\end{aligned}
$$

$\eta = m_1/m_2; \eta_1 = \sqrt{\eta}.$

Conclusions

- The flavor non-singlet heavy quark contributions to unpolarized and polarized structure functions up to 3 loop order are fully understood at large enough virtualities.
- The "tagged flavor" picture is inapplicable from 3-loop order onward.
- The non-singlet variable flavor transition coefficients are known to 3-loop order for single flavor transitions.
- All effects are of the order of a few per cent, due to the fact that the effects first emerge at $O(a_s^2)$.
- The case of two different quark masses has also been dealt with ([J.B., F. Wißbrock, et al., 2015]).
- The associated sum rules do not yield logarithmic contributions. Asymptotically in Q^2, the heavy quark contributes by shifting N_F by one unit. At lower scales there are power corrections, which are currently completed at $O(a_s^2)$.