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$O(\alpha)$ QED Corrections to Neutral Current Polarized Deep–Inelastic Lepton–Nucleon Scattering

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1. BORN CROSS SECTION

2. O(α) CORRECTION

3. LLA

4. NUMERICAL RESULTS

1. The Born Cross Section

The Feynman diagram describing neutral current deep-inelastic lepton–nucleon scattering

$$l(k_1) + p(p) \to l(k_2) + X(p') \tag{1}$$

is shown in Figure 1. The particle 4–momenta are given in parentheses.

The matrix element for the Born cross section reads

$$\mathcal{M}_{\rm Born} = ie^2\langle p'|\mathcal{J}_\mu|p\rangle \frac{1}{Q^2}\bar{u}(k_2)\left[Q_l\gamma^\mu + s_l\gamma^\mu\left(v_l + a_l\gamma_5\right)\chi(Q^2)\right]u(k_1)\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2k_1^0 2k_2^0}}. \tag{2}$$

For later use we rewrite the differential Born cross section (5) in terms of the leptonic and hadronic tensors, $L^{\mu\nu}$ and $W_{\mu\nu}$, integrating over the phase space up to two variables:

$$d\sigma_{\rm Born} = \frac{2\alpha^2}{\sqrt{\lambda_S}}\frac{1}{Q^4}\left[L^{\mu\nu}W_{\mu\nu}\right]\frac{d\vec{k}_2}{k_2^0} = \frac{2\pi\alpha^2}{\lambda_S}\frac{S^2 y}{Q^4}\left[L^{\mu\nu}W_{\mu\nu}\right]dx\,dy. \tag{9}$$

$$\begin{aligned} L^{\mu\nu} = {} & \left[2\left(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu\right) - g^{\mu\nu}Q^2\right]L_S(Q^2,\lambda_l) - 2ik_{1\alpha}k_{2\beta}\varepsilon^{\alpha\beta\mu\nu}L_A(Q^2,\lambda_l) \\ & +4\frac{m^2}{S}\Big\{i\,\varepsilon^{\alpha\beta\mu\nu}\left[p_\alpha q_\beta L_v(Q^2,\lambda_l) - p_\alpha\left(k_{1\beta}+k_{2\beta}\right)L_a(Q^2,\lambda_l)\right] \\ & +\Big[p^\mu\left(k_1^\nu + k_2^\nu\right) + p^\nu\left(k_1^\mu + k_2^\mu\right) - p^\mu q^\nu - p^\nu q^\mu \\ & -g^{\mu\nu}\left[p.(k_1+k_2) - p.q\right]\Big]L_\chi(Q^2,\lambda_l)\Big\} + 4g^{\mu\nu}m^2\left[L_\chi(Q^2,\lambda_l) - L_a(Q^2,\lambda_l)\right]. \end{aligned} \tag{14}$$

The symmetric (S) and antisymmetric (A) parts are

$$L_S(Q^2,\lambda_l) = Q_l^2 + 2|Q_l|\left(v_l - p_l\lambda_l a_l\right)\chi(Q^2) + \left(v_l^2 + a_l^2 - 2p_l\lambda_l v_l a_l\right)\chi^2(Q^2), \tag{15}$$

$$L_A(Q^2,\lambda_l) = -\lambda_l Q_l^2 + 2|Q_l|\left(p_l a_l - \lambda_l v_l\right)\chi(Q^2) + \left(2p_l v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right)\chi^2(Q^2). \tag{16}$$

The form factors contributing to the terms $\propto m^2$ are

$$\begin{aligned} L_v(Q^2,\lambda_l) &= \lambda_l\left[|Q_l| + v_l\chi(Q^2)\right]^2, \\ L_a(Q^2,\lambda_l) &= \lambda_l a_l^2\chi^2(Q^2), \\ L_\chi(Q^2,\lambda_l) &= \lambda_l p_l a_l\chi(Q^2)\left[|Q_l| + v_l\chi(Q^2)\right]. \end{aligned} \tag{17}$$

The particle label p_l takes the values $p_l = 1$ for particles and -1 for antiparticles.

The hadronic tensor reads

$$W_{\mu\nu} = p^0(2\pi)^6 \sum \int \langle p'|\mathcal{J}_\mu|p\rangle\langle p|\mathcal{J}_\nu|p'\rangle \delta^4(p' - \sum_i p_i') \prod_i d\vec{p}_i\,'. \tag{18}$$

For its representation in terms of structure functions we follow the convention of ref. [11],

$$\begin{aligned} W_{\mu\nu} = & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(x,Q^2) + \frac{\widehat{p_\mu}\widehat{p_\nu}}{p.q}\mathcal{F}_2(x,Q^2) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2p.q}\mathcal{F}_3(x,Q^2) \\ & + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda s^\sigma}{p.q}\mathcal{G}_1(x,Q^2) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(p.qs^\sigma - s.qp^\sigma)}{(p.q)^2}\mathcal{G}_2(x,Q^2) \\ & + \left[\frac{\widehat{p_\mu}\widehat{s_\nu} + \widehat{s_\mu}\widehat{p_\nu}}{2} - s.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{p.q}\right]\frac{1}{p.q}\mathcal{G}_3(x,Q^2) \\ & + s.q\frac{\widehat{p_\mu}\widehat{p_\nu}}{(p.q)^2}\mathcal{G}_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{s.q}{p.q}\mathcal{G}_5(x,Q^2), \end{aligned} \tag{19}$$

where

$$\widehat{p_\mu} = p_\mu - \frac{p.q}{q^2}q_\mu, \qquad \widehat{s_\mu} = s_\mu - \frac{s.q}{q^2}q_\mu. \tag{20}$$

s denotes the polarization 4–vector of the nucleon. In the nucleon rest frame it is given by

$$s = M(0, \vec{n}_\lambda). \tag{21}$$

$\vec{n}_\lambda$ is an unit 3-vector.

For a short-hand notation we have introduced the combined neutral current structure functions $\mathcal{F}_i$ and $\mathcal{G}_i$ in eq. (19). In terms of the structure functions $F_i^{J_1J_2}$ and $g_i^{J_1J_2}$, which are associated with the respective currents, they read:

$$\begin{aligned} \mathcal{F}_{1,2}(x,Q^2) = & Q_l^2 F_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_l^2|(v_l - p_l\lambda_l a_l)\chi(Q^2)F_{1,2}^{\gamma Z}(x,Q^2) \\ & + \left(v_l^2 + a_l^2 - 2p_l\lambda_l v_l a_l\right)\chi^2(Q^2)F_{1,2}^{ZZ}(x,Q^2), \end{aligned} \tag{22}$$

$$\begin{aligned} \mathcal{F}_3(x,Q^2) = & 2|Q_l|(p_l a_l - \lambda_l v_l)\chi(Q^2)F_3^{\gamma Z}(x,Q^2) \\ & + \left[2p_l v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2)F_3^{ZZ}(x,Q^2), \end{aligned} \tag{23}$$

$$\begin{aligned} \mathcal{G}_{1,2}(x,Q^2) = & -Q_l^2\lambda_l g_{1,2}^{\gamma\gamma}(x,Q^2) + 2|Q_l|(p_l a_l - \lambda_l v_l)\chi(Q^2)g_{1,2}^{\gamma Z}(x,Q^2) \\ & + \left[2p_l v_l a_l - \lambda_l\left(v_l^2 + a_l^2\right)\right]\chi^2(Q^2)g_{1,2}^{ZZ}(x,Q^2), \end{aligned} \tag{24}$$

$$\begin{aligned} \mathcal{G}_{3,4,5}(x,Q^2) = & 2|Q_l|(v_l - p_l\lambda_l a_l)\chi(Q^2)g_{3,4,5}^{\gamma Z}(x,Q^2) \\ & + \left(v_l^2 + a_l^2 - 2p_l\lambda_l v_l a_l\right)\chi^2(Q^2)g_{3,4,5}^{ZZ}(x,Q^2). \end{aligned} \tag{25}$$

The twist-2 contributions to the structure functions can be expressed in terms of parton densities. In lowest order QCD one obtains [12, 13, 11] :

$$F_1^{J_1J_2}(x,Q^2) = \sum_q \alpha^q_{J_1J_2}\left[q(x,Q^2)+\bar{q}(x,Q^2)\right], \tag{26}$$

$$F_2^{J_1J_2}(x,Q^2) = 2xF_1^{J_1J_2}(x,Q^2), \tag{27}$$

$$F_3^{J_1J_2}(x,Q^2) = \sum_q \beta^q_{J_1J_2}\left[q(x,Q^2)-\bar{q}(x,Q^2)\right], \tag{28}$$

$$g_1^{J_1J_2}(x,Q^2) = \frac{1}{2}\sum_q \alpha^q_{J_1J_2}\left[\Delta q(x,Q^2)+\Delta\bar{q}(x,Q^2)\right], \tag{29}$$

$$g_2^{J_1J_2}(x,Q^2) = -g_1^{J_1J_2}(x,Q^2)+\int_x^1\frac{dy}{y}g_1^{J_1J_2}(y,Q^2), \tag{30}$$

$$g_3^{J_1J_2}(x,Q^2) = 4x\int_x^1\frac{dy}{y}g_5^{J_1J_2}(y,Q^2), \tag{31}$$

$$g_4^{J_1J_2}(x,Q^2) = 2xg_5^{J_1J_2}(x,Q^2), \tag{32}$$

$$g_5^{J_1J_2}(x,Q^2) = \sum_q \beta^q_{J_1J_2}\left[\Delta q(x,Q^2)-\Delta\bar{q}(x,Q^2)\right], \tag{33}$$

where

$$\alpha^q_{J_1J_2} = \alpha^q_{\gamma\gamma,\gamma Z,ZZ} = \left[e_q^2,\ 2e_qv_q,\ v_q^2+a_q^2\right], \tag{34}$$
$$\beta^q_{J_1J_2} = \beta^q_{\gamma\gamma,\gamma Z,ZZ} = [0,\ 2e_qa_q,\ 2v_qa_q], \tag{35}$$

and the electroweak couplings are

$$\begin{aligned} e_u &= +\frac{2}{3}, & e_d &= -\frac{1}{3}, \\ v_u &= \frac{1}{2}-\frac{4}{3}\sin^2\theta_W, & v_d &= -\frac{1}{2}+\frac{2}{3}\sin^2\theta_W, \\ a_u &= \frac{1}{2}, & a_d &= -\frac{1}{2}. \end{aligned} \tag{36}$$

For the numerical results, which are presented in section 5 below, the structure functions eq. (29–33) are parametrized using a partonic description for the structure functions $F_1(x,Q^2), F_3(x,Q^2), g_1(x,Q^2)$ and $g_5(x,Q^2)$. The other structure functions are calculated using the relations above. $q(x,Q^2)$, $\bar{q}(x,Q^2)$, $\Delta q(x,Q^2)$, and $\Delta\bar{q}(x,Q^2)$ denote the unpolarized and polarized quark and antiquark densities, respectively.

It appears to be convenient to rewrite the Born cross section in terms of the following two contributions:

$$\frac{d^2\sigma_{\rm Born}}{dxdy} = \frac{d^2\sigma^{\rm unpol}_{\rm Born}}{dxdy}+\frac{d^2\sigma^{\rm pol}_{\rm Born}}{dxdy}, \tag{37}$$

with

$$\frac{d^2\sigma^{\rm unpol}_{\rm Born}}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4} S \sum_{i=1}^{3} S_i^U(x,y)\mathcal{F}_i(x,Q^2), \tag{38}$$

and

$$\frac{d^2\sigma^{\rm pol}_{\rm Born}}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\lambda_N^p f^p S \sum_{i=1}^{5} S_{gi}^p(x,y)\mathcal{G}_i(x,Q^2). \tag{39}$$

λ_N^p denotes the degree of nucleon polarization. For unpolarized deep-inelastic scattering only the first term, $d^2\sigma^{\rm unpol}$, contributes. Eq. (39) applies both to the case of longitudinal (L) and transversal (T) nucleon polarization, where

$$f^L = 1, \tag{40}$$

$$f^T = \cos\varphi\,\frac{d\varphi}{2\pi}\sqrt{\frac{4M^2x}{Sy}\left(1-y-\frac{M^2xy}{S}\right)} \equiv \cos\varphi\,\frac{d\varphi}{2\pi}\frac{1-y}{y}\sin\theta_2. \tag{41}$$

θ_2 is the angle between the incoming and outgoing leptons. φ denotes the angle between the nucleon spin vector s and the plane of the incoming and outgoing lepton in the nucleon rest frame (see figure 4 in appendix A). The polarization 3–vectors for longitudinal and transverse polarization are given by

$$\vec{n}^L = \lambda_N^L\frac{\vec{k}_1}{|\vec{k}_1|}, \tag{42}$$

$$\vec{n}^T = \lambda_N^T\vec{n}_\perp, \quad \text{with} \quad \vec{n}_\perp\vec{k}_1 = 0. \tag{43}$$

Finally, the kinematic coefficients in eqs. (38) and (39) are:

$$\begin{aligned}
S_1^U(x,y) &= 2xy^2,\\
S_2^U(x,y) &= 2\left[(1-y)-xy\frac{M^2}{S}\right],\\
S_3^U(x,y) &= x[1-(1-y)^2],
\end{aligned} \tag{44}$$

$$\begin{aligned}
S_1^L(x,y) &= 2xy\left[(2-y)-2xy\frac{M^2}{S}\right],\\
S_2^L(x,y) &= -8x^2y\frac{M^2}{S},\\
S_3^L(x,y) &= 4x\frac{M^2}{S}\left[(1-y)-xy\frac{M^2}{S}\right],\\
S_4^L(x,y) &= -2\left(1+\frac{2xM^2}{S}\right)\left[(1-y)-xy\frac{M^2}{S}\right],\\
S_5^L(x,y) &= -2xy\left(y+2xy\frac{M^2}{S}\right),
\end{aligned} \tag{45}$$

$$\begin{aligned}
S_1^T(y,Q^2) &= S_5^T(y,Q^2) = S_1^U(y,Q^2),\\
S_2^T(y,Q^2) &= 4xy,\\
S_3^T(y,Q^2) &= -\frac{S_1^L(y,Q^2)}{2xy},\\
S_4^T(y,Q^2) &= S_2^U(y,Q^2).
\end{aligned} \tag{46}$$

3. The $O(\alpha)$ Leptonic Correction

The $O(\alpha)$ leptonic radiative corrections to the scattering cross sections are given by

$$\frac{d^2\sigma_{\rm rad}^{\rm QED,1}}{dx_l dy_l} = \frac{\alpha}{\pi}\delta_{\rm VR}\frac{d^2\sigma_{\rm Born}}{dx_l dy_l} + \frac{d^2\sigma_{\rm Brems}}{dx_l dy_l} = \frac{d^2\sigma_{\rm rad}^{\rm unpol}}{dx_l dy_l} + \frac{d^2\sigma_{\rm rad}^{\rm pol}}{dx_l dy_l}. \tag{47}$$

The corrections are represented by the finite parts of the virtual and soft (VR) and Bremsstrahlung terms, respectively. Since we integrate over the phase space of the radiated photon, the differential cross sections (47) depend on the way in which the Bjorken variables x and y are determined kinematically. Because in all the polarized deep-inelastic scattering experiments performed so far the kinematic variables were measured using the scattered lepton[1] the present calculation refers to this set of variables. The kinematic variables are defined by

$$x_l = \frac{Q_l^2}{Sy_l}, \quad y_l = \frac{p.(k_1 - k_2)}{p.k_1}, \quad \text{and} \quad Q_l^2 = -(k_1 - k_2)^2. \tag{48}$$

The hadronic structure functions depend on the hadronic variables

$$x_h = \frac{Q_h^2}{Sy_h}, \quad y_h = \frac{p.(p' - p)}{p.k_1}, \quad \text{and} \quad Q_h^2 = -(p' - p)^2\ . \tag{49}$$

over which is integrated.

3.1 Virtual and Soft Corrections

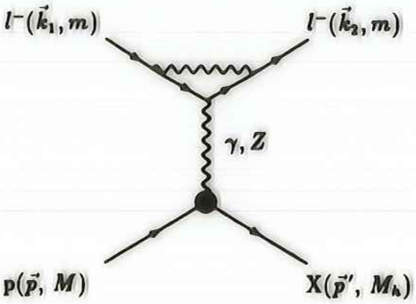


Figure 2: Diagram of the leptonic virtual photon correction for deep-inelastic scattering.

The virtual and soft correction, $\delta_{\rm VR}$, can be written as

$$\begin{aligned}\delta_{\rm VR}(y_l, Q_l^2) &= \delta_{\rm inf}(y_l, Q_l^2) - \frac{1}{2}\ln^2\left[\frac{1 - y_l(1 - x_l)}{1 - y_l x_l}\right] + {\rm Li}_2\left[\frac{1 - y_l}{(1 - y_l x_l)(1 - y_l(1 - x_l))}\right] \\ &+ \frac{3}{2}\ln\left(\frac{Q_l^2}{m^2}\right) - {\rm Li}_2(1) - 2,\end{aligned} \tag{50}$$

[1] Other choices of kinematic variables were also investigated for the case of unpolarized deep-inelastic scattering [14, 15, 12, 13].

with

$$\delta_{\rm inf}(y_l, Q_l^2) = \left[\ln\left(\frac{Q_l^2}{m^2}\right) - 1\right] \ln\left[\frac{y_l^2(1-x_l)^2}{(1-y_l x_l)[1-y_l(1-x_l)]}\right] \tag{51}$$

and

$$\mathrm{Li}_2(x) = -\int_0^1 dz \frac{\ln(1-xz)}{z}. \tag{52}$$

These expressions are the same in the unpolarized and polarized case. They were derived in refs. [12, 13].

2.2 Bremsstrahlung Corrections

The differential Bremsstrahlung cross section for the scattering of polarized electrons off polarized protons, originating from the diagrams in figure 3, is

$$\frac{d\sigma_{\rm Brem}}{dx_l dy_l} = 2\alpha^3 S y_l \int dy_h dQ_h^2 \frac{1}{Q_h^4}\left[\frac{1}{2\pi}\frac{d\varphi_k}{\sqrt{\lambda_q}}\frac{1}{4}\left(L_{\rm rad}^{\mu\nu} W_{\mu\nu}\right)\right]. \tag{53}$$

Figure 3: The leptonic Bremsstrahlung diagrams for for deep-inelastic scattering.

$W_{\mu\nu}$ denotes the hadronic tensor (19). The invariant λ_q is defined in (A .14). The Bremsstrahlung correction to the leptonic tensor, $L_{\rm rad}^{\mu\nu}$ has the following form :

$$\begin{aligned}
L_{\rm rad}^{\mu\nu} &= 2L_s(Q_h^2)\Bigg\{4\left(-\frac{2m^2}{z_2^2}+\frac{Q_h^2}{z_1 z_2}\right)k_1^\mu k_1^\nu + 4\left(-\frac{2m^2}{z_1^2}+\frac{Q_h^2}{z_1 z_2}\right)k_2^\mu k_2^\nu \\
&\quad + \left[2m^2 Q_h^2\left(\frac{1}{z_1^2}+\frac{1}{z_2^2}\right) - \frac{Q_l^4+Q_h^4}{z_1 z_2} - 2\right] g_{\mu\nu}\Bigg\} \\
&\quad + 4i\varepsilon^{\alpha\beta\mu\nu}\Bigg\{-L_A(Q_h^2,\lambda_l)\left[\left(\frac{2m^2}{z_2^2} - \frac{Q_l^2}{z_1 z_2} - \frac{1}{z_2}\right)k_{1\alpha} q_\beta + \left(\frac{2m^2}{z_1^2} - \frac{Q_l^2}{z_1 z_2} + \frac{1}{z_1}\right)k_{2\alpha} q_\beta\right] \\
&\quad + 2\left[L_v(Q_h^2,\lambda_l) + L_a(Q_h^2,\lambda_l)\right]\frac{m^2}{z_1^2}\left[y_{lh} k_{1\alpha} k_{2\beta} + (1+y_{lh})\, k_{2\alpha} k_\beta\right] \\
&\quad - 2\left[L_v(Q_h^2,\lambda_l) - L_a(Q_h^2,\lambda_l)\right]\frac{m^2}{z_1^2} k_{1\alpha} k_\beta\Bigg\} \\
&\quad + 8L_\chi(Q_h^2,\lambda_l)\frac{m^2}{z_1^2}\left[2\,(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu) - 4\,(1+y_{lh})\, k_2^\mu k_2^\nu - (Q_{lh}^2 - y_{lh} Q_h^2)\, g^{\mu\nu}\right],
\end{aligned}$$

3. The Leading Log Approximation

$$\frac{d^2\sigma^{\rm LLA}_{\rm rad}}{dxdy} = \frac{d^2\sigma_i}{dxdy} + \frac{d^2\sigma_f}{dxdy} + \frac{d^2\sigma_{\rm Comp}}{dxdy}. \quad (93)$$

ISR, FSR:

$$\frac{d^2\sigma^k_{i,f}}{dxdy} = \frac{\alpha}{2\pi}\left(\ln\frac{Q^2}{m^2}-1\right)\int_0^1 dz\frac{1+z^2}{1-z}\left\{\theta(z-z_0)\,\mathcal{J}\,\frac{d^2\sigma^k_{\rm Born}}{dxdy}\bigg|^{i,f}_{x=\hat{x},y=\hat{y},S=\hat{S}} - \frac{d^2\sigma^k_{\rm Born}}{dxdy}\right\}, \quad (94)$$

where the rescaled variables are

$$\hat{S} = zS, \quad \hat{y} = \frac{y+z-1}{z}, \quad \hat{Q}^2 = zQ^2, \quad \hat{x} = \frac{\hat{Q}^2}{\hat{y}\hat{S}}, \quad (95)$$

for initial-state radiation and

$$\hat{S} = S, \quad \hat{y} = \frac{y+z-1}{z}, \quad \hat{Q}^2 = \frac{Q^2}{z}, \quad \hat{x} = \frac{\hat{Q}^2}{\hat{y}\hat{S}}, \quad (96)$$

for final-state radiation. The Jacobian $\mathcal{J}$ is given by

$$\mathcal{J} \equiv \mathcal{J}(x,y,Q^2) = \left|\frac{\partial(\hat{x},\hat{y})}{\partial(x,y)}\right|, \quad (97)$$

cf. refs. [7, 16]–[19, 9]. The lower integration boundaries z_0 derive from the conditions

$$\hat{x}(z_0) \le 1, \qquad \hat{y}(z_0) \le 1, \quad (98)$$

and are given by

$$z_0^i = \frac{1-y}{1-yx}, \qquad z_0^f = 1-y+xy\,. \quad (99)$$

The structure of eq. (94) is the same in the unpolarized and polarized case in leading order QED, since, as is well–known [20], the fermion–fermion splitting function

$$P_{ff}(z) = \left(\frac{1+z^2}{1-z}\right)_+ \quad (100)$$

COMPTON:

$$\frac{d^2\sigma^U_{\rm Comp}}{dx_l dy_l} = \frac{\alpha^3}{Sx_l^2}\frac{Y_+}{y_{l_1}}\int_{x_l}^1 dz \int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}} \frac{dQ_h^2}{Q_h^2}\left[\frac{Z_+}{z}F_2^{\gamma\gamma}(x_h,Q_h^2) - zF_L^{\gamma\gamma}(x_h,Q_h^2)\right], \quad \text{cf. [17, 19, 16, 23]},$$

$$\frac{d^2\sigma^L_{\rm Comp}}{dx_l dy_l} = \left(-2\lambda_l\lambda_N^L\right)\frac{\alpha^3}{Sx_l^2}\frac{Y_-}{y_{l_1}}\int_{x_l}^1 dz \int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}} \frac{dQ_h^2}{Q_h^2}\frac{Z_-}{z}x_h g_1^{\gamma\gamma}(x_h,Q_h^2),$$

$$\frac{d^2\sigma^T_{\rm Comp}}{dx_l dy_l} = \left(-2\lambda_l\lambda_N^T\right)\frac{\alpha^3}{Sx_l^2}\cos\varphi\frac{d\varphi}{2\pi}\frac{y_l}{y_{l_1}^2}\sqrt{\frac{4M^2x_l}{Sy_l}\left(y_{l_1} - \frac{M^2x_ly_l}{S}\right)}$$
$$\times \int_{x_l}^1 \frac{dz}{z}\int_{(Q_h^2)^{\rm min}}^{(Q_h^2)^{\rm max}} \frac{dQ_h^2}{Q_h^2}\left\{(Y_- - y_l z)\, z x_h g_1^{\gamma\gamma}(x_h,Q_h^2) + 2\left[Y_+(1-z) + y_{l_1}\right]x_h g_2^{\gamma\gamma}(x_h,Q_h^2)\right\}. \quad (101)$$

Here we used the abbreviations

$$Y_\pm = 1 \pm (1-y_l)^2, \qquad Z_\pm = 1 \pm (1-z)^2, \quad (102)$$

$$z = \frac{x_l}{x_h}. \quad (103)$$

Part of the structure of eqs. (101) can be understood, as in the case of Bremsstrahlung, in terms of partonic splitting functions. In the unpolarized case

U: →
$$P^U_{\gamma f}(z) = \frac{Z_+}{z} = \frac{1+(1-z)^2}{z} \quad (104)$$

emerges [19, 9]. The longitudinal structure function is convoluted by the coefficient function

U: →
$$c_L^q(z) = z, \quad (105)$$

cf. [24]. In the case of longitudinal nucleon polarization one has [20]

L: →
$$P^L_{\gamma f}(z) = \frac{Z_-}{z} = \frac{1-(1-z)^2}{z}, \quad (106)$$

and the Compton term can as well be understood in the collinear parton model. Such an interpretation is, however, not as straightforwar in the case of transverse nucleon polarization[2].

For the boundaries of the Q_h^2-integral in eqs. (101) we used

$$(Q_h^2)^{\rm max} = \frac{x_l}{x_h}(1-y_l)Q_l^2,$$
$$(Q_h^2)^{\rm min} = \max\left\{(Q_h^2)^{\rm min}_{\rm kin}, \overline{Q}_h^2, \hat{Q}_h^2\right\}, \quad (107)$$

where

$$\hat{Q}_h^2 = \left(\overline{M}_h^2 - M^2\right)\frac{x_h}{1-x_h},$$
$$(Q_h^2)^{\rm min}_{\rm kin} = \frac{x_h(x_hy_lS - Q_l^2)(y_lS - \sqrt{\lambda_q}) + 2Q_l^2(x_hM)^2}{2\left[x_hy_lS - Q_l^2 + (x_hM)^2\right]}. \quad (108)$$

4. NUMERICAL RESULTS

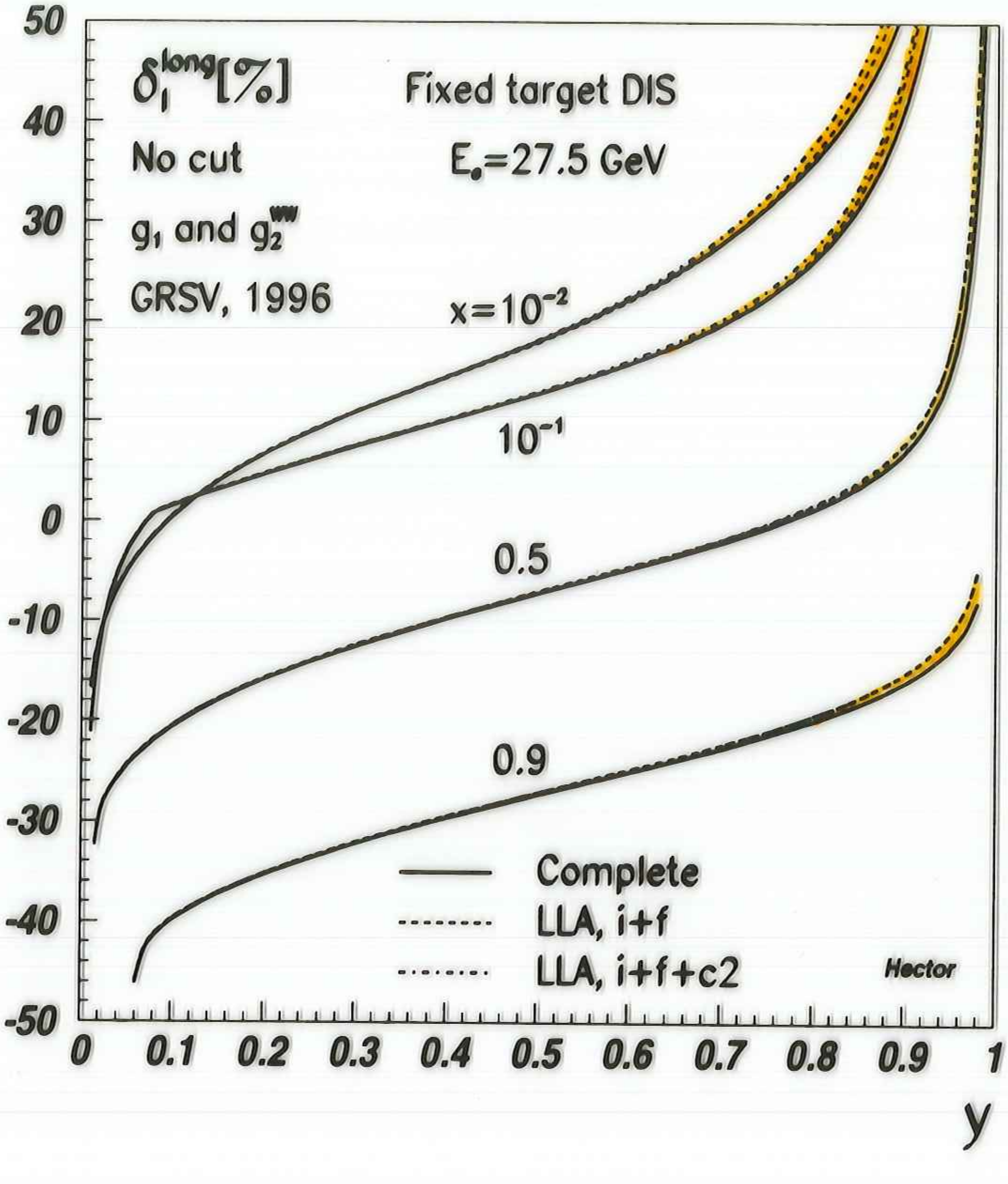


Figure 5 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : complete corrections; dashed lines : initial and final-state Bremsstrahlung contributions in LLA; dash-dotted lines : complete LLA contributions, eq. (94).

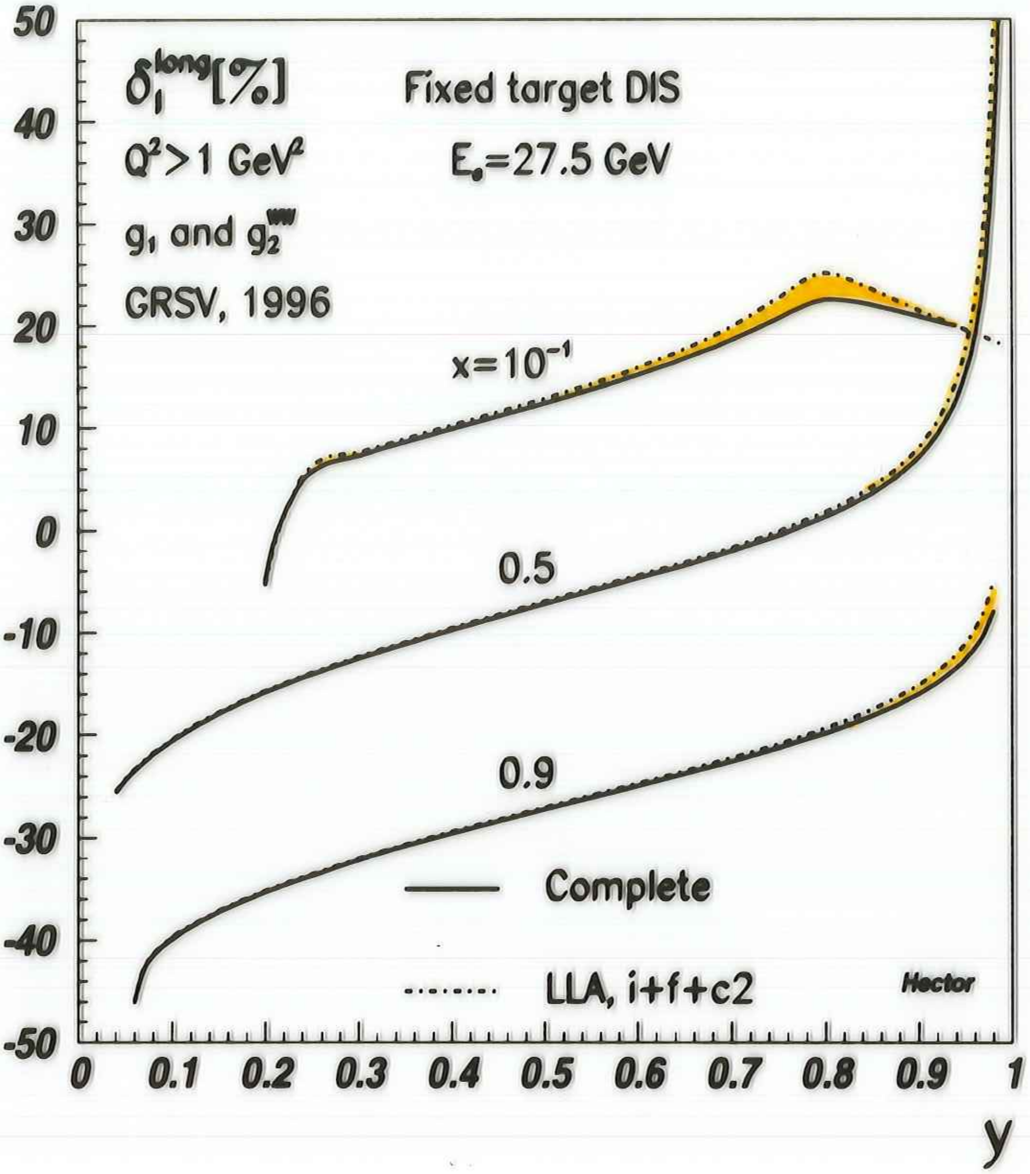


Figure 6 : The same as in figure 5, but for a Q^2-cut of $Q_h^2 > 1$ GeV. Full lines : complete corrections; dash–dotted lines : complete LLA corrections.

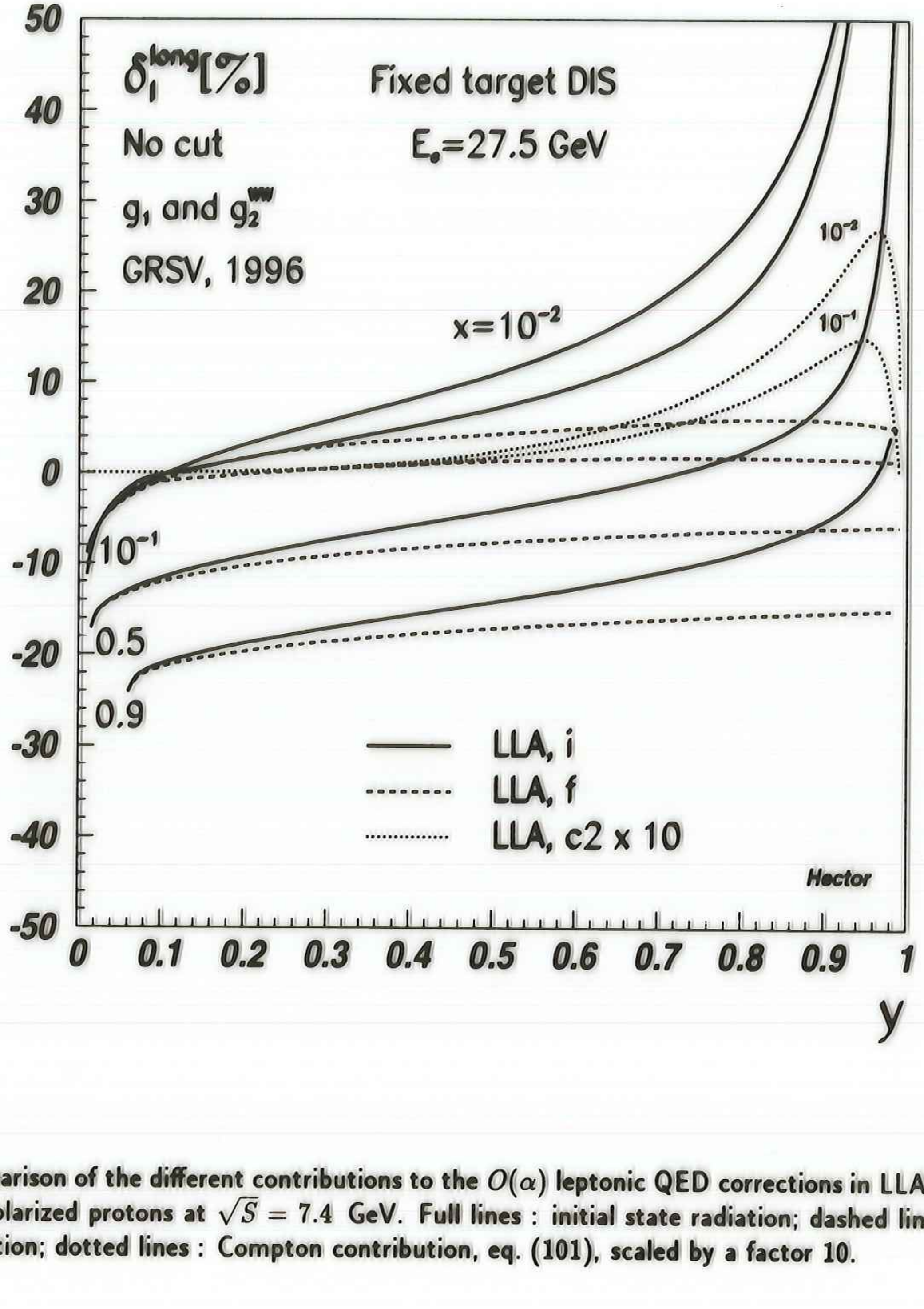


Figure 7 : Comparison of the different contributions to the $O(\alpha)$ leptonic QED corrections in LLA for longitudinally polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : initial state radiation; dashed lines : final state radiation; dotted lines : Compton contribution, eq. (101), scaled by a factor 10.

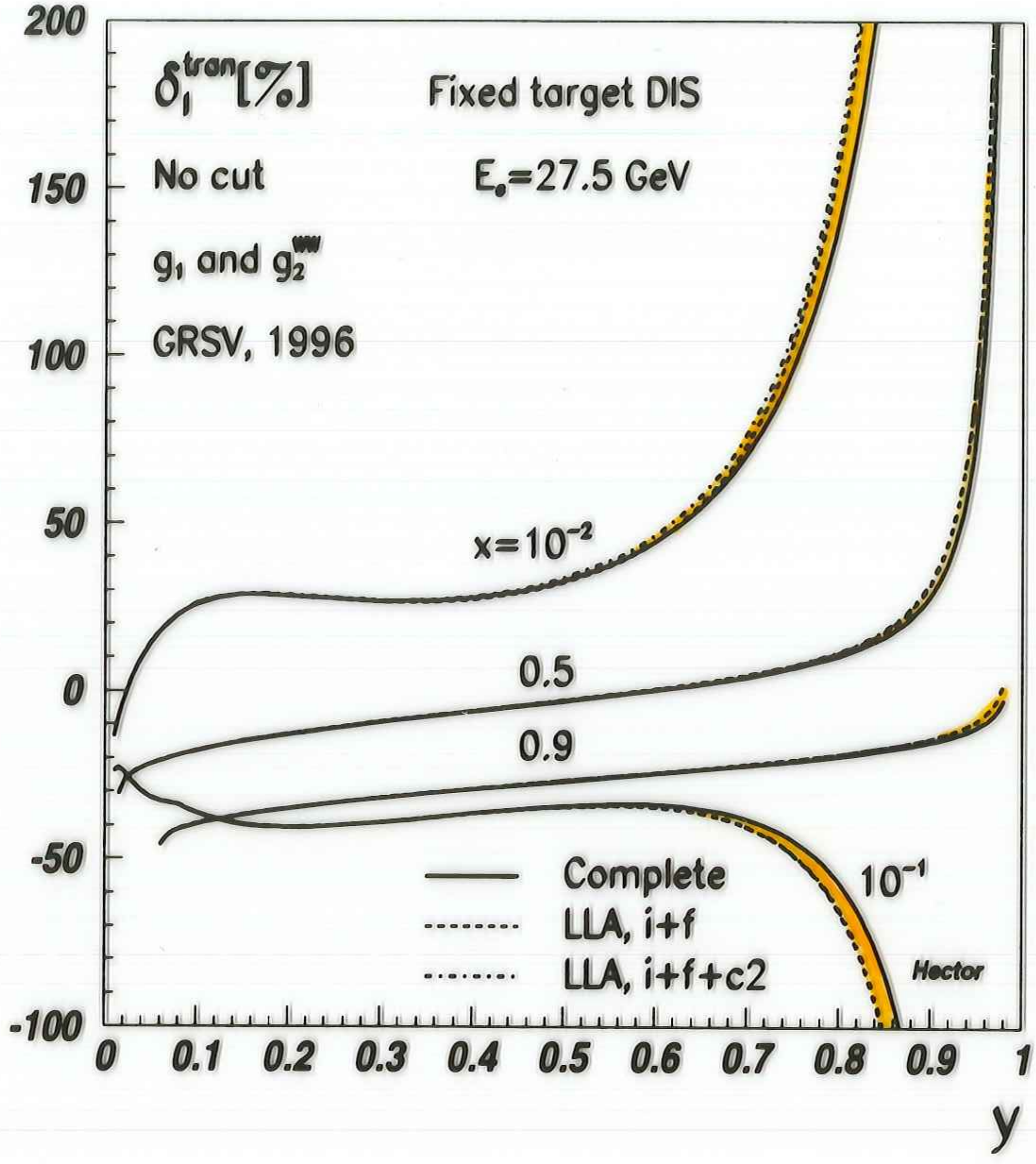


Figure 8 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for transversely polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : complete corrections; dashed lines : initial and final-state Bremsstrahlung contributions in LLA; dash-dotted lines : complete LLA contributions, eq. (94).

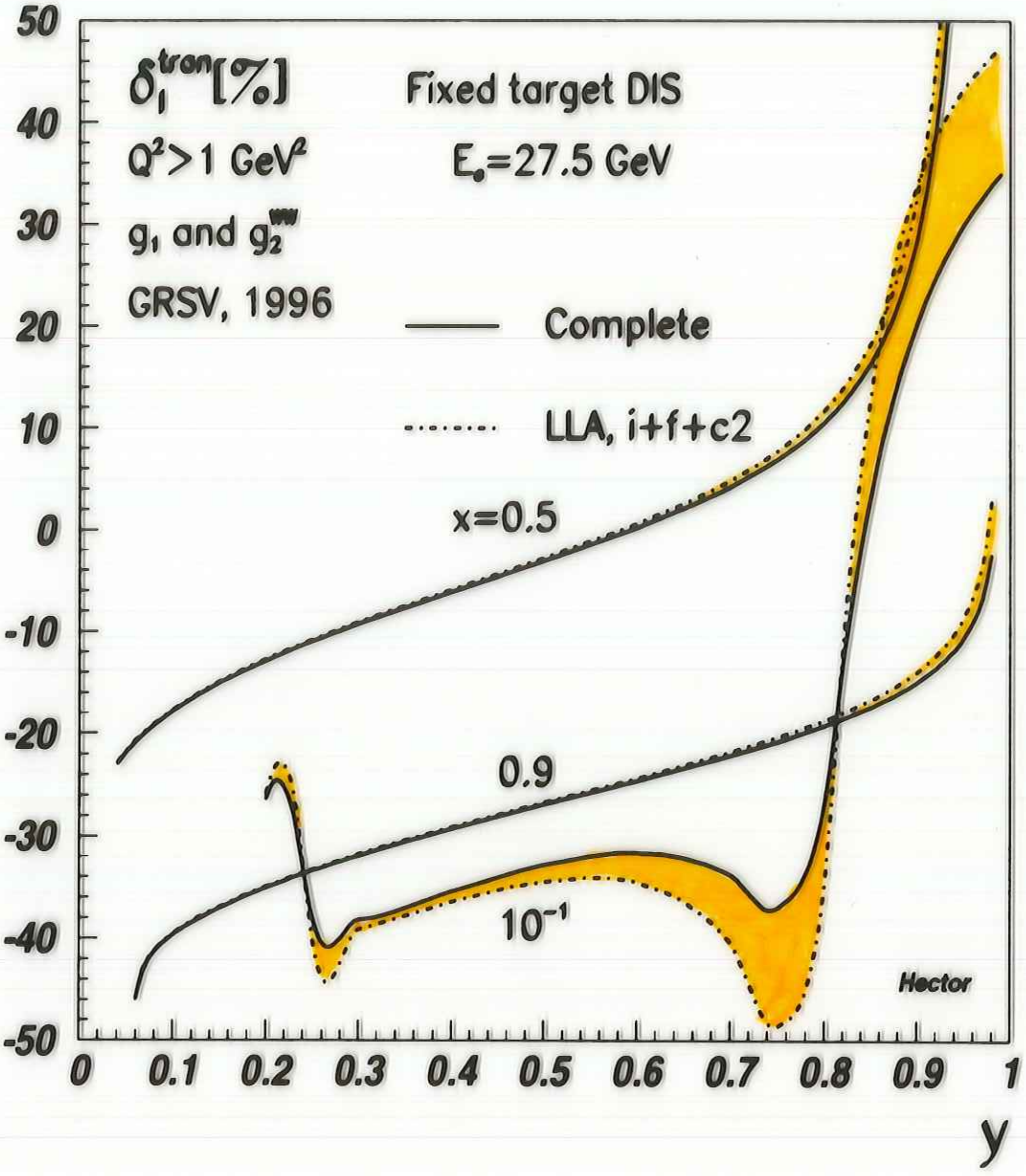


Figure 9 : The same as in figure 8 applying a Q^2-cut of $Q_h^2 > 1\,\mathrm{GeV}$. Full lines : complete corrections; dash–dotted lines : complete LLA corrections.

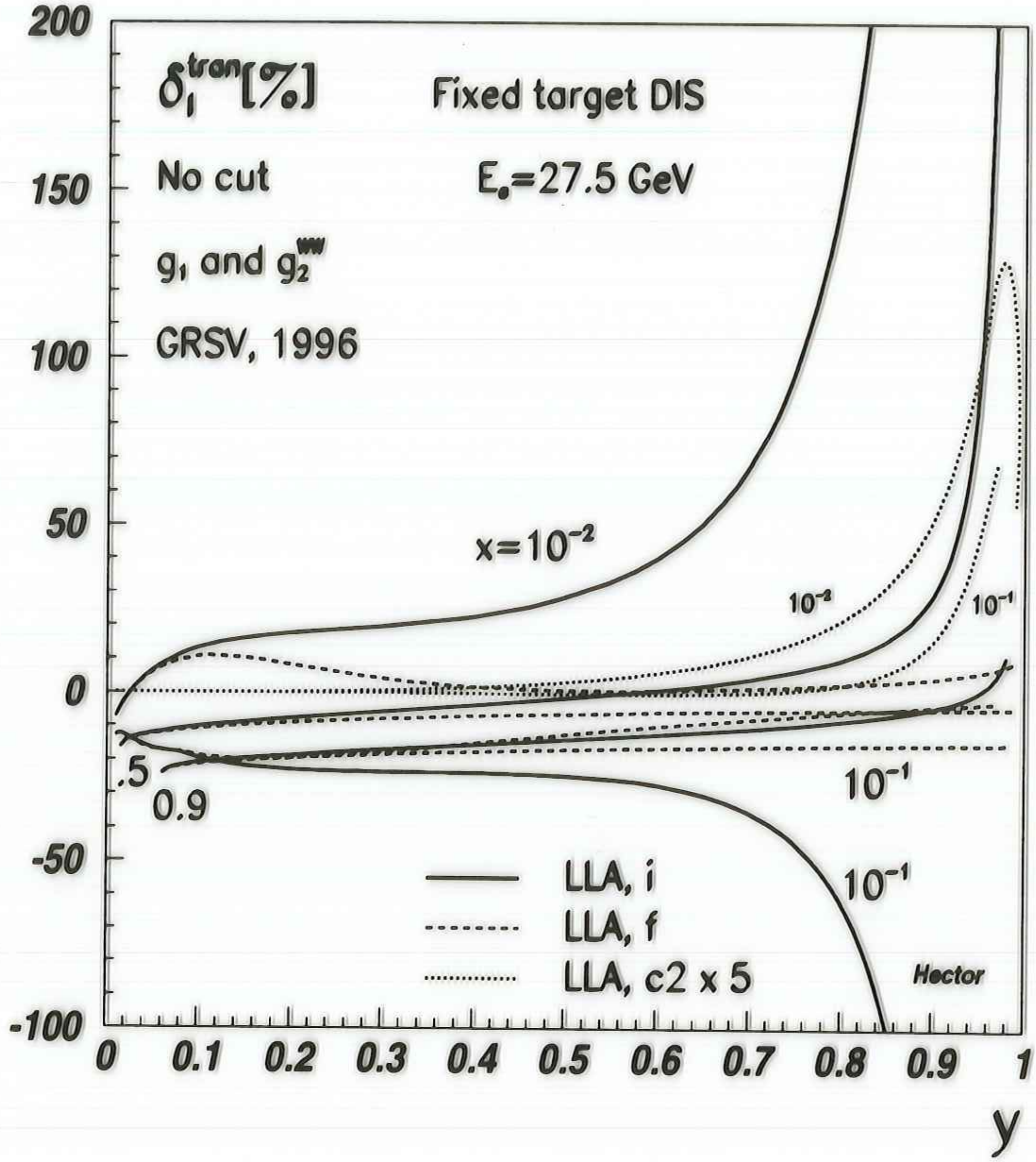


Figure 10 : Comparison of the different contributions to the $O(\alpha)$ leptonic QED corrections in LLA for transversely polarized protons at $\sqrt{S} = 7.4$ GeV. Full lines : initial state radiation; dashed lines : final state radiation; dotted lines : Compton contribution, eq. (101), scaled by a factor 5.

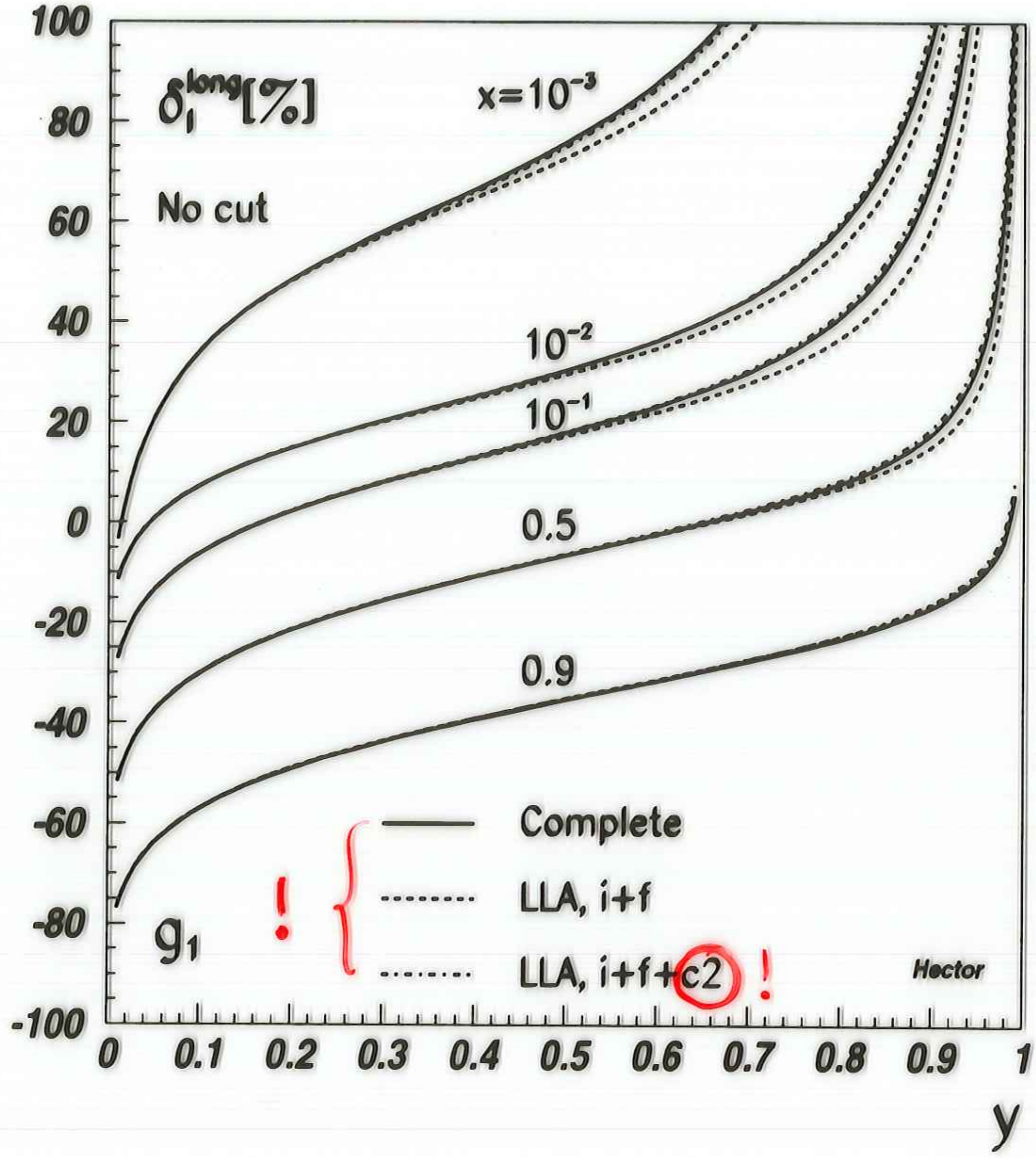
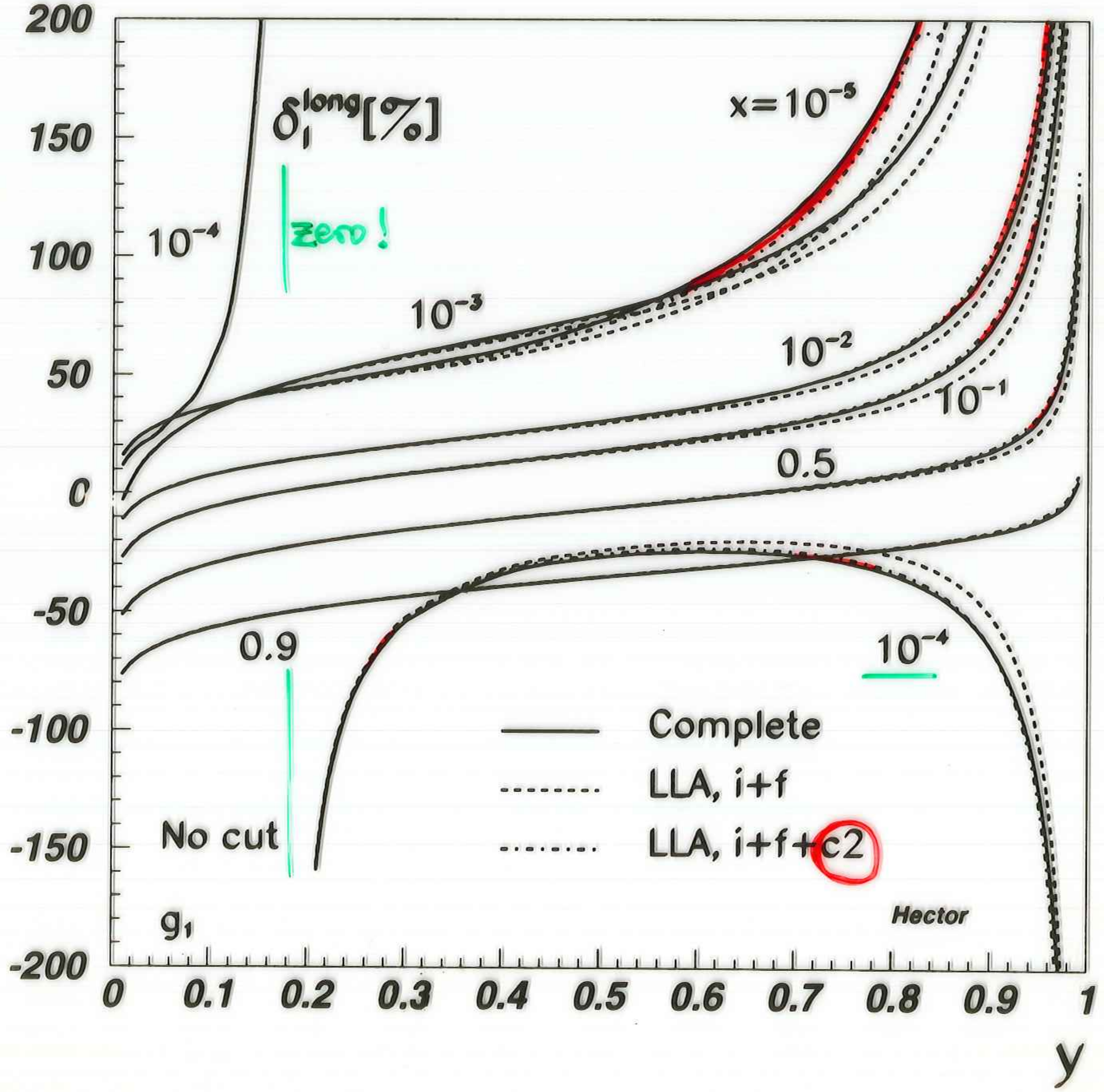


Figure 11 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 314$ GeV. Full lines : complete corrections; dashed lines : LLA terms, eq. (94).



$\delta_l^{long}[\%]$
zero!
x=10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
0.5
0.9
10^{-4}
Complete
LLA, i+f
LLA, i+f+c2
No cut
g$_1$
Hector
200
150
100
50
0
-50
-100
-150
-200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
y

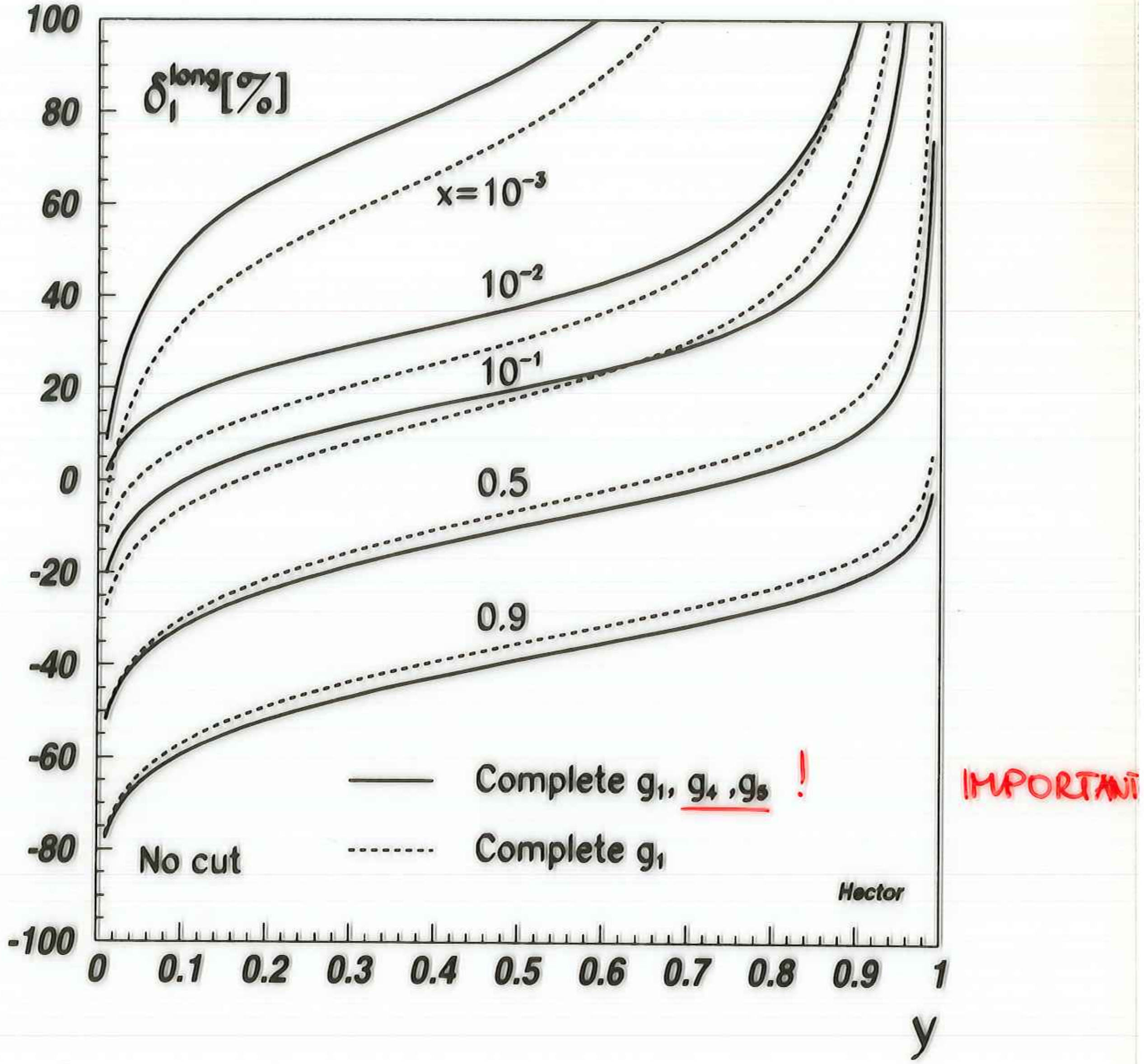
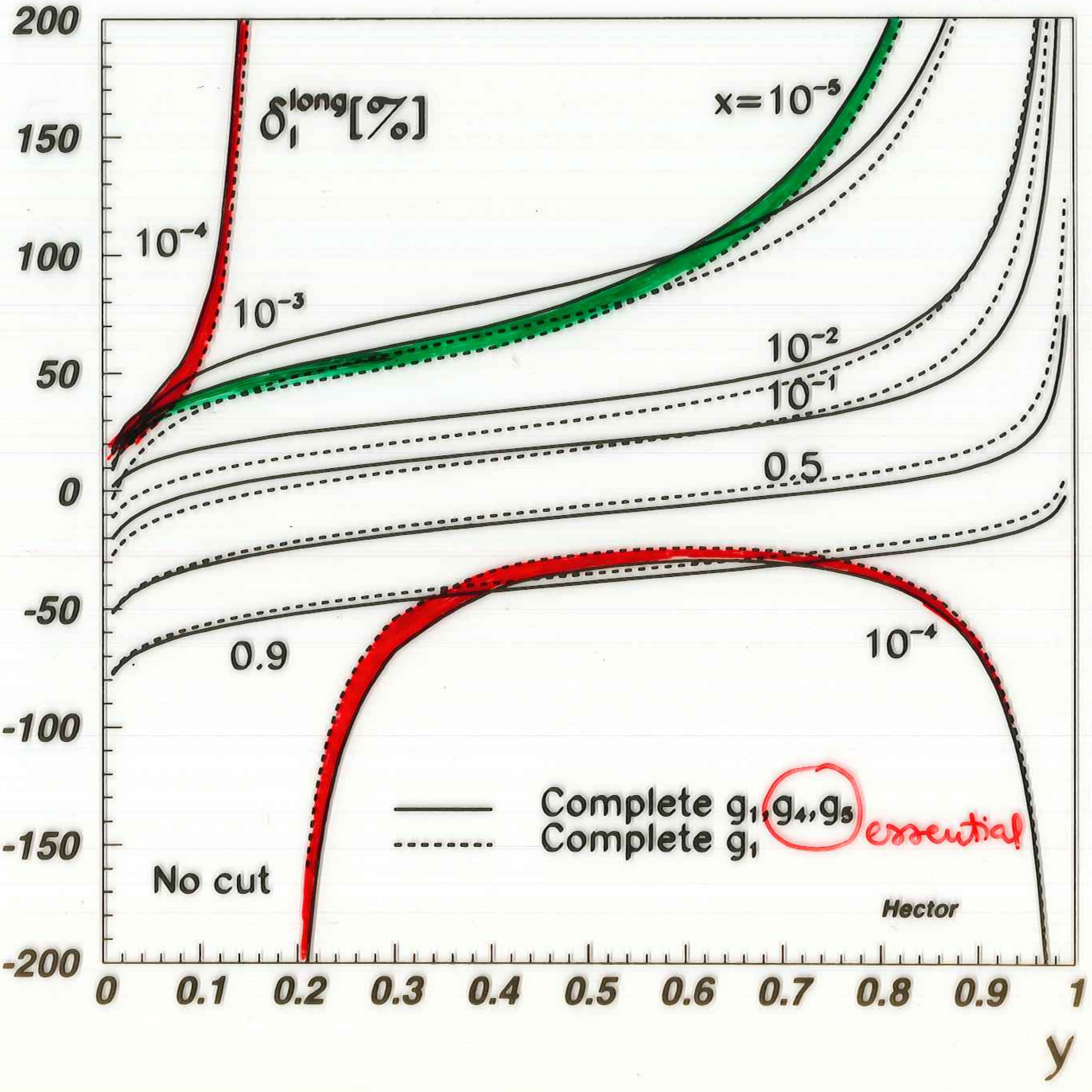


Figure 12 : $O(\alpha)$ leptonic QED correction, eq. (47), to the polarized part of the differential deep-inelastic scattering cross section for longitudinally polarized protons at $\sqrt{S} = 314$ GeV. Dashed lines : $\delta_1^{\rm long}$ for only the structure function g_1; full lines : complete correction. The contributions due to the structure functions g_2 and g_3 are of $O(M^2/S)$ and are not included.



200
150
100
50
0
-50
-100
-150
-200
$\delta_1^{long}[\%]$
$x=10^{-5}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
0.5
0.9
10^{-4}
Complete g_1, g_4, g_5
Complete g_1
essential
No cut
Hector
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
y