# THE POLARIZED STRUCTURE FUNCTION

# $g_1(x, Q^2)$

- PHENOMENOLOGY — DESY 164-95
- ACCESS AT HERA: $e\uparrow p\uparrow$
- RESUMMATION OF $(\alpha_s \ln^2 x)^n$ TERMS — DESY 175-95
  WITH A. VOGT.

# SUMMARY ON SMALL $x$ RESUMMATIONS IN TWIST 2

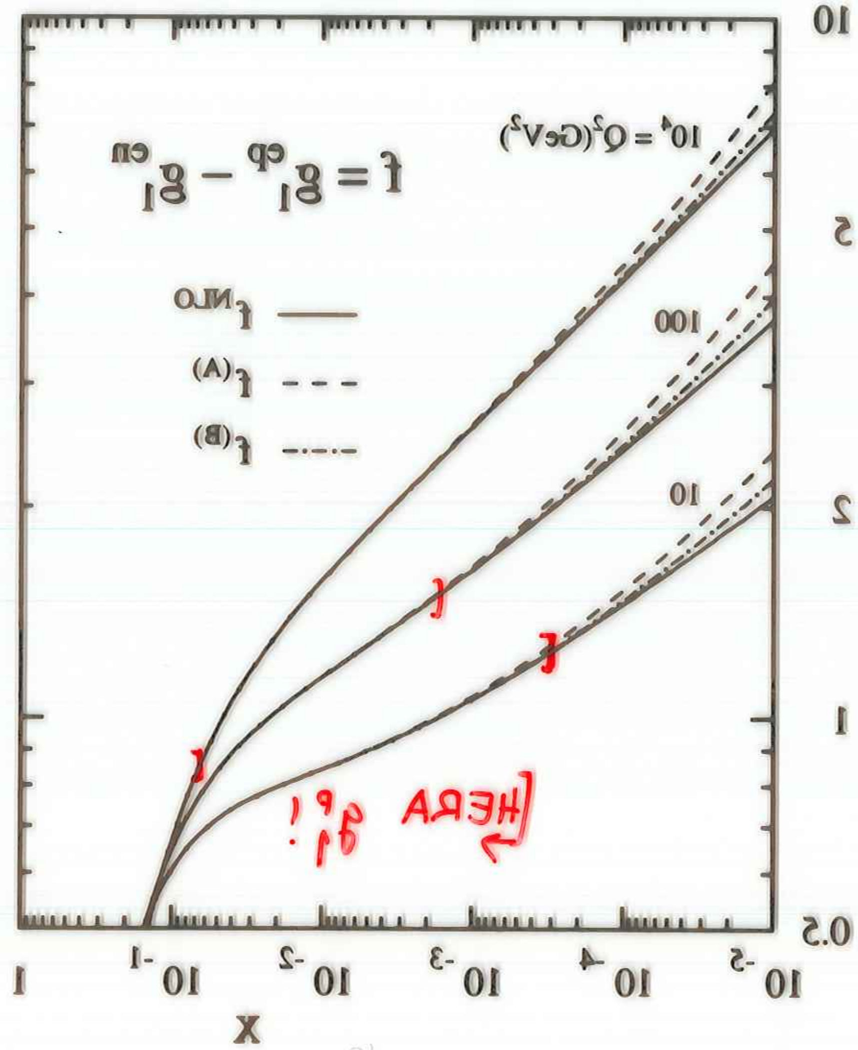
- PREDICTIVE : e.g. 3LOOP TERMS TO BE EXPECTED IN LEADING TERMS
- COMPACT ALL ORDER RESUMMATION
- PROBLEM: CONSERVED QUANTITIES

F-NUMBER; ENERGY-MOMENTUM

$NS^-$      $S$

↳ SUBLEADING TERMS MAY AS WELL BE IMPORTANT QUANTITATIVELY.

↳ NEED FOR COMPLETE CALCULATIONS AT SMALL $x$ !

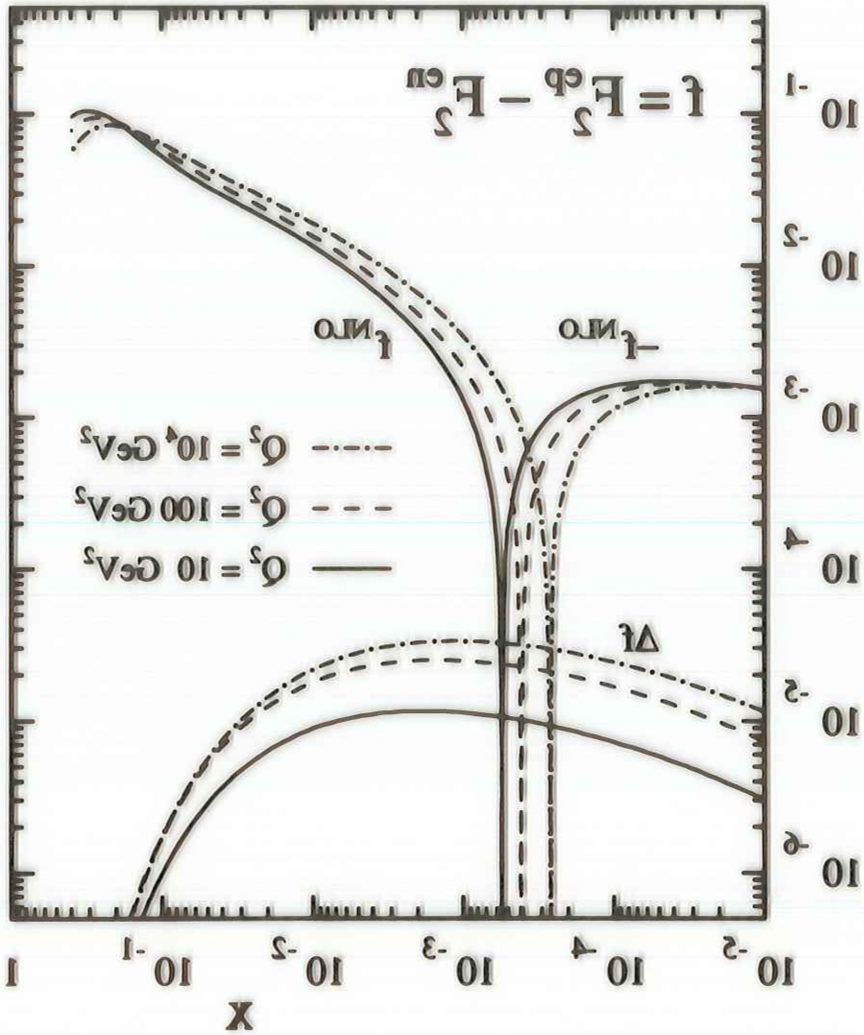



$-A\,\delta(1-x)$ : 'A'

$\gamma_N^- \cdot (1-N)$: 'B'

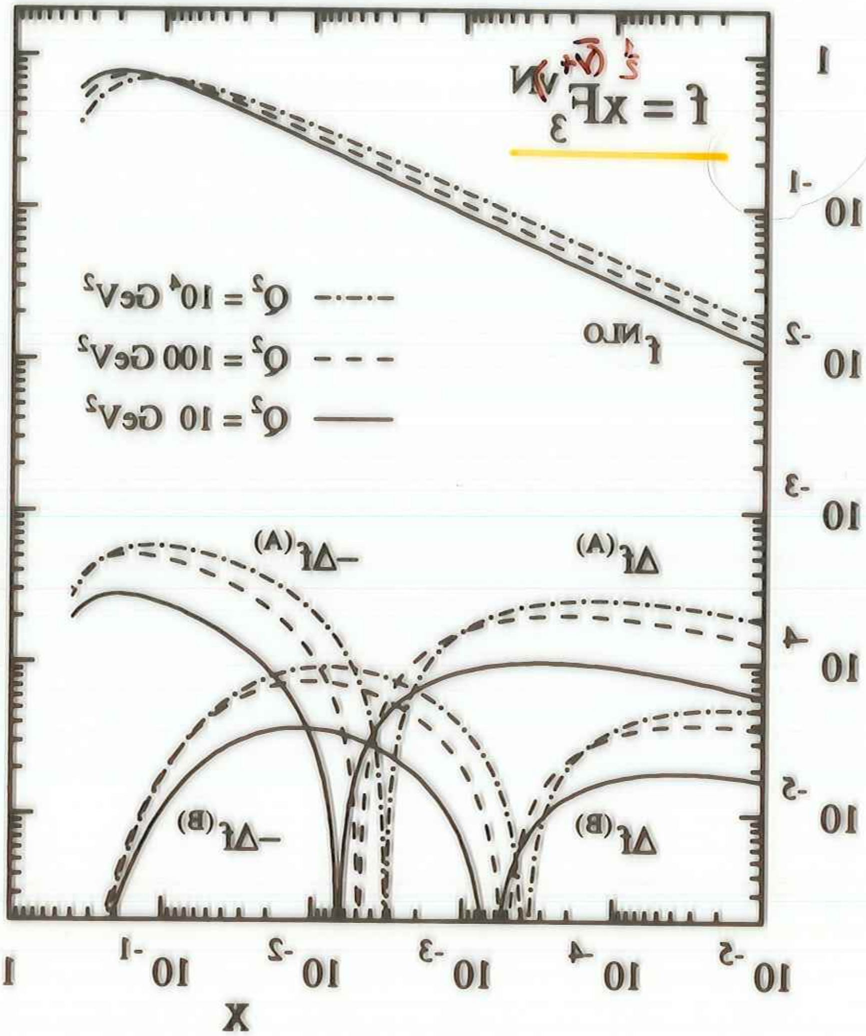
P+

no f-winder constraint.



NLO + $O(\alpha_s^3 \ln^4 x \ldots)$ resummed

p-




$\Delta f^{(B)}$ : f-number restored by $\gamma_N^0 (1-N)$ renorm.

$\Delta f^{(A)}$ : f-number restored by $-A \cdot \delta(1-x)$

PREDICTION OF THE CORRESPONDING TERM IN THE 3-LOOP ANOMALOUS DIMENSION FOR $q^2<0$:

$$P^-_{NS}(x,\alpha_s) \underset{x\to 0}{=} \left(\frac{\alpha_s}{4\pi}\right) 2C_F + \left(\frac{\alpha_s}{4\pi}\right)^2 \ln^2 x \left[4C_F C_A - 6C_F^2\right]$$

$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \ln^4 x \left[-C_F C_A^2 + 4C_F^2 C_A - \frac{10}{3} C_F^3\right]$$

$$P^+_{NS}(x,\alpha_s) \underset{x\to 0}{=} \left(\frac{\alpha_s}{4\pi}\right) 2C_F + \left(\frac{\alpha_s}{4\pi}\right)^2 \ln^2 x \; 2C_F^2$$

$$+ \left(\frac{\alpha_s}{4\pi}\right)^3 \ln^4 x \; \frac{2}{3} C_F^3$$

NOTE THAT A DIFFERENT RESULT $O(\alpha_s^2)$
IS OBTAINED FOR THE TIMELIKE REGION $q^2>0$:

(VIOL. OF GRIBOV - LIPATOV rel.)

- NO CONTRIBUTION DUE TO $\frac{\partial C_{\pm}^{T}}{\partial \ln q^2} \otimes C_{\pm}^{T^{-1}}$
- $P^T_{\pm, x\to 0} = P^S_{\pm, x\to 0} - 2\left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x$.

THE RESUMMATION DUE TO K&L DOES NOT
APPLY FOR $q^2>0$.

IRE : LIPATOV 1982

* KIRSCHNER, LIPATOV 1983

CF ALSO: RYSKIN et al. '95, BARTELS et al. '95

LO resummation for $F_{NS}^{\pm}$ (not for $f_{NS}^{\pm}$!)

Mellin transform:

$$M[P_{+,x\to 0}](\omega) = \frac{\omega}{2}\left\{1-\sqrt{1-\frac{2\alpha_s C_F}{\pi\omega^2}}\right\} \qquad (*)$$

$$M[P_{-,x\to 0}](\omega) = \frac{\omega}{2}\left\{1-\sqrt{1-\frac{2\alpha_s C_F}{\pi\omega^2}\left[1-\frac{2N_c\,\alpha_s}{\pi\omega}\frac{d}{d\omega}\phi\right]}\right\}$$

$$\phi := \ln\left(e^{z^2/4} D_{-\frac{1}{2N_c^2}}(z)\right)$$

$z = \omega/\omega_r$, $\omega_r = \sqrt{\bar\alpha_s/2}$, $\bar\alpha_s = \frac{N_c\alpha_s}{\pi}$.

EXPAND & USE: $M[\ln^k \frac{1}{x}](\omega) = \frac{k!}{\omega^{k+1}}$.

$$\boxed{\begin{aligned} P_{+,x\to 0} &= \frac{\alpha_s C_F}{2\pi} + \frac{1}{2}\left(\frac{\alpha_s C_F}{2\pi}\right)^2 \ln^2 x + \dots \\ P_{-,x\to 0} &= \underbrace{\frac{\alpha_s C_F}{2\pi}}_{\text{known}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 C_F\left[\frac{1}{N_c}+\frac{1}{2}C_F\right]\ln^2 x}_{\text{new}} + \dots \end{aligned}}$$

SINCE:

$$C_G - \frac{3}{2}C_F \equiv \frac{1}{N_c} + \frac{1}{2}C_F$$

IN SU(N),

(*) AGREES TO ALL (UTLO) KNOWN ORDERS WITH THE RESULT ON $F_{NS}$ EVOLUTION FOR $Q^2>0$.

## COEFF. FKT.:

$-Q^2 \equiv q^2 < 0$:

$$C^+_{NS} = \delta(1-x) + \frac{\alpha_s}{2\pi} C_F \left[ \frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x)\right]_+ + \dots$$

$$C^-_{NS} = C^+_{NS} - C_F \frac{\alpha_s}{2\pi}(1+x) + \dots$$

$$\frac{\partial C^\pm_{NS}}{\partial \log Q^2} \otimes C^{\pm\,-1}_{NS} = -\frac{\alpha_s^2}{(2\pi)^2}\frac{\beta_0}{2} C_F \left[\frac{1+x^2}{1-x}\left(\ln\frac{1-x}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x) - \delta_-(1+x)\right] + O(\alpha_s^3)$$

$$\lim_{x\to 0} \frac{\partial C^\pm_{NS}}{\partial \log Q^2} \otimes C^{\pm\,-1}_{NS} = -\frac{\alpha_s^2}{(2\pi)^2}\frac{\beta_0}{2} C_F \cdot \underline{0} \cdot \ln^2 x.$$

## ASYMPTOTIC PART OF THE EVOLUTION EQ:

$$\boxed{\begin{aligned}
\frac{\partial F^\pm_{NS,x\to 0}}{\partial \log Q^2} &= P^\pm_{NS,x\to 0}(x,\alpha_s) \otimes F^\pm_{NS,x\to 0} \\
P_{+,x\to 0} &= \frac{\alpha_s}{2\pi} C_F + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x \\
P_{-,x\to 0} &= \frac{\alpha_s}{2\pi} C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[-\frac{3}{2} C_F^2 + C_F C_A\right] \ln^2 x
\end{aligned}}$$

## RESUMMATION OF $O(\alpha_s \ln^2 x)$ TERMS :

$NS_\pm$ EVOLUTION — J.B., DESY 95-175 & A. VOGT.

- NS : MOST SINGULAR TERMS AS $x \to 0$ (TWIST 2)
- S : " " " " FOR $g_1$.

## EVOLUTION EQU. FOR STRUCTURE FUNCTIONS

$$F^{\pm}_{NS,i}(x,Q^2) = C^{(i)\pm}_{NS}(x,\alpha_s) \otimes f^{\pm}_{NS}(x,\alpha_s) \qquad (\overline{MS})$$

$$\frac{\partial F^{\pm}_{NS}}{\partial \log Q^2} = \left[\frac{\partial C^{\pm}_{NS}}{\partial \log Q^2} \otimes C^{\pm\,-1}_{NS} + P^{\pm}_{NS}(x,\alpha_s)\right] \otimes F^{\pm}_{NS}$$

SPLITTING FCT.: $x<1$ $\quad q^2<0$ :

$$P^{\pm}_{NS} = P_{qq} \pm P_{q\bar q}$$

$$P_{qq} = \frac{\alpha_s}{2\pi} C_F \frac{1+x^2}{1-x} + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[C_F^2 P_F + \frac{1}{2} C_F C_G P_G + C_F N_f T_R P_{N_f}(x)\right] + O(\alpha_s^3)$$

$$P_{q\bar q} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left(C_F^2 - \frac{1}{2} C_F C_G\right) P_A + O(\alpha_s^3)$$

$$P_F = -\frac{1}{2}\log^2 x + \dots \qquad P_A = \log^2 x$$

$$P_G = \log^2 x + \dots$$

$$P_{N_f} = + \dots + \underline{0} \log^2 x + O(\alpha_s^3)$$

$$\lim_{x\to 0} P_\pm(x) =:$$

$$P_+ \to \frac{\alpha_s}{2\pi} C_F + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 C_F^2 \ln^2 x$$

$$P_- \to \frac{\alpha_s}{2\pi} C_F + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(-\frac{3}{2} C_F^2 + C_F C_G\right) \ln^2 x .$$

KIRSCHNER, LIPATOV 1983

RESUMMATION OF TERMS $\propto \alpha_s \ln^2 x$

RYSKIN, ERMOLAEV, MANAYENKOV
BARTELS, RYSKIN, ERMOLAEV $\Big\}$ Appl.

NS:

p+

DESY 95-047

Table 1.

| $\mu^2$, GeV$^2$ | $x$ | $R = fns/fns$ | $\alpha_s$ |
|---|---|---|---|
| 0.1 | 0.001 | 5.56 | 0.3* |
| 0.1 | 0.01 | 2.94 | 0.3* |
| 0.1 | 0.1 | 1.64 | 0.3* |
| 1.0 | 0.001 | 10.26 | 0.394 |
| 1.0 | 0.01 | 4.52 | 0.394 |
| 1.0 | 0.1 | 2.10 | 0.394 |
| 4.0 | 0.001 | 12.71 | 0.283 |
| 4.0 | 0.01 | 5.94 | 0.283 |
| 4.0 | 0.1 | 2.75 | 0.283 |

structure function): for the non-singlet quark structure function with the initial condition $q(x, Q_0^2) \propto \delta(1-x)$ [5] a comparison has been made of the GLAP-evolution at small-x and the double logarithmic formula derived in [2], and a difference of up to a factor of 10 (at typical HERA values for $x$ and $Q^2$) was found. Together with (4.5), this leads to a slightly stronger discrepancy in $g_1$: a factor of ten or even more.

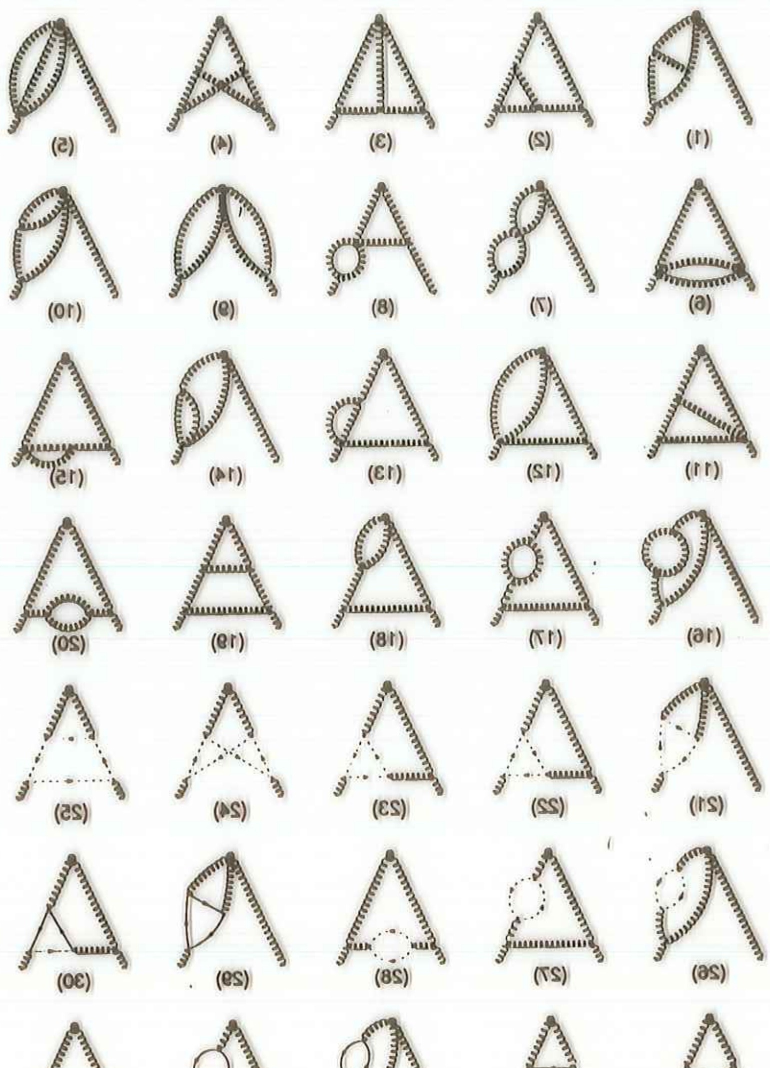
! $\hat{=}$ p$^-$

DESY 95-124

[5] It does not seem very plausible to assume that $g_1(x, Q_0^2)$ is singular at $x \to 0$, therefore we think that the somewhat oversimplified $\delta$-function ansatz is justified.

HERTIG, VAN NEERVEN: NLO ANOM. FOR $g_1$ EVOL.

∴ $P_{gg}$ DIAGRAMS:



...

$$P^{(1)}_{PS,qq} = C_F T_f \left[-16(1+x)\ln^2 x - 16(1-3x)\ln x + 16(1-x)\right].$$

• LEAD. SING. STRUCT. : D. GROSS' CONG. '74
NLO CONFIRMED.

$$\begin{aligned}
P^{(1)}_{S,qg} = {} & 4C_A T_f \Big[-8(1+2x)\mathrm{Li}_2(-x) - 8\zeta(2) - 8(1+2x)\ln x\ln(1+x) \\
& + 4(1-2x)\ln^2(1-x) - 4(1+2x)\ln^2 x \\
& - 16(1-x)\ln(1-x) + 4(1+8x)\ln x - 44x + 48\Big] \\
& + 4C_F T_f \Big[8(1-2x)\zeta(2) - 4(1-2x)\ln^2(1-x) \\
& + 8(1-2x)\ln x\ln(1-x) - 2(1-2x)\ln^2 x \\
& + 16(1-x)\ln(1-x) - 2(1-16x)\ln x + 4 + 6x\Big],
\end{aligned} \tag{3.66}$$

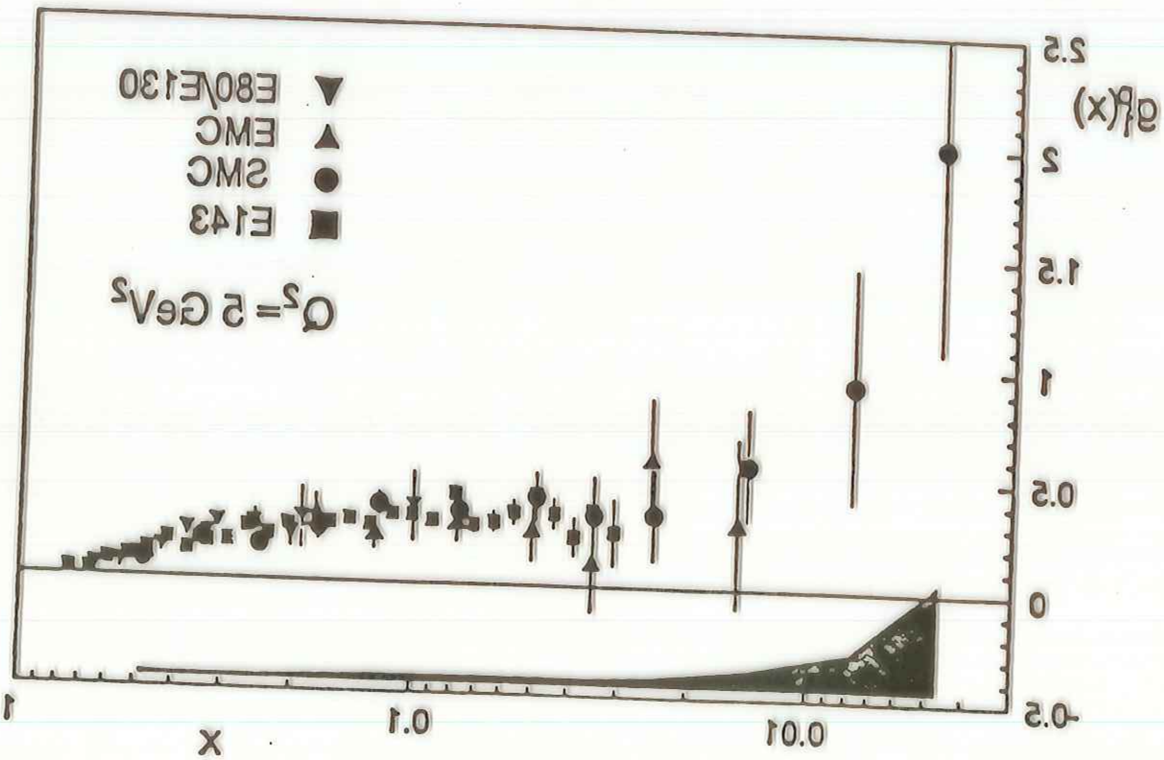
$$\begin{aligned}
P^{(1)}_{S,gq} = {} & C_A C_F \Big[16(2+x)\mathrm{Li}_2(-x) + 16x\zeta(2) + 8(2-x)\ln^2(1-x) \\
& + 16(2+x)\ln x\ln(1+x) + 8(2+x)\ln^2 x \\
& + 16(x-2)\ln x\ln(1-x) + \left(\frac{80}{3} + \frac{8}{3}x\right)\ln(1-x) \\
& + 8(4-13x)\ln x + \frac{328}{9} + \frac{280}{9}x\Big] \\
& + C_F^2 \Big[8(x-2)\ln^2(1-x) - 4(x-2)\ln^2 x - 164 + 128x \\
& - 8(x+2)\ln(1-x) - 4(20+7x)\ln x\Big] \\
& + C_F T_f \Big[-\frac{32}{9}(4+x) + \frac{32}{3}(x-2)\ln(1-x)\Big],
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
P^{(1)}_{S,gg} = {} & C_A^2 \Bigg[\left(64x + \frac{32}{1+x} + 32\right)\mathrm{Li}_2(-x) + \left(64x - 16\left(\frac{1}{1-x}\right)_+ + \frac{16}{1+x}\right)\zeta(2) \\
& + \left(\frac{8}{1-x} - \frac{8}{1+x} + 32x\right)\ln^2 x + \left(64x + \frac{32}{1+x} + 32\right)\ln x\ln(1+x) \\
& + \left(64x - \frac{32}{1-x} - 32\right)\ln x\ln(1-x) + \left(\frac{232}{3} - \frac{536}{3}x\right)\ln x \\
& + \frac{536}{9}\left(\frac{1}{1-x}\right)_+ - \frac{388}{9}x - \frac{148}{9} + \delta(1-x)\left(24\zeta(3) + \frac{64}{3}\right)\Bigg] \\
& + C_A T_f \Bigg[-\frac{160}{9}\left(\frac{1}{1-x}\right)_+ - \frac{32}{3}(1+x)\ln x - \frac{448}{9} + \frac{608}{9}x \\
& - \frac{32}{3}\delta(1-x)\Bigg] \\
& + C_F T_f \Big[-16(1+x)\ln^2 x + 16(x-5)\ln x - 80(1-x) \\
& - 8\delta(1-x)\Big].
\end{aligned} \tag{3.68}$$

R. Voss

# From $A_1$ to $g_1$

- Remember: $g_1 \propto A_1 F_2 / 2x[1 + R]$
- Assumptions:
  - $R(x,Q^2)$: use SLAC parametrisation
  - $F_2(x,Q^2)$: use NMC parametrisation
  - Assume $A_1$ to be $Q^2$-independent
- QCD predicts $Q^2$-dependence of $A_1$ – formalism exists but depends on polarised gluon distribution
- Present status of $g_1{}^p$:




- Observe a rise at small x???

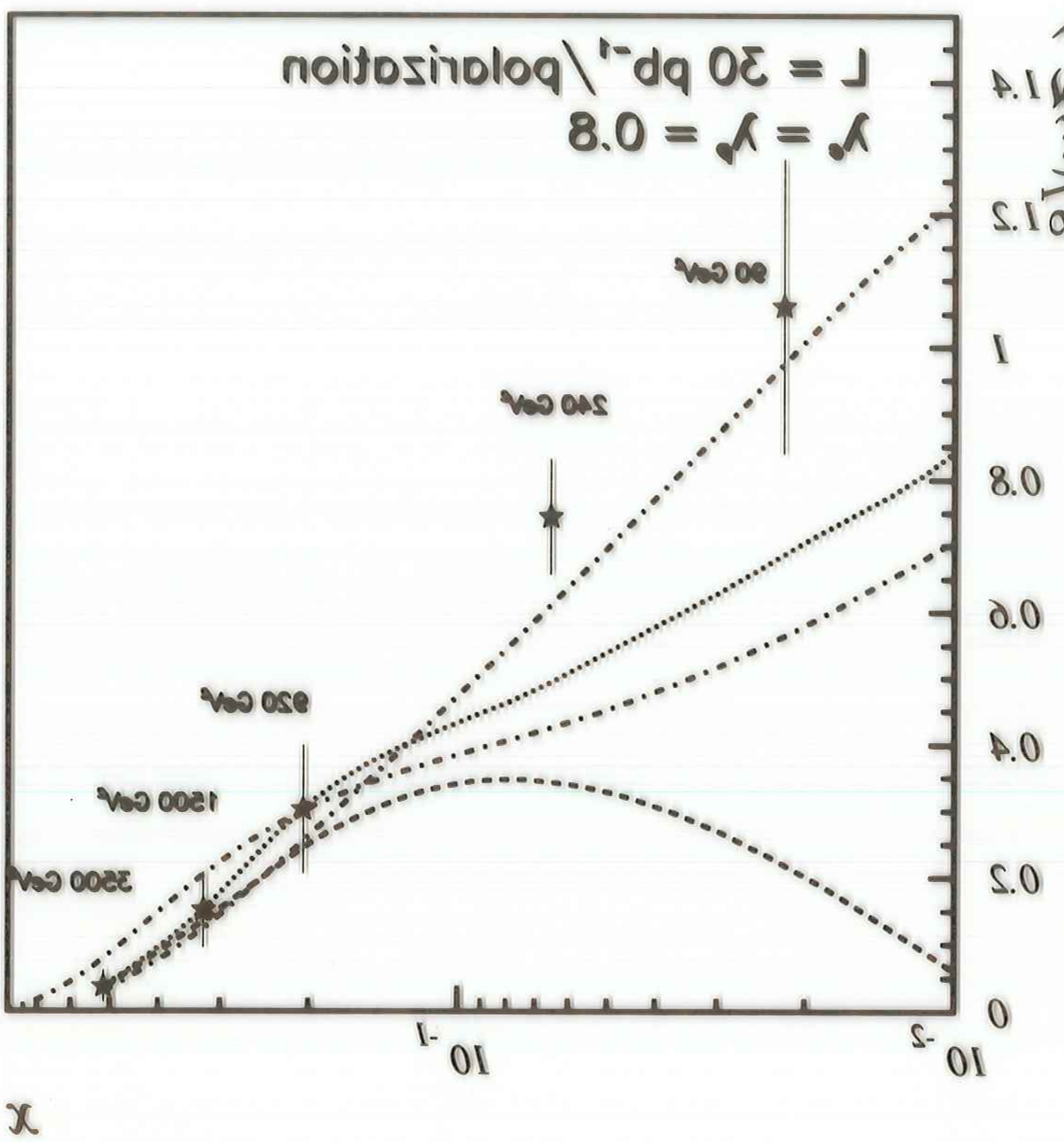



Figure 6: Statistical precision of a measurement of $g_1^p(x,Q^2)$ in the kinematical domain of HERA at larger values of $x$. The data points represent averages over the accessible $Q^2$ range and were calculated using the parametrization [6]. The dashed, dotted, and upper dash–dotted line correspond to the values of $g_1^p(x,\langle Q^2\rangle)$ for the parametrizations [8], [7], and [5], respectively. The lower dash–dotted line shows $g_1^p(x,Q_0^2)$ for $Q_0^2 = 4\,\mathrm{GeV}^2$ for parametrization [5].

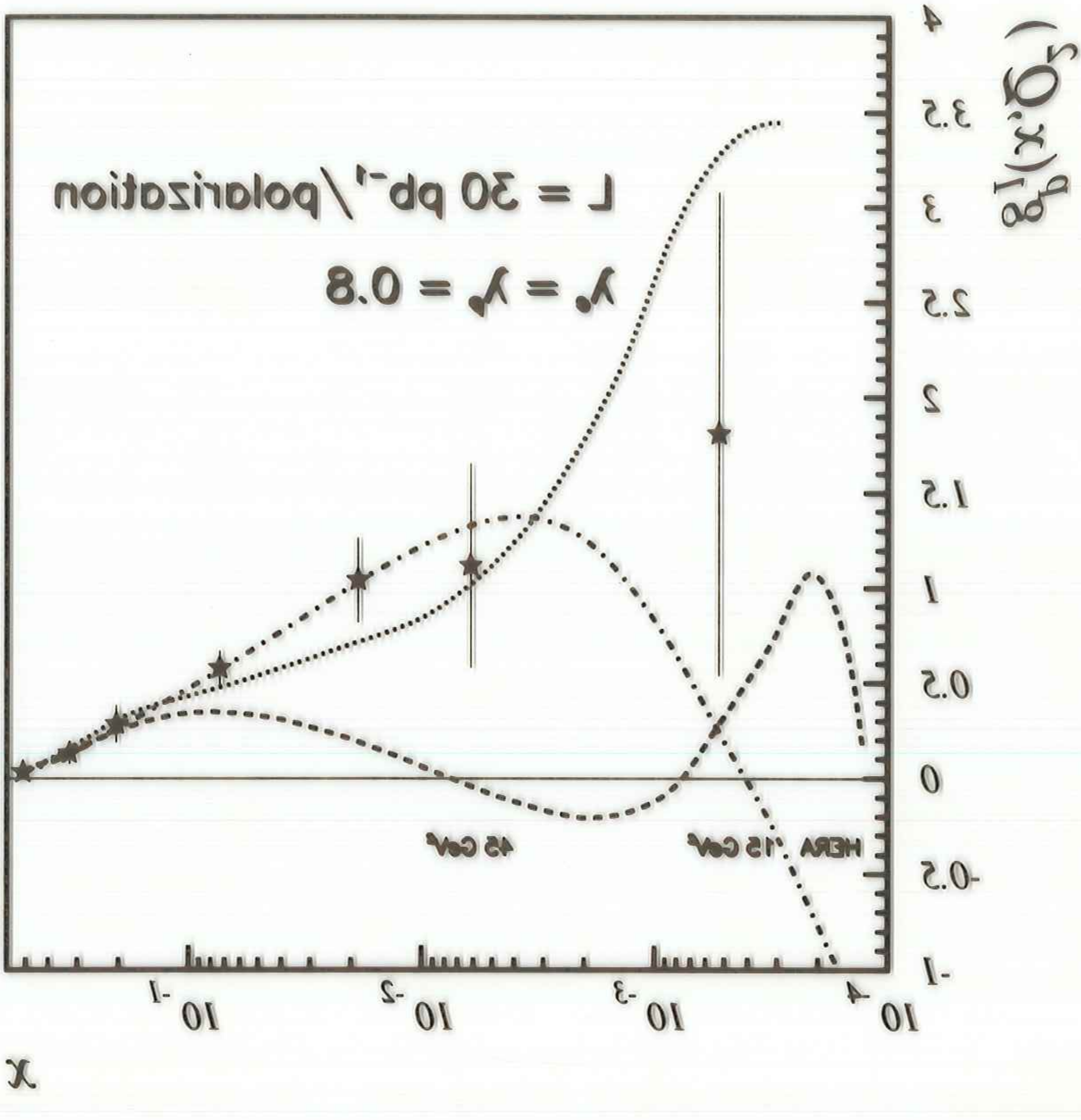



Figure 5: Statistical precision of a measurement of $g_1^p(x,Q^2)$ in the kinematical domain of HERA. The data points represent averages over the accessible $Q^2$ range and were calculated using the parametrization [6]. The dashed, dotted line, and dash–dotted line correspond to the values of $g_1^p(x,\langle Q^2\rangle)$ for the parametrizations [8], [7], and [5], respectively.

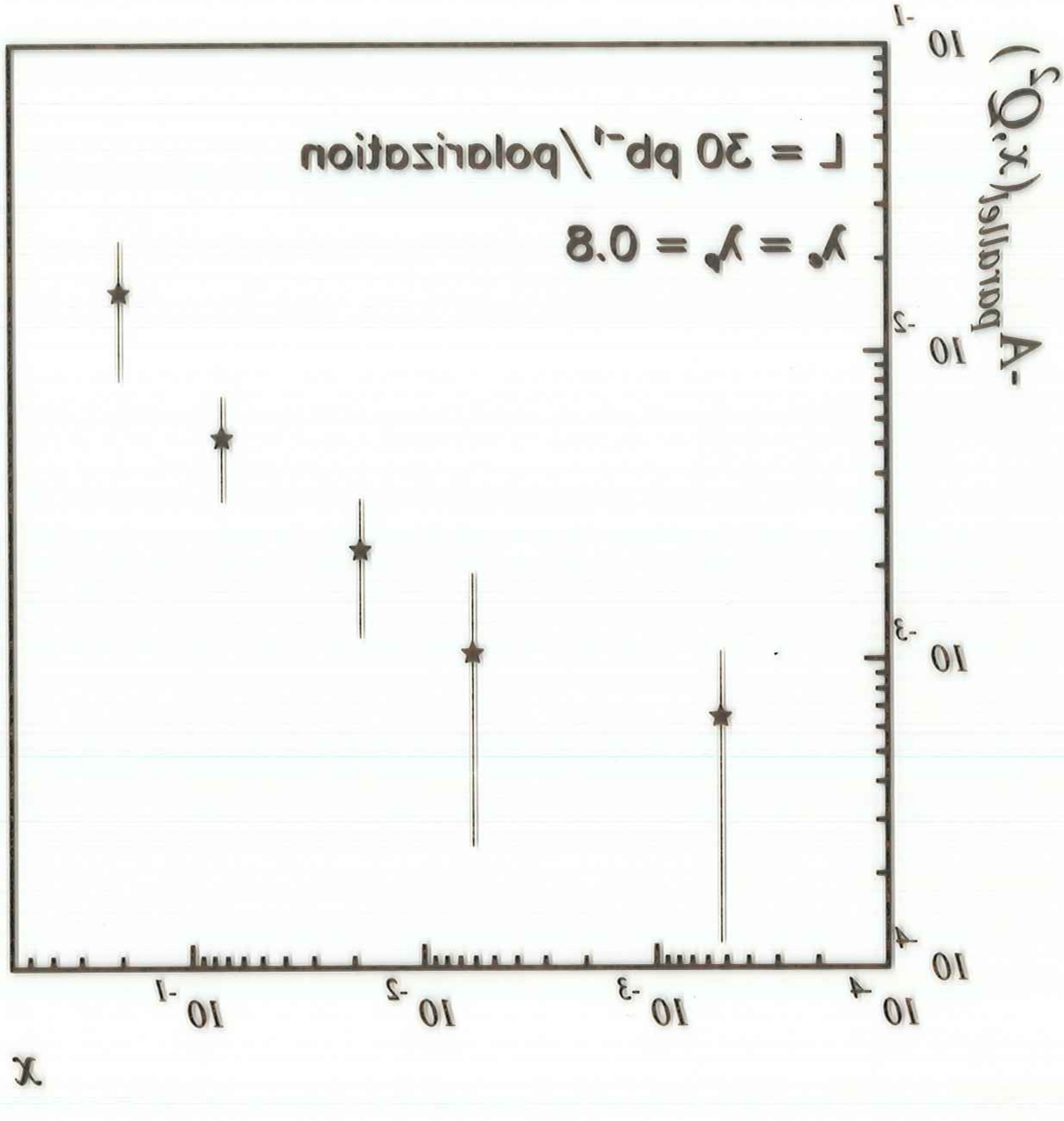



Figure 4: Statistical precision of a measurement of $-A_\parallel(x, \langle Q^2 \rangle)$ in the kinematical domain of HERA. The data points represent averages over the accessible $Q^2$ range and were calculated using the parametrizations [6, 9].

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$g_1(x,Q^2)$ AT HERA

$\lambda_e \equiv \lambda_p = 0.8$ ; $\int \mathcal{L}dt = 60\,pb^{-1}$ (30 per pol.)

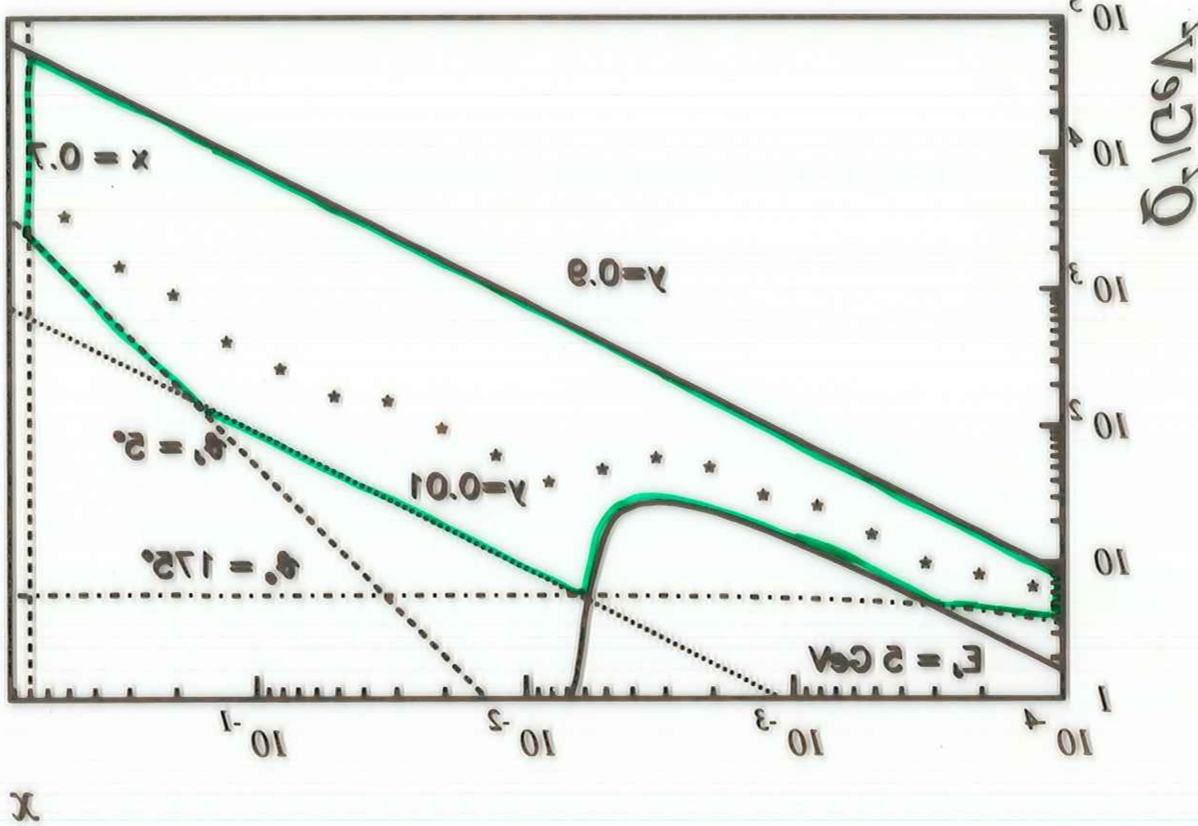


**Figure 3:** The accessible kinematical range for neutral current deep inelastic scattering at HERA; $E_p$ = 820 GeV, $E_e$ = 27.6 GeV. The stars indicate the values of $\langle Q^2 \rangle$ at a given value of $x$ for neutral current deep inelastic scattering.

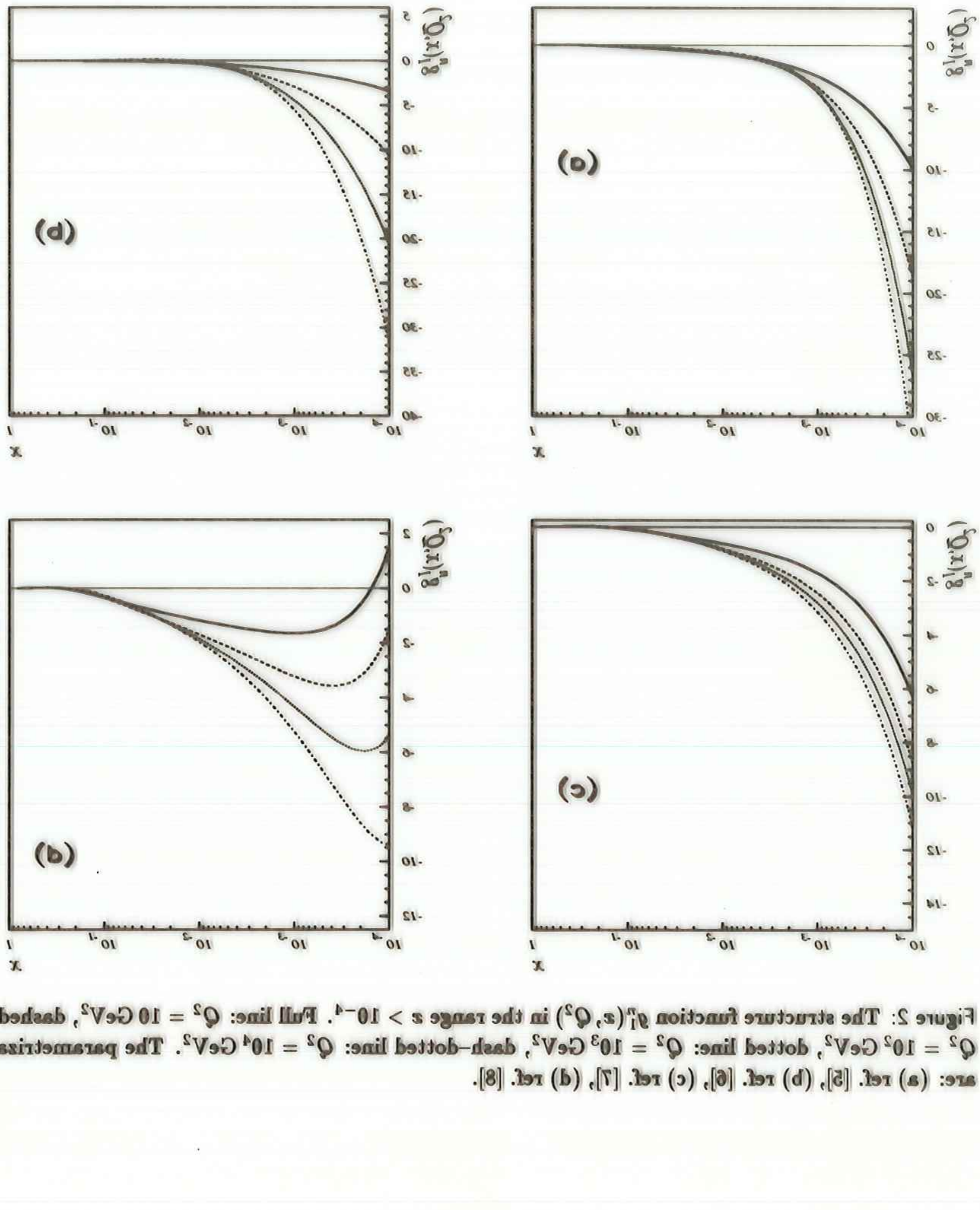

Figure 2: The structure function $g_1^\gamma(x, Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].

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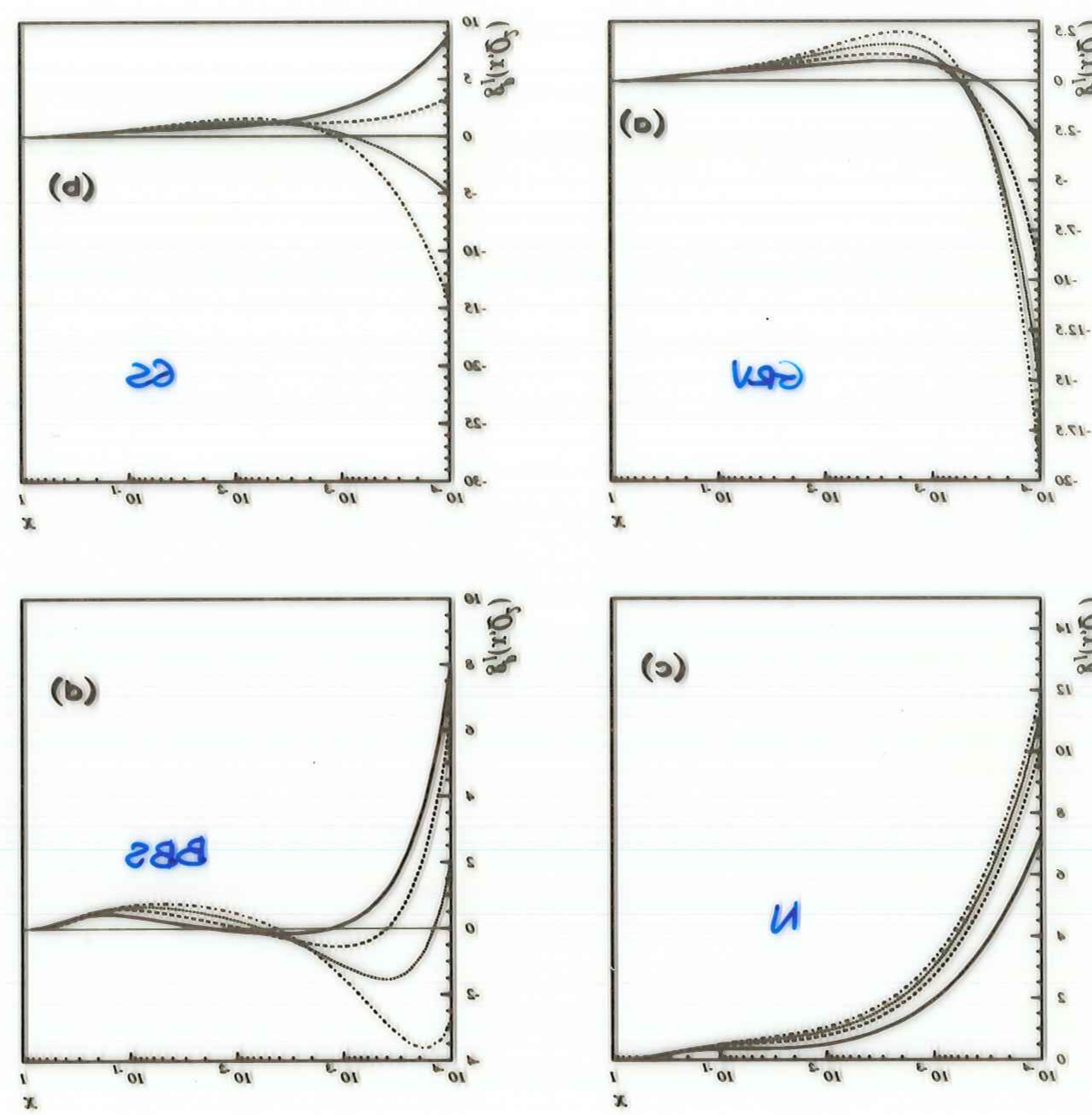


Figure 1: The structure function $g_1^p(x,Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].