DESY-HH
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Scalar and Vector Leptoquark Pair Production in *ep* Collisions

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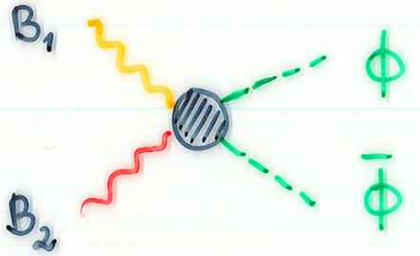
DESY

1 Introduction

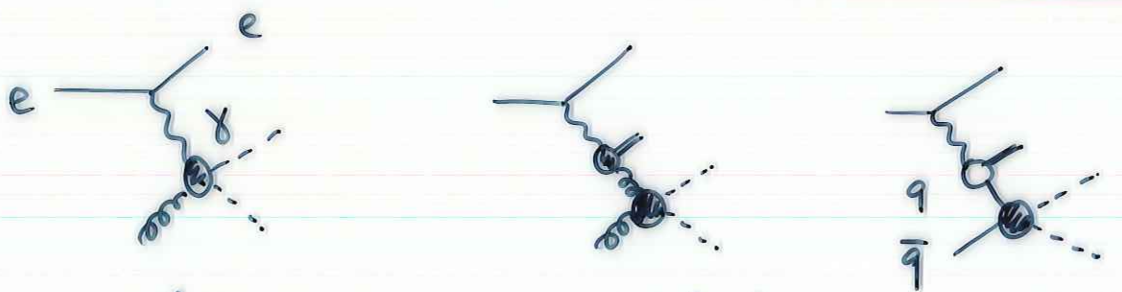
- WHY PAIR PRODUCTION?

→ SINGE PRODUCTION (real or virtual!) $\propto \lambda_{eq} \ll e$

FOR MASSES IN THE HERA RANGE.



ONLY GAUGE & 'ANOMALOUS' COUPLINGS !



dir — resolved_g — resolved_q

NATURE OF V-LEPTOQUARKS? $\kappa_{A,G}$; $\lambda_{A,G}$

→ MASS LIMITS

&

CONSTRAINS ON $\kappa_{A,G}$; $\lambda_{A,G}$.

BASED ON WORK WITH
E. BOOS, A. KRYUKOV, A. PUKHOV

2 Basic Notation

$$\mathcal{L} = \mathcal{L}_S^g + \mathcal{L}_V^g,$$

$$\mathcal{L}_S^g = \sum_{scalars} \left[\left(D_{ij}^{\mu} \Phi^j \right)^{\dagger} \left(D_{\mu}^{ik} \Phi_k \right) - M_S^2 \Phi^{i\dagger} \Phi_i \right],$$

$$\mathcal{L}_V^g = \sum_{vectors} \left\{ -\frac{1}{2} G_{\mu\nu}^{i\dagger} G_i^{\mu\nu} + M_V^2 \Phi_{\mu}^{i\dagger} \Phi_i^{\mu} - i g_s \left[(1-\kappa_G) \Phi_{\mu}^{i\dagger} t_{ij}^a \Phi_{\nu}^j \mathcal{G}_a^{\mu\nu} + \frac{\lambda_G}{M_V^2} G_{\sigma\mu}^{i\dagger} t_{ij}^a G_{\nu}^{j\mu} \mathcal{G}_a^{\nu\sigma} \right] \right\}.$$

$$\begin{aligned} \mathcal{G}_{\mu\nu}^a &= \partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + g_s f^{abc} \mathcal{A}_{\mu b} \mathcal{A}_{\nu c}, \\ G_{\mu\nu}^i &= D_\mu^{ik} \Phi_{\nu k} - D_\nu^{ik} \Phi_{\mu k}, \end{aligned}$$

$$D_\mu^{ij} = \partial_\mu \delta^{ij} - i g_s t_a^{ij} \mathcal{A}_\mu^a.$$

$$\mu_{\Phi,G} = \frac{g_s}{2M_\Phi} (2 - \kappa_G + \lambda_G),$$

$$q_{\Phi,G} = -\frac{g_s}{M_\Phi^2} (1 - \kappa_G - \lambda_G).$$

4.2 *ep* scattering

In the case of ep scattering there are two contributions[10] to the production cross section: the direct process $\gamma^* g \to \Phi\overline{\Phi}$ [9] and the resolved process. Due to the photon-leptoquark coupling the direct contribution σ_{dir} scales as $\propto Q_\Phi^2$ while the resolved one σ_{res} does not depend on the leptoquark charge. The total cross section is

$$\sigma_{S,V}^{ep,tot} = \sigma_{S,V}^{ep,dir} + \sigma_{S,V}^{ep,res}, \tag{42}$$

with (cf. [9])

$$\sigma_{S,V}^{ep,dir} = \int_{y_{min}}^{y_{max}} dy \int_{x_{min}}^{x_{max}} dx \phi_{\gamma/e}(y) G_p(x,\mu^2) \hat{\sigma}_{S,V}^{dir}(\hat{s}, M_\Phi)\theta(\hat{s} - 4M_\Phi^2), \tag{43}$$

and $x_{min} = 4M_\Phi^2/yS$, $\hat{s} = Sxy$, $S = 4E_e E_p$, $x_{max} = 1$, $y_{min,max}$ is given in eq. (46). $\phi_{\gamma/e}(y)$ denotes the Weizsäcker-Williams distribution [25] :

$$\phi_{WWA}(y) = \frac{\alpha}{2\pi}\left[2m_e^2 y\left(\frac{1}{Q_{max}^2} - \frac{1}{Q_{min}^2}\right) + \frac{1+(1-y)^2}{y}\log\frac{Q_{max}^2}{Q_{min}^2}\right]. \tag{44}$$

To parametrize the scales $Q_{min,max}^2$ we choose the kinematical limits

$$Q_{min}^2 = \frac{m_e^2 y^2}{1-y}, \qquad Q_{max}^2 = yS - 4M_\Phi^2 - 4M_\Phi m_p, \tag{45}$$

and

$$y_{min,max} = \frac{S + \widetilde{W}^2 \pm \sqrt{(S - \widetilde{W}^2)^2 - 4m_e^2\widetilde{W}^2}}{2(S + m_e^2)}, \tag{46}$$

where $\widetilde{W}^2 = (2M_\Phi + m_p)^2 - m_p^2$, m_e and m_p are the electron and proton mass, respectively, $y = P.q/P.l_e$, with $q = l_e - l'_e$, and P, l_e, l'_e the four momenta of the proton, the incoming and outgoing electron. The cross sections in the γg subsystem are

$$\hat{\sigma}_{S,V}^{dir}(\hat{s}, M_\Phi) = \frac{\pi\alpha\alpha_s(\mu^2)}{\hat{s}} Q_\Phi^2 R_{S,V}(\hat{s}, M_\Phi) \tag{47}$$

[10]Note that we neglect fermionic couplings λ_{lq} due to their smallness.

where

$$R_S = (2-\beta^2)\beta - \frac{1}{2}(1-\beta^4)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{48}$$

$$R_V = \sum_{j=0}^{20} \chi_j^*(\kappa_{A,G}, \lambda_{A,G})\widetilde{H}_j(\hat{s}, \beta). \tag{49}$$

Fig.

The functions $\chi_j^*(\kappa_A, \kappa_G, \lambda_A, \lambda_G)$ and $\widetilde{H}_j$ were given in [9] in eqs. (12;A.2) where we defined $\widetilde{H}_j \equiv (M_V^2/\hat{s})\widetilde{F}_j^*$ [11].

The cross sections due to the resolved photon contributions read:

$$\begin{aligned}\sigma_{S,V}^{ep,res}(s, M_\Phi) &= \int_{y_{min}}^{y_{max}} dy \int_{4M_\Phi^2/Sy}^{1} dz \int_{4M_\Phi^2/Syz}^{1} dx \phi_{\gamma/e}(y)\theta(\hat{s} - 4M_\Phi^2) \\ &\times \left\{\sum_{f=1}^{N_f}\left[q_f^\gamma(z,\mu_1)\overline{q}_f^p(x,\mu_2) + \overline{q}_f^\gamma(z,\mu_1)q_f^p(x,\mu_2)\right]\hat{\sigma}_{S,V}^q(\hat{s}, M_\Phi)\right. \\ &+ \left. G^\gamma(z,\mu_1)G^p(x,\mu_2)\hat{\sigma}_{S,V}^g(\hat{s}, M_\Phi)\right\}. \end{aligned} \tag{50}$$

$q^{\gamma,p}$ and $G^{\gamma,p}$ denote the quark and gluon densities in the photon and proton, respectively, and $\hat{s} = xyzS$. The factorization scales μ_1 and μ_2 are different in general.

3. Production Cross Sections

σ_{res} :

Scalar Leptoquarks

The differential and integral pair production cross sections for gg and $q\bar{q}$ scattering are

gg

$$\frac{d\hat{\sigma}^{gg}_{S\bar{S}}}{d\cos\theta} = \frac{\pi\alpha_s^2}{6\hat{s}}\beta\left\{\frac{1}{32}\left[25+9\beta^2\cos^2\theta-18\beta^2\right] - \frac{1}{16}\frac{(25-34\beta^2+9\beta^4)}{1-\beta^2\cos^2\theta}+\frac{(1-\beta^2)^2}{(1-\beta^2\cos^2\theta)^2}\right\},$$

$$\hat{\sigma}^{gg}_{S\bar{S}} = \frac{\pi\alpha_s^2}{96\hat{s}}\left\{\beta\left(41-31\beta^2\right)-\left(17-18\beta^2+\beta^4\right)\log\left|\left|\frac{1+\beta}{1-\beta}\right|\right|\right\},$$

and

$q\bar{q}$

$$\frac{d\hat{\sigma}^{q\bar{q}}_{S\bar{S}}}{d\cos\theta} = \frac{\pi\alpha_s^2}{18\hat{s}}\beta^3\sin^2\theta,$$

$$\hat{\sigma}^{q\bar{q}}_{S\bar{S}} = \frac{2\pi\alpha_s^2}{27\hat{s}}\beta^3,$$

with $\alpha_s = g_s^2/4\pi$, $\beta = \sqrt{1-4M_\Phi^2/\hat{s}}$

Vector Leptoquarks

The differential and integral pair production cross sections for gg scattering are

gg:

$$\frac{d\hat{\sigma}^{gg}_{V\bar{V}}}{d\cos\theta} = \frac{\pi\alpha_s^2}{192\hat{s}}\beta\sum_{i=0}^{14}\chi_i^g(\kappa_G,\lambda_G)\frac{F_i(\hat{s},\beta,\cos\theta)}{(1-\beta^2\cos^2\theta)^2},$$

with

$$\begin{aligned}\sum_{i=0}^{14}\chi_i^g(\kappa_G,\lambda_G)F_i = \; & F_0 + \kappa_G F_1 + \lambda_G F_2 + \kappa_G^2 F_3 + \kappa_G\lambda_G F_4 \\ & \lambda_G^2 F_5 \quad \kappa_G^3 F_6 + \kappa_G^2\lambda_G F_7 + \kappa_G\lambda_G^2 F_8 + \lambda_G^3 F_9 \\ & \kappa_G^4 F_{10} + \kappa_G^3\lambda_G F_{11} + \kappa_G^2\lambda_G^2 F_{12} + \kappa_G\lambda_G^3 F_{13} + \lambda_G^4 F_{14},\end{aligned}$$

$$\hat{\sigma}^{gg}_{V\bar{V}} = \frac{\pi\alpha_s^2}{96M_V^2}\sum_{i=0}^{14}\chi_i(\kappa_G,\lambda_G)\tilde{F}_i(\hat{s},\beta),$$

$$\tilde{F}_i = \frac{M_V^2}{\hat{s}}\int_0^\beta d\xi\frac{F_i(\xi=\beta\cos\theta)}{(1-\xi^2)^2}.$$

$q\bar{q}$

$$\frac{d\hat{\sigma}^{q\bar{q}}_{V\bar{V}}}{d\cos\theta} = \frac{2\pi\alpha_s^2}{9M_V^2}\beta^3\sum_{i=0}^{5}\chi_i^q(\kappa_G,\lambda_G)G_i(\hat{s},\beta,\cos\theta)$$

with

$$\sum_{i=0}^{5}\chi_i^q(\kappa_G,\lambda_G)G_i = G_0+\kappa_G G_1+\lambda_G G_2+\kappa_G^2 G_3+\kappa_G\lambda_G G_4+\lambda_G^2 G_5$$

$$\hat{\sigma}^{q\bar{q}}_{V\bar{V}} = \frac{4\pi\alpha_s^2}{9M_V^2}\beta^3\sum_{i=0}^{5}\chi_i^q(\kappa_G,\lambda_G)\tilde{G}_i(\hat{s},\beta)$$

$$\tilde{G}_i = \int_0^1 d\cos\theta\, G_i(\hat{s},\beta,\cos\theta)$$

B Coefficients of the production cross section of vector leptoquarks

The functions $F_i(\hat{s}, \beta, \cos\theta)$ of (21) which determine the differential pair production cross section for $gg \to V\overline{V}$ are:

$$F_0 = \left[19 - 6\beta^2 + 6\beta^4 + \left(16 - 6\beta^2\right)\beta^2\cos^2\theta + 3\beta^4\cos^4\theta\right] \cdot \left(7 + 9\beta^2\cos^2\theta\right) \quad (85)$$

$$F_1 = -4 \cdot \left(77 + 143\beta^2\cos^2\theta + 36\beta^4\cos^4\theta\right) \quad (86)$$

$$F_2 = -8 \cdot \left(7 + 11\beta^2\cos^2\theta - 18\beta^4\cos^4\theta\right) \quad (87)$$

$$F_3 = 2 \cdot \left(117 + 185\beta^2\cos^2\theta + 18\beta^4\cos^4\theta\right) + 2\frac{\hat{s}}{M_\Phi^2}\left(8 - \beta^2\cos^2\theta - 7\beta^4\cos^4\theta\right) + \frac{7}{4}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)^2 \quad (88)$$

$$F_4 = -4 \cdot \left(19 + 27\beta^2\cos^2\theta + 18\beta^4\cos^4\theta\right) + 10\frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(7 - \beta^2\cos^2\theta\right) \quad (89)$$

$$F_5 = 2 \cdot \left(19 + 27\beta^2\cos^2\theta + 18\beta^4\cos^4\theta\right) - \frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(65 + 29\beta^2\cos^2\theta\right) + \frac{1}{8}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)\left(97 + 2\beta^2\cos^2\theta - 115\beta^4\cos^4\theta\right) + \frac{\hat{s}^3}{M_\Phi^6}\frac{9}{4}\left(1 - \beta^2\cos^2\theta\right)^3 \quad (90)$$

$$F_6 = -61 - 67\beta^2\cos^2\theta - \frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(39 + 14\beta^2\cos^2\theta\right) - \frac{7}{4}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)^2 \quad (91)$$

$$F_7 = 127 + 129\beta^2\cos^2\theta - \frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(89 + 3\beta^2\cos^2\theta\right) + \frac{1}{4}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)^2\left(-23 + 18\beta^2\cos^2\theta\right) \quad (92)$$

$$F_8 = -71 - 57\beta^2\cos^2\theta + \frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(170 + 21\beta^2\cos^2\theta\right) + \frac{1}{4}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)\left(-59 + 40\beta^2\cos^2\theta + 27\beta^4\cos^4\theta\right) - \frac{9}{4}\frac{\hat{s}^3}{M_\Phi^6}\left(1 - \beta^2\cos^2\theta\right)^3 \quad (93)$$

$$F_9 = 5\left(1 - \beta^2\cos^2\theta\right) - \frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(21 + 2\beta^2\cos^2\theta\right) + \frac{1}{4}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)^2\left(74 + 9\beta^2\cos^2\theta\right) + \frac{1}{4}\frac{\hat{s}^3}{M_\Phi^6}\left(1 - \beta^2\cos^2\theta\right)^2\left(-15 + 8\beta^2\cos^2\theta\right) \quad (94)$$

$$F_{10} = 3 + 5\beta^2\cos^2\theta + \frac{5}{4}\frac{\hat{s}}{M_\Phi^2}\left(1 - \beta^2\cos^2\theta\right)\left(4 - \beta^2\cos^2\theta\right) + \frac{1}{32}\frac{\hat{s}^2}{M_\Phi^4}\left(1 - \beta^2\cos^2\theta\right)^2\left(25 + 13\beta^2\cos^2\theta\right) \quad (95)$$

The functions $F_i(\hat{s},\beta)$, which describe the different contributions to the integrated cross-section (13), are:

$$\widetilde{F}_0 = \beta\left(\frac{11}{2} - \frac{9}{4}\beta^2 + \frac{3}{4}\beta^4\right) - \frac{3}{8}\left(1 - \beta^2 - \beta^4 + \beta^6\right)\ln\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_1 = -4\beta - \frac{3}{4}(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_2 = \frac{1}{16}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{3-\beta^2}{4}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_3 = 3\beta + \frac{1}{8}\beta\frac{\hat{s}}{M_\Phi^2} + \left(2 - \frac{3}{2}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_4 = -\frac{1}{8}\beta\frac{\hat{s}}{M_\Phi^2} + \left(-1 + \frac{3}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_5 = -\frac{1}{96}\beta + \frac{5}{48}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{4-\beta^2}{16}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_6 = -\frac{1}{2}(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_7 = \frac{7}{12}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{1}{24}\beta\frac{\hat{s}^2}{M_\Phi^4} - \frac{5+\beta^2}{4}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_8 = -\frac{1}{6}\beta + \frac{1}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1}{12}\beta\frac{\hat{s}^2}{M_\Phi^4} + \left(-\frac{1}{2} + \frac{1}{2}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_9 = -\frac{1}{2}\beta + \frac{11}{12}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1}{6}\beta\frac{\hat{s}^2}{M_\Phi^4} - \frac{3+\beta^2}{8}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{10} = -\frac{1}{96}\beta + \frac{59}{80}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{113}{320}\beta\frac{\hat{s}^2}{M_\Phi^4} + \frac{43}{960}\beta\frac{\hat{s}^3}{M_\Phi^6} + \left(-\frac{1}{2} - \frac{1}{16}\beta^2 + \frac{1}{8}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{11} = \frac{1}{2}(1+\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{12} = \beta + \frac{1}{2}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{13} = \beta - \frac{5}{12}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{1}{24}\beta\frac{\hat{s}^2}{M_\Phi^4} + \left[-\frac{1}{4}\frac{\hat{s}}{M_\Phi^2} + \left(\frac{3}{8} + \frac{1}{4}\beta^2\right)\right]\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{14} = -\frac{11}{24}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1}{24}\beta\frac{\hat{s}^2}{M_\Phi^4} + \frac{9+3\beta^2}{8}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{15} = \frac{1}{48}\beta - \frac{59}{96}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{5}{64}\beta\frac{\hat{s}^2}{M_\Phi^4} + \frac{5+\beta^2}{8}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{16} = -\frac{1}{2}\beta - \frac{1}{8}\beta^2\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{17} = -\frac{1}{96}\beta + \frac{1}{48}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{1}{48}\beta\frac{\hat{s}^2}{M_\Phi^4} - \frac{2+\beta^2}{16}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{18} = -\frac{1}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1-6\beta^2}{8}\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{19} = -\frac{1}{24}\beta + \frac{7}{96}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{3}{64}\beta\frac{\hat{s}^2}{M_\Phi^4} + \left[\frac{1}{8}\frac{\hat{s}}{M_\Phi^2} - \frac{2+\beta^2}{4}\right]\log\left|\frac{1+\beta}{1-\beta}\right|,$$

$$\widetilde{F}_{20} = \frac{1}{48}\beta + \frac{1}{6}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1}{8}(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right|.$$

$$F_{12} = \frac{\hat{s}}{M_\Phi^2}(2 - 3\beta^2\cos^2\theta + 4\beta^4\cos^4\theta)\,,$$

$$F_{13} = -2(1-\beta^2\cos^2\theta) + \frac{\hat{s}}{M_\Phi^2}\frac{9 - 13\beta^2\cos^2\theta + 4\beta^4\cos^4\theta}{4} - \frac{\hat{s}^2}{M_\Phi^4}\frac{2 - 3\beta^2\cos^2\theta + \beta^4\cos^4\theta}{2} + \frac{\hat{s}^3}{M_\Phi^6}\frac{(1-\beta^2\cos^2\theta)^3}{16}\,,$$

$$F_{14} = -5 + \beta^2\cos^2\theta + \frac{\hat{s}}{M_\Phi^2}\frac{7 - 8\beta^2\cos^2\theta + \beta^4\cos^4\theta}{2} - 3\frac{\hat{s}^2}{M_\Phi^4}\frac{(1-\beta^2\cos^2\theta)^2}{8} - \frac{(1-\beta^2\cos^2\theta)^3}{16}\,,$$

$$F_{15} = -\frac{3-\beta^2\cos^2\theta}{2} + \frac{\hat{s}}{M_\Phi^2}\frac{13 - 14\beta^2\cos^2\theta + \beta^4\cos^4\theta}{8} - \frac{\hat{s}^2}{M_\Phi^4}\frac{41 - 81\beta^2\cos^2\theta + 39\beta^4\cos^4\theta + \beta^6\cos^6\theta}{64} + \frac{\hat{s}^3}{M_\Phi^6}\frac{11 - 25\beta^2\cos^2\theta + 17\beta^4\cos^4\theta - 3\beta^6\cos^6\theta}{128}\,,$$

$$F_{16} = 1 - \beta^2\cos^2\theta - \frac{\hat{s}}{M_\Phi^2}\frac{3 - 5\beta^2\cos^2\theta + 2\beta^4\cos^4\theta}{4}\,,$$

$$F_{17} = \frac{3-\beta^2\cos^2\theta}{4} - \frac{\hat{s}}{M_\Phi^2}\frac{7 - 8\beta^2\cos^2\theta + \beta^4\cos^4\theta}{16} - \frac{\hat{s}^2}{M_\Phi^4}\frac{3 - 7\beta^2\cos^2\theta + 5\beta^4\cos^4\theta - \beta^6\cos^6\theta}{128} + \frac{\hat{s}^3}{M_\Phi^6}\frac{(1-\beta^2\cos^2\theta)^3}{32}\,,$$

$$F_{18} = 2(5 - \beta^2\cos^2\theta) - \frac{\hat{s}}{M_\Phi^2}\frac{11 - 15\beta^2\cos^2\theta + 4\beta^4\cos^4\theta}{4} - \frac{\hat{s}^2}{M_\Phi^4}\frac{(1-\beta^2\cos^2\theta)^2}{4}\,,$$

$$F_{19} = 3 - \beta^2\cos^2\theta - \frac{\hat{s}}{M_\Phi^2}\frac{7 - 8\beta^2\cos^2\theta + \beta^4\cos^4\theta}{4} + \frac{\hat{s}^2}{M_\Phi^4}\frac{11 - 13\beta^2\cos^2\theta + \beta^4\cos^4\theta + \beta^6\cos^6\theta}{32} + \frac{\hat{s}^3}{M_\Phi^6}\frac{5 - 7\beta^2\cos^2\theta - \beta^4\cos^4\theta + 3\beta^6\cos^6\theta}{128}\,, \qquad \text{(A.1)}$$

$$F_{20} = -\frac{3-\beta^2\cos^2\theta}{2} + \frac{\hat{s}}{M_\Phi^2}\frac{(1-\beta^2\cos^2\theta)^2}{8} + \frac{\hat{s}^2}{M_\Phi^4}\frac{11 - 23\beta^2\cos^2\theta + 13\beta^4\cos^4\theta - \beta^6\cos^6\theta}{64}\,.$$

Appendix

The functions $F_i(\hat{s}, \beta, \cos\theta)$ of (12) are:

$$F_0 = 19 - 6\beta^2 + 6\beta^4 + (16 - 6\beta^2)\beta^2\cos^2\theta + 3\beta^4\cos^4\theta\,,$$

$$F_1 = -22 - 10\beta^2\cos^2\theta\,,$$

$$F_2 = 4 + \frac{\hat{s}}{M_\Phi^2}\frac{1 - \beta^4\cos^4\theta}{2} + \frac{\hat{s}^2}{M_\Phi^4}\frac{(1 - \beta^2\cos^2\theta)^2}{16}\,,$$

$$F_3 = 28 + 4\beta^2\cos^2\theta + \frac{\hat{s}}{M_\Phi^2}\beta^2\cos^2\theta(1 - \beta^2\cos^2\theta) + \frac{\hat{s}^2}{M_\Phi^4}\frac{(1 - \beta^2\cos^2\theta)^2}{8}\,,$$

$$F_4 = -5 + \beta^2\cos^2\theta + \frac{\hat{s}}{M_\Phi^2}\frac{-3 + \beta^2\cos^2\theta + 2\beta^4\cos^4\theta}{4} - \frac{\hat{s}^2}{M_\Phi^4}\frac{(1 - \beta^2\cos^2\theta)^2}{8}\,,$$

$$F_5 = \frac{3 - \beta^2\cos^2\theta}{4} + \frac{\hat{s}}{M_\Phi^2}\frac{5 - 4\beta^2\cos^2\theta - \beta^4\cos^4\theta}{16} + \frac{\hat{s}^2}{M_\Phi^4}\frac{13 - 25\beta^2\cos^2\theta + 11\beta^4\cos^4\theta + \beta^6\cos^6\theta}{128}\,,$$

$$F_6 = -4 + 4\beta^2\cos^2\theta\,,$$

$$F_7 = 4 + \frac{\hat{s}}{M_\Phi^2}\frac{-7 + 8\beta^2\cos^2\theta - \beta^4\cos^4\theta}{2} + \frac{\hat{s}^2}{M_\Phi^4}\frac{(1 - \beta^2\cos^2\theta)^2}{2} + \frac{\hat{s}^3}{M_\Phi^6}\frac{(1 - \beta^2\cos^2\theta)^3}{16}\,,$$

$$F_8 = -\frac{\hat{s}}{M_\Phi^2}(1 - \beta^2\cos^2\theta) + \frac{\hat{s}^2}{M_\Phi^4}\frac{11 - 13\beta^2\cos^2\theta + \beta^4\cos^4\theta + \beta^6\cos^6\theta}{8} - \frac{\hat{s}^2}{M_\Phi^4}\frac{(1 - \beta^2\cos^2\theta)^3}{8}\,,$$

$$F_9 = 1 - \beta^2\cos^2\theta + \frac{\hat{s}}{M_\Phi^2}\frac{-3 + 4\beta^2\cos^2\theta - \beta^4\cos^4\theta}{2} + \frac{\hat{s}^2}{M_\Phi^4}(1 - \beta^2\cos^2\theta)^2 + \frac{\hat{s}^3}{M_\Phi^6}\frac{-3 + 7\beta^2\cos^2\theta - 5\beta^4\cos^4\theta + \beta^6\cos^6\theta}{16}\,,$$

$$F_{10} = \frac{3 - \beta^2\cos^2\theta}{4} + \frac{\hat{s}}{M_\Phi^2}\frac{-19 + 20\beta^2\cos^2\theta - \beta^4\cos^4\theta}{16} + \frac{\hat{s}^2}{M_\Phi^4}\frac{141 - 249\beta^2\cos^2\theta + 107\beta^4\cos^4\theta + \beta^6\cos^6\theta}{128} + \frac{\hat{s}^3}{M_\Phi^6}\frac{-53 + 119\beta^2\cos^2\theta - 79\beta^4\cos^4\theta + 13\beta^6\cos^6\theta}{128} + \frac{\hat{s}^4}{M_\Phi^8}\frac{27 - 68\beta^2\cos^2\theta + 58\beta^4\cos^4\theta - 20\beta^6\cos^6\theta + 3\beta^8\cos^8\theta}{512}\,,$$

$$F_{11} = -8 + \frac{\hat{s}}{M_\Phi^2}(3 - 4\beta^2\cos^2\theta + \beta^4\cos^4\theta)\,,$$

$$\tilde{F}_8 = -\frac{35}{2}\beta - 22\beta\frac{\hat{s}}{M_\Phi^2} - \frac{3}{2}\beta\frac{\hat{s}^2}{M_\Phi^4} + \left(\frac{375}{8} + \frac{7}{8}\beta^2 + \frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{107}$$

$$\tilde{F}_9 = -\beta + \frac{199}{12}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{37}{12}\beta\frac{\hat{s}^2}{M_\Phi^4} - \left(\frac{87}{8} + \frac{5}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{108}$$

$$\tilde{F}_{10} = \frac{41}{24}\beta + \frac{11}{12}\beta\frac{\hat{s}}{M_\Phi^2} + \left(\frac{7}{4} + \frac{1}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{109}$$

$$\tilde{F}_{11} = -\frac{41}{6}\beta + \frac{23}{6}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{1}{2}(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{110}$$

$$\tilde{F}_{12} = \frac{41}{4}\beta + \frac{43}{48}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{145}{96}\beta\frac{\hat{s}^2}{M_\Phi^4} - \left(12 - \frac{3}{4}\beta^2 + \frac{1}{4}\frac{\hat{s}}{M_V^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{111}$$

$$\tilde{F}_{13} = -\frac{41}{6}\beta - \frac{355}{24}\beta\frac{\hat{s}}{M_V^2} + \frac{37}{16}\beta\frac{\hat{s}^2}{M_\Phi^4} + \left(\frac{31}{2} - \frac{1}{2}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \tag{112}$$

$$\tilde{F}_{14} = \frac{41}{24}\beta + \frac{37}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{113}{32}\beta\frac{\hat{s}^2}{M_\Phi^4} + \frac{49}{96}\beta\frac{\hat{s}^3}{M_\Phi^6} - \left(\frac{23}{4} - \frac{1}{8}\beta^2 + \frac{1}{4}\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right|. \tag{113}$$

(114)

Finally, the coefficients for the differential and the integrated cross section for $q\bar{q} \to V\overline{V}$, $G_i(\hat{s}, \beta, \cos\theta)$ and $\tilde{G}_i(\hat{s}, \beta)$, are given by

$$G_0 = 1 + \frac{1}{16}\left[\frac{\hat{s}}{M_\Phi^2} - (1+3\beta^2)\right]\sin^2\theta \tag{115}$$

$$G_1 = -1 - \frac{1}{8}\left[\frac{\hat{s}}{M_\Phi^2} - 2\right]\sin^2\theta \tag{116}$$

$$G_2 = 1 \tag{117}$$

$$G_3 = \frac{1}{4} + \frac{1}{16}\left[\frac{\hat{s}}{M_\Phi^2} - 2\right]\sin^2\theta \tag{118}$$

$$G_4 = -\frac{1}{2} + \frac{1}{4}\sin^2\theta \tag{119}$$

$$G_5 = \frac{1}{4} + \frac{1}{8}\left[\frac{\hat{s}}{M_\Phi^2} - 1\right]\sin^2\theta \tag{120}$$

$$\tilde{G}_0 = \frac{1}{24}\frac{\hat{s}}{M_\Phi^2} + \frac{23-3\beta^2}{24} \tag{121}$$

$$\tilde{G}_1 = -\frac{1}{12}\frac{\hat{s}}{M_\Phi^2} - \frac{5}{6} \tag{122}$$

$$\tilde{G}_2 = 1 \tag{123}$$

$$\tilde{G}_3 = \frac{1}{24}\frac{\hat{s}}{M_\Phi^2} + \frac{1}{6} \tag{124}$$

$$\tilde{G}_4 = -\frac{1}{3} \tag{125}$$

$$\tilde{G}_5 = \frac{1}{12}\frac{\hat{s}}{M_\Phi^2} + \frac{1}{6}. \tag{126}$$

$$F_{11} = -4\cdot\left(3+5\beta^2\cos^2\theta\right) - 5\frac{\hat{s}}{M_\Phi^2}\left(1-\beta^2\cos^2\theta\right)^2$$
$$+ \frac{1}{8}\frac{\hat{s}^2}{M_\Phi^4}\left(1-\beta^2\cos^2\theta\right)^2\left(35-13\beta^2\cos^2\theta\right) \quad (96)$$

$$F_{12} = 6\cdot\left(3+5\beta^2\cos^2\theta\right) - \frac{15}{2}\frac{\hat{s}}{M_\Phi^2}\left(1-\beta^2\cos^2\theta\right)\left(2+\beta^2\cos^2\theta\right)$$
$$+ \frac{1}{16}\frac{\hat{s}^2}{M_\Phi^4}\left(1-\beta^2\cos^2\theta\right)\left(-23+54\beta^2\cos^2\theta-39\beta^4\cos^4\theta\right)$$
$$+ \frac{1}{64}\frac{\hat{s}^3}{M_\Phi^6}\left(1-\beta^2\cos^2\theta\right)^2\left(113-49\beta^2\cos^2\theta\right) \quad (97)$$

$$F_{13} = -4\cdot\left(3+5\beta^2\cos^2\theta\right) + 5\frac{\hat{s}}{M_\Phi^2}\left(1-\beta^2\cos^2\theta\right)\left(5+\beta^2\cos^2\theta\right)$$
$$- \frac{1}{8}\frac{\hat{s}^2}{M_\Phi^4}\left(1-\beta^2\cos^2\theta\right)^2\left(119+13\beta^2\cos^2\theta\right)$$
$$+ \frac{1}{32}\frac{\hat{s}^3}{M_\Phi^6}\left(1-\beta^2\cos^2\theta\right)^2\left(79-15\beta^2\cos^2\theta\right) \quad (98)$$

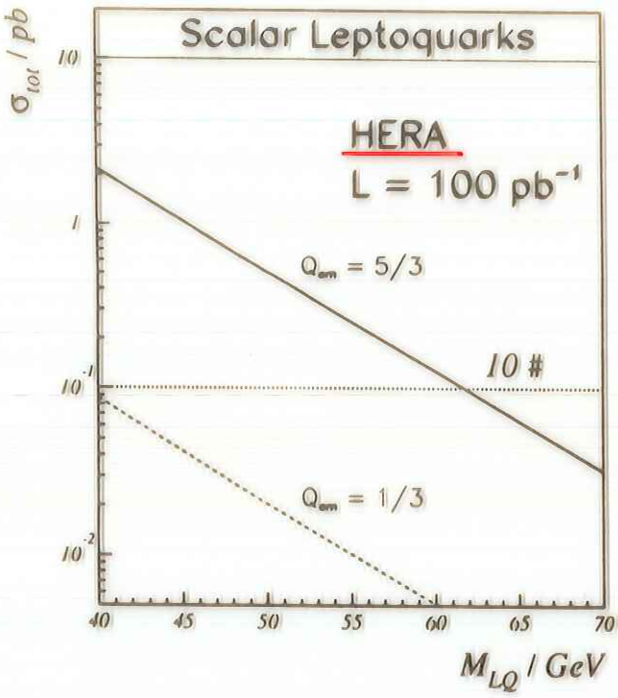
$$F_{14} = 3+5\beta^2\cos^2\theta - \frac{5}{4}\frac{\hat{s}}{M_\Phi^2}\left(1-\beta^2\cos^2\theta\right)\left(8+\beta^2\cos^2\theta\right)$$
$$+ \frac{1}{32}\frac{\hat{s}^2}{M_\Phi^4}\left(1-\beta^2\cos^2\theta\right)\left(321-324\beta^2\cos^2\theta-13\beta^4\cos^4\theta\right)$$
$$+ \frac{11}{64}\frac{\hat{s}^3}{M_\Phi^6}\left(1-\beta^2\cos^2\theta\right)^2\left(-23+7\beta^2\cos^2\theta\right)$$
$$+ \frac{1}{256}\frac{\hat{s}^4}{M_\Phi^8}\left(1-\beta^2\cos^2\theta\right)^2\left(135-22\beta^2\cos^2\theta+15\beta^4\cos^4\theta\right). \quad (99)$$

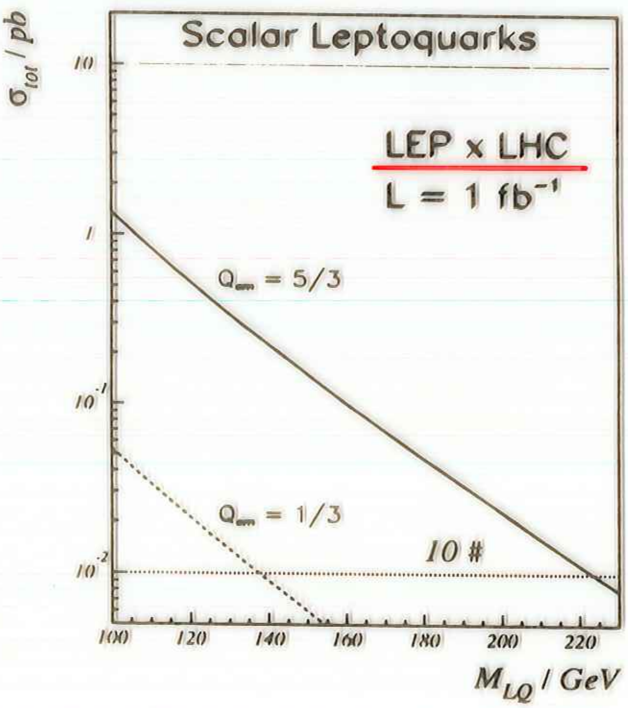
The coefficients $\tilde{F}_i(\hat{s},\beta)$ for the integrated cross section for $gg \to V\overline{V}$ are:

$$\tilde{F}_0 = \beta\left(\frac{523}{4}-90\beta^2+\frac{93}{4}\beta^4\right) - \frac{3}{4}(65-83\beta^2+19\beta^4-\beta^6)\log\left|\frac{1+\beta}{1-\beta}\right|$$
$$\tilde{F}_1 = -4\beta(41-9\beta^2) - \frac{87}{2}(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (100)$$
$$\tilde{F}_2 = 36\beta(1-\beta^2) - 25(1-\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (101)$$
$$\tilde{F}_3 = \beta(75-9\beta^2) + \frac{7}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \frac{1}{4}(1-61\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (102)$$
$$\tilde{F}_4 = -2\beta(20-9\beta^2) + \frac{1}{2}(91-31\beta^2)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (103)$$
$$\tilde{F}_5 = \beta\left(\frac{209}{6}-9\beta^2\right) + \frac{263}{12}\beta\frac{\hat{s}}{M_\Phi^2} + \frac{3}{2}\beta\frac{\hat{s}^2}{M_\Phi^4} - \left(\frac{219}{4}-\frac{31}{4}\beta^2+\frac{\hat{s}}{M_\Phi^2}\right)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (104)$$
$$\tilde{F}_6 = -9\beta - \frac{7}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \left(\frac{103}{8}+\frac{3}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (105)$$
$$\tilde{F}_7 = \frac{55}{2}\beta - \frac{17}{4}\beta\frac{\hat{s}}{M_\Phi^2} - \left(\frac{185}{8}-\frac{1}{8}\beta^2\right)\log\left|\frac{1+\beta}{1-\beta}\right| \quad (106)$$

4. NUMERICAL RESULTS

σ_{dir}:





σ_{dir}:

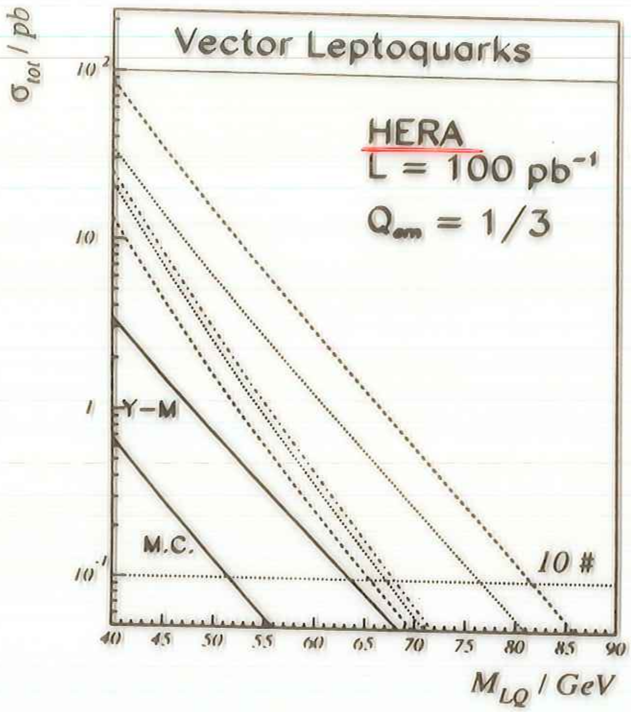


Fig. 3a. Integrated cross-sections for vector leptoquark pair production for $\sqrt{S} = 314$ GeV and different values of $\kappa_{A,G}$ and $\lambda_{A,G}$. Full lines: minimal coupling (M.C.) $\kappa_{A,G} = 1$, $\lambda_{A,G} = 0$ and Yang–Mills coupling (Y–M) $\kappa_{A,G} = \lambda_{A,G} = 0$. Upper dashed line: $\kappa_{A,G} = \lambda_{A,G} = -1$, lower dashed line: $\kappa_{A,G} = \lambda_{A,G} = 1$; upper dotted line: $\kappa_{A,G} = -1$, $\lambda_{A,G} = 1$; lower dotted line: $\kappa_{A,G} = 1$, $\lambda_{A,G} = -1$; dash-dotted line: $\kappa_A = 1$, $\kappa_G = -1$, $\lambda_A = 1$, $\lambda_G = -1$.

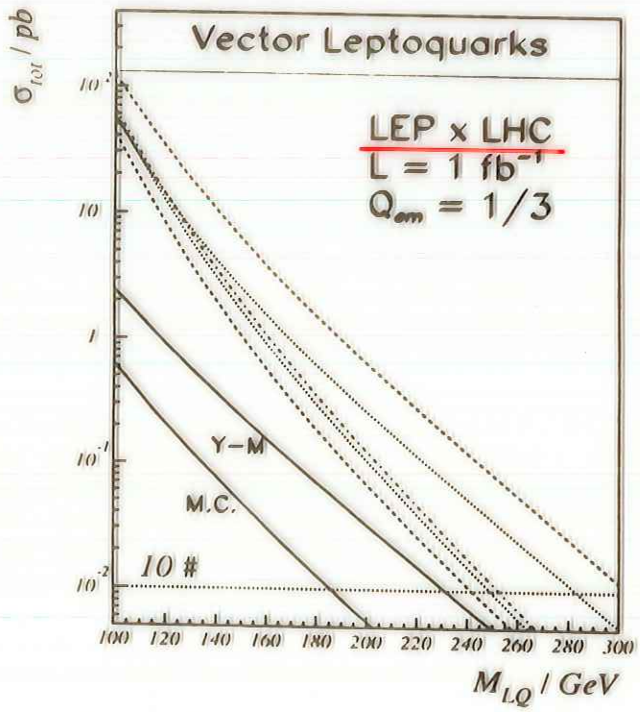


Fig. 3b. Integrated cross-sections for vector leptoquark pair production for $\sqrt{S} = 1260$ GeV. The other parameters are the same as in Fig. 3a.

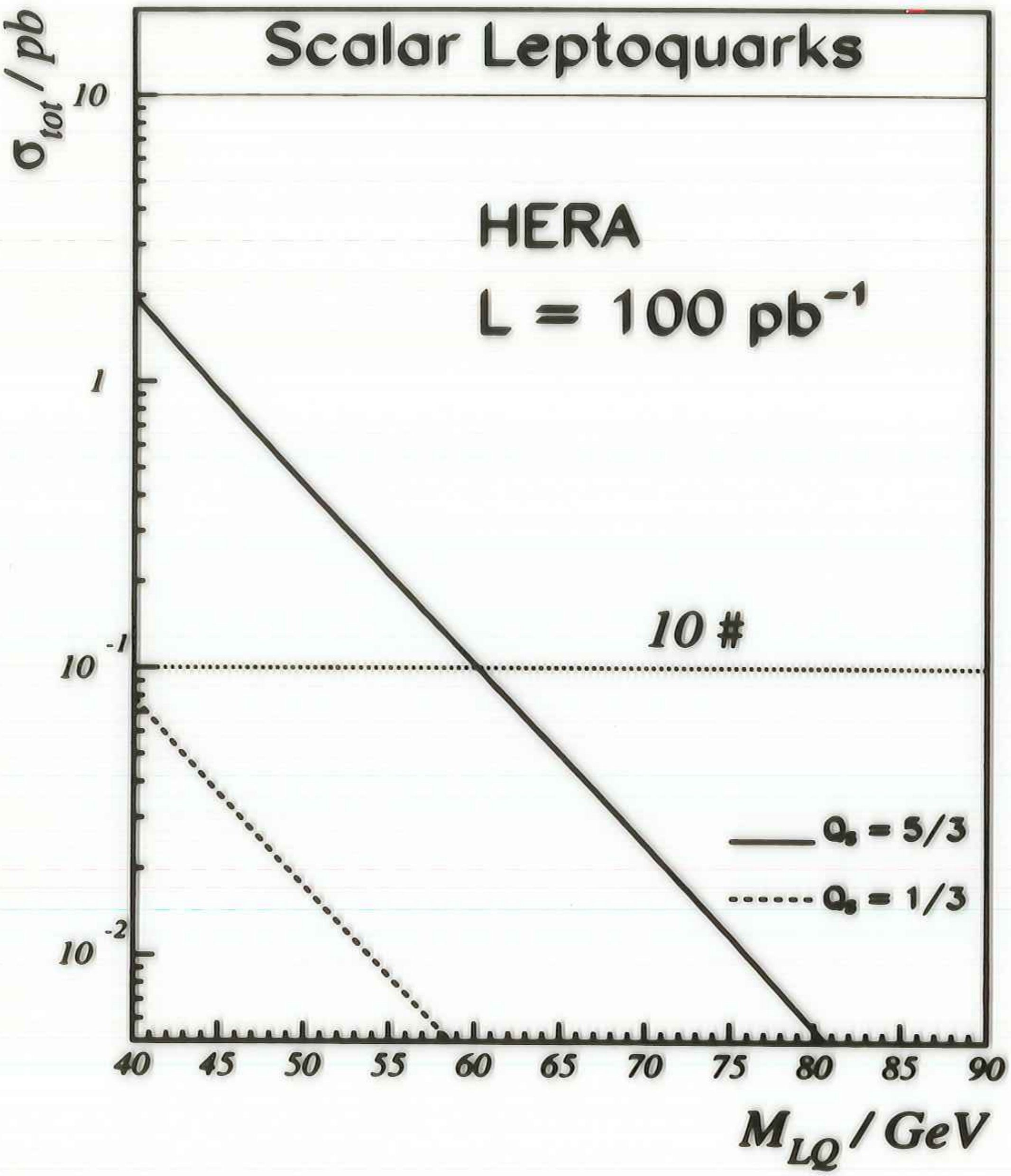


Figure 9a: Integrated cross sections for scalar leptoquark pair production at HERA, $\sqrt{S}$ = 314 GeV. Full line: σ_{tot} for $|Q_\Phi| = 5/3$; dotted line: σ_{dir} for $|Q_\Phi| = 5/3$; dashed line: σ_{tot} for $|Q_\Phi| = 1/3$; dash–dotted line: σ_{dir} for $|Q_\Phi| = 1/3$.

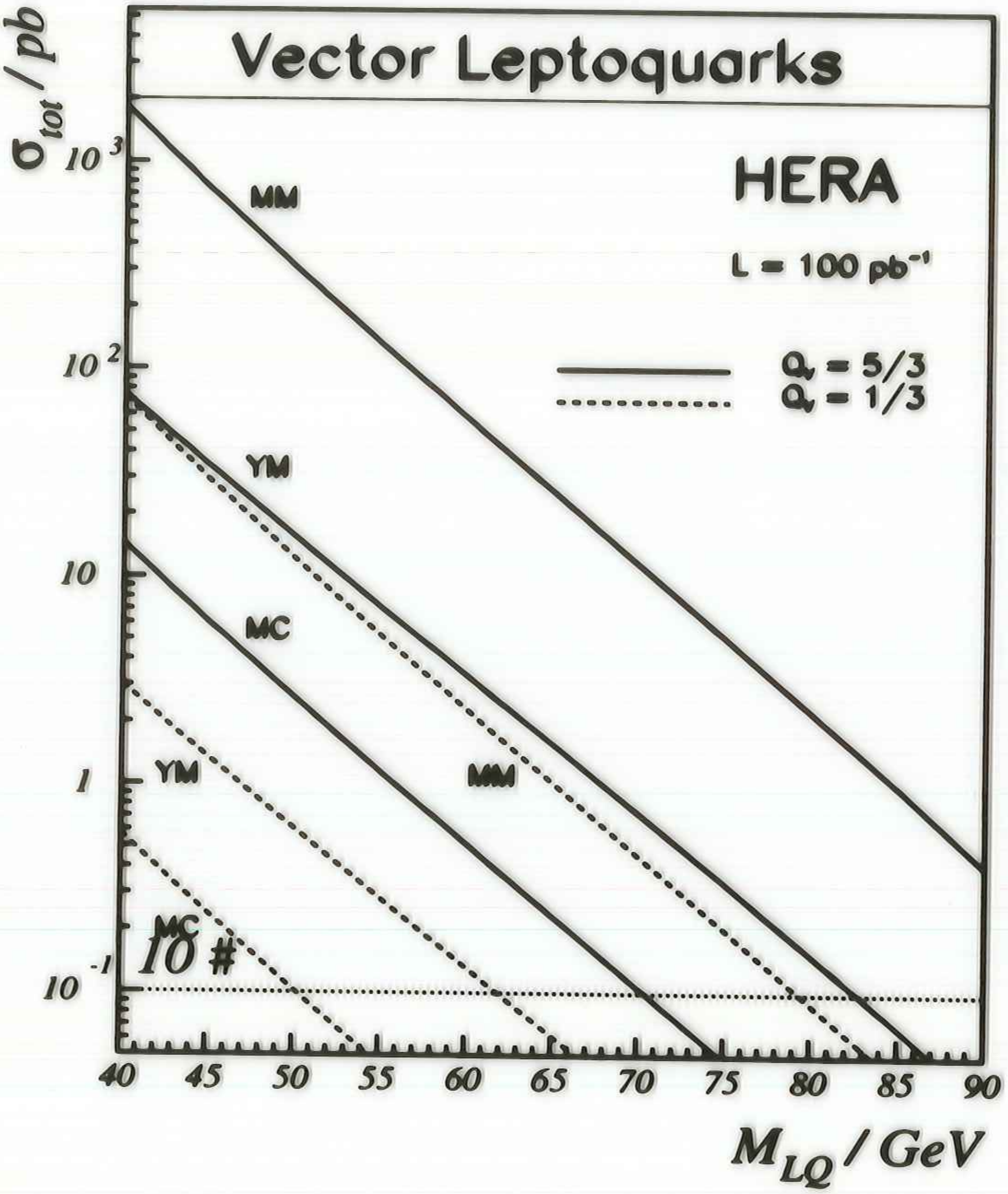


Figure 10a: Integrated cross sections $\sigma_{tot} = \sigma_{dir} + \sigma_{res}$ for vector leptoquark pair production at HERA, $\sqrt{S} = 314$ GeV. Upper full line: $|Q_\Phi| = 5/3, \kappa_{A,G} = \lambda_{A,G} = -1$ (MM5); Upper dashed line: $|Q_\Phi| = 5/3, \kappa_{A,G} = \lambda_{A,G} = 0$ (YM5); Upper dotted line: $|Q_\Phi| = 5/3, \kappa_{A,G} = 1, \lambda_{A,G} = 0$ (MC5); The corresponding lower lines are those for $|Q_\Phi = 1/3$.

SUMMARY OF LIMITS @ HERA:

$\int dt \, \mathcal{L} = 100 \, pb^{-1}$

$|Q_\phi| = \frac{1}{3}$:

$M_S > 48$ GeV

$M_V^{MC} > 63$ GeV

$M_V^{YM} > 74$ GeV

$|Q_\phi| = \frac{5}{3}$:

$M_S > 62$ GeV

$M_V^{MC} > 72$ GeV

$M_V^{YM} > 85$ GeV

- ONE MAY TRY TO CONSTRAIN κ_A, λ_A

  NOT POSSIBLE @ TEVATRON !

- ONE MAY LOOK FOR $2^{nd}, 3^{rd}$ FAMILY LQ's

  ↳ τ jet (b) !

  ↳ μ jet

  ↑ DIFFICULT FOR TEVATRON !