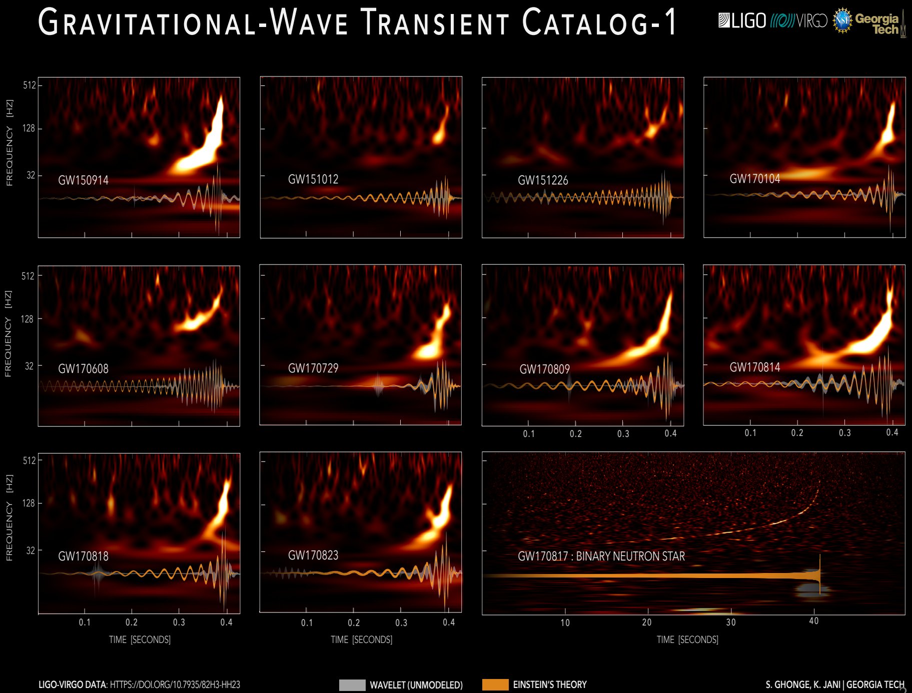
Binary black hole systems in a non-relativistic effective theory

Johannes Blümlein

Kitzbühel, Austria, 25 June 2019

J. Blümlein, A. Maier, P. Marquard, arXiv:1902.11180 [gr-qc]

GRAVITATIONAL-WAVE TRANSIENT CATALOG-1

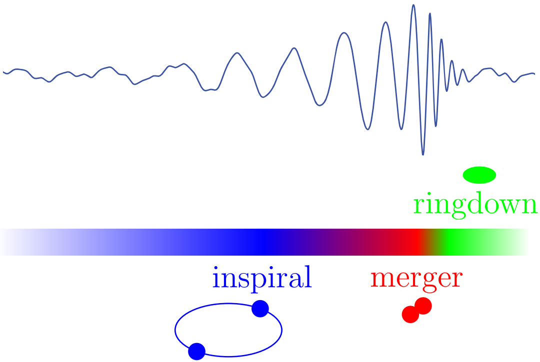


LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23

S. GHONGE, K. JANI | GEORGIA TECH

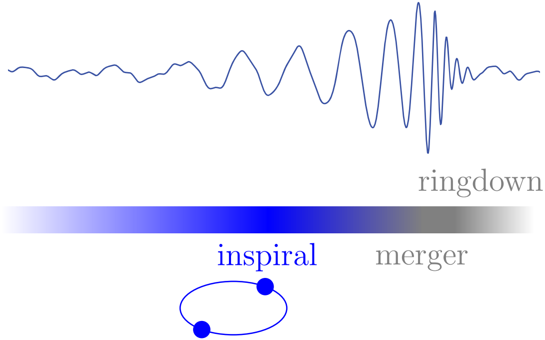
Gravitational waves

[LIGO Scientific Collaboration and Virgo Collaboration 2016]



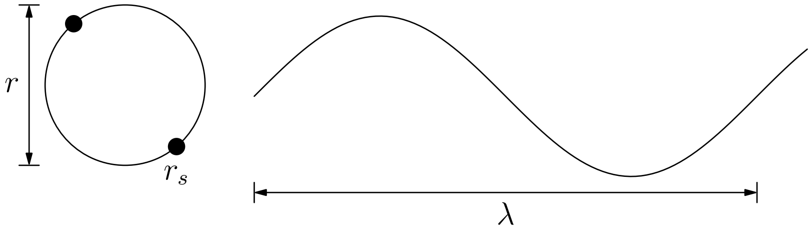
Gravitational waves

[LIGO Scientific Collaboration and Virgo Collaboration 2016]



Compact binary systems

Power counting

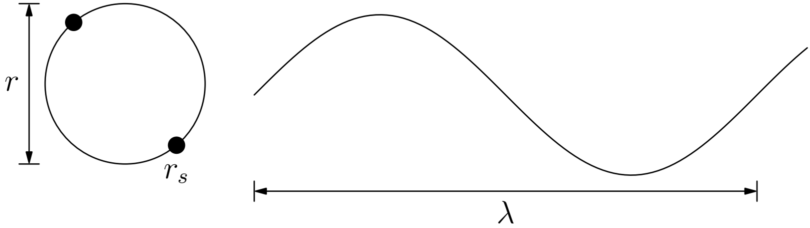


- Masses comparable: $m \equiv m_1 \sim m_2$
  Generalisation to different masses straightforward
- Nonrelativistic system: $v \ll 1$
- Virial theorem: $mv^2 \sim \frac{Gm^2}{r}$

> *Post-Newtonian (PN) expansion*:
> Combined expansion in $v \sim \sqrt{Gm/r} \ll 1$

Post-Newtonian expansion

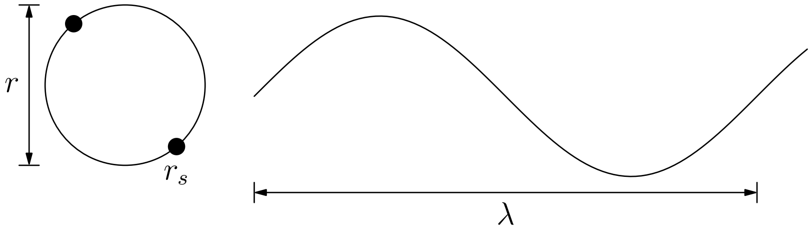
Scales



- $\omega \approx \frac{2v}{r} \Rightarrow$ $\boxed{\lambda \sim \frac{r}{v}}$
- $r_s = 2GM \Rightarrow$ $\boxed{r_s \sim rv^2}$

Post-Newtonian expansion

Scales at LIGO/VIRGO



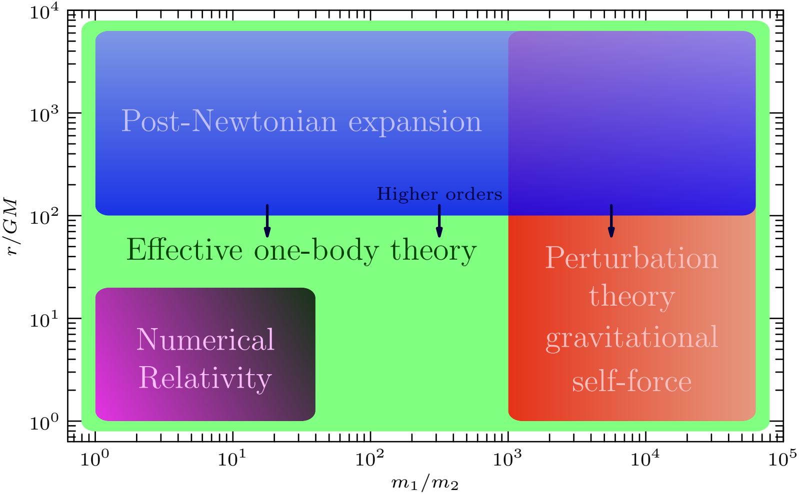
- $10\,\text{km} \lesssim \lambda \lesssim 10\,000\,\text{km}$
- $10\,\text{km} \lesssim r_s \lesssim 100\,\text{km}$
- Inspiral: $0.1 \lesssim v \lesssim 0.5$, $100\,\text{km} \lesssim r \lesssim 1000\,\text{km}$

	black holes	neutron stars
masses	$\sim 10\text{–}50\, m_\odot$	$\sim 1 m_\odot$
radiated energy	$\sim 1\text{–}5\, m_\odot$	$\geq 0.04\, m_\odot$
redshift	$\sim 0.1\text{–}0.5$	~ 0.01

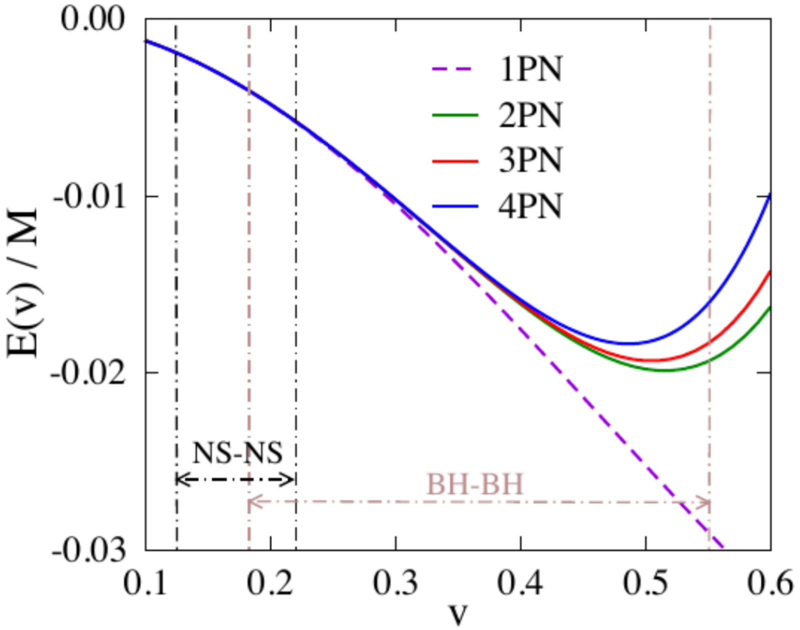
[LIGO Scientific Collaboration and the Virgo Collaboration, O1/O2 Catalog, 2018]

Post-Newtonian expansion

Other approaches



adapted from [Buonanno, Sathyaprakash arXiv:1410.7832]



[Buonanno, QCD Meets Gravity IV, 2018]

Post-Newtonian expansion

Theory status

- 1PN (v^2): [Lorentz, Droste 1917; Einstein, Infeld, Hoffmann 1938]
- 2PN (v^4): [Chandrasekhar, Esposito, Nutku 1969–1970; Okamura, Ohta, Kimura, Hiida 1973–1974]
- 2.5PN (v^5): [Damour, Deruelle 1981–1983]
- 3PN (v^6): [Damour, Jaranowski, Schäfer, 1997–2001; Andrade, Blanchet, Iyer, Faye 2000–2002]
- 3.5PN (v^7): [Iyer, Will 1993–1995]
- 4PN (v^8):
  - ADM Hamiltonian formalism [Damour, Jaranowski, Schäfer 2014-2016]
  - Fokker Lagrangian in harmonic coordinates [Bernard, Blanchet, Bohé, Faye, Marchant, Marsat 2017]
  - Non-relativistic effective field theory [Foffa, Mastrolia, Sturani, Sturm 2017–2019]
- Many more contributions, e.g. spin effects, radiation

Method in this talk:
non-relativistic effective field theory [Goldberger, Rothstein 2004]

General Relativity

- Here: point-like objects
  - No spin
  - No finite-size effects (neutron stars: 5PN, black holes: 6PN)
- Harmonic gauge fixing: $\partial_\mu(\sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu}) = 0$
  $g = \det(g^{\mu\nu})$
- Dimensional regularisation: $d = 3 - 2\epsilon$

$$
\begin{aligned}
S_{\text{GR}} &= S_{\text{EH}} + S_{\text{GF}} + S_{pp} \\
S_{\text{EH}} &= \frac{1}{16\pi G} \int d^{d+1}x \, \sqrt{-g} R \\
S_{\text{GF}} &= -\frac{1}{32\pi G} \int d^{d+1}x \, \sqrt{-g} \, \Gamma_\mu \Gamma^\mu \\
S_{\text{pp}} &= -\sum_i m_i \int d\tau_i = -\sum_i m_i \int dt \, \sqrt{-g_{\mu\nu} \frac{\partial x_i^\mu}{\partial t} \frac{\partial x_i^\nu}{\partial t}} \\
R &= g^{\mu\nu} R_{\mu\nu} \\
\Gamma^\mu &= g^{\alpha\beta} {\Gamma^\mu}_{\alpha\beta}
\end{aligned}
$$

Why can QFT methods be of help here ?

- Any dynamical physical theory is based on the Lagrange formalism and can be derived from the principle of the least action.
- Near the non-relativistic limit, Einstein gravity possesses an expansion in Newton's constant and the velocities of the involved macroscopic bodies $v_i \ll c$.
- One can consider the associated path integral representation [Feynman & Hibbs 1965, Zinn-Justin 2002] and derive the perturbative expansions from it.
- This will lead to the associated effective field theory representation, not necessarily built on the (flat space) gravitons.
- From order to order (potentially) new interaction vertices will appear.
- This systematic approach will, however, perturbatively represent the full theory.
- The key-issue is to reduce to the associated master integrals and to calculate them.
- Current level: 5–loop integrals in $d = 3 - 2\varepsilon$.

Non-relativistic effective theory

[Goldberger, Rothstein 2004]

Similar to non-relativistic QCD

[Caswell, Lepage 1985; Pineda, Soto 1997; Luke, Manohar, Rothstein 2000; ...]

Full theory: General relativity		Effective theory: NRGR
$S_{\text{GR}} = S_{\text{EH}} + S_{\text{GF}} + S_{pp}$	$\longrightarrow$	$S_{\text{NRGR}} = \int dt \frac{1}{2} m_i v_i^2 + \frac{G m_1 m_2}{r} + \dots$
potential gravitons: $k_0 \sim \frac{v}{r}, \vec{k} \sim \frac{1}{r}$		classical potentials
radiation gravitons: $k_0 \sim \frac{v}{r}, \vec{k} \sim \frac{v}{r}$		radiation gravitons

Potential matching

Expansion of action

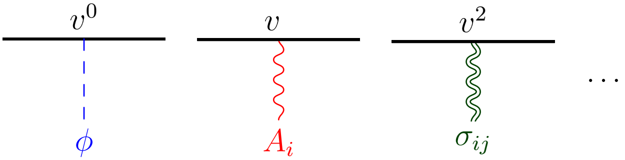
Expand $S_{\rm GR}$ in $v \sim \sqrt{Gm/r} \ll 1$, e.g.

$$S_{\rm pp} = -\sum_i m_i \int dt \sqrt{-g_{\mu\nu}\frac{\partial x_i^\mu}{\partial t}\frac{\partial x_i^\nu}{\partial t}} = -\sum_i m_i \int dt \sqrt{-g_{00}} + \mathcal{O}(v_i)$$

Coupling to spatial components of metric suppressed

Temporal Kaluza-Klein decomposition [Kol, Smolkin 2010]: 10 fields.

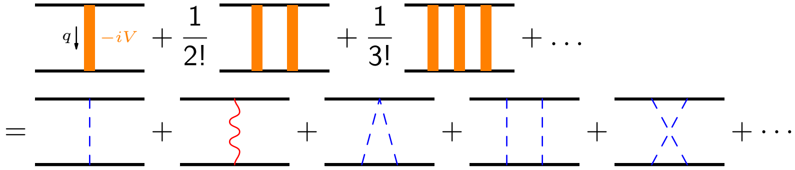
$$g^{\mu\nu} = e^{2\phi}\begin{pmatrix} -1 & A_j \\ A_i & e^{-2\frac{d-1}{d-2}\phi}(\delta_{ij}+\sigma_{ij}) - A_i A_j \end{pmatrix}$$



Potential matching

Diagrammatic expansion

Equate amplitude in effective and full theory:

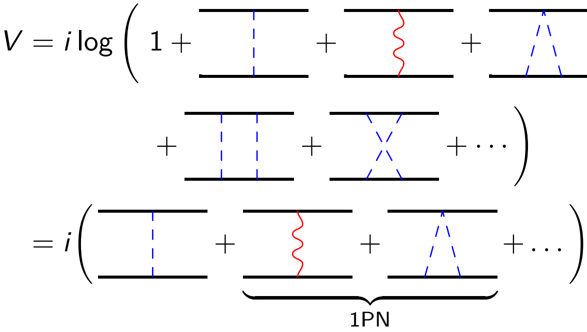


All momenta potential, $p_0 \sim \frac{v}{r} \ll p_i \sim \frac{1}{r}$
$\hookrightarrow$ expand propagators:

$$\frac{1}{\vec{p}^{\,2} - p_0^2} = \frac{1}{\vec{p}^{\,2}} + \frac{p_0^2}{\vec{p}^{\,4}} + \mathcal{O}(v^4)$$

Potential matching

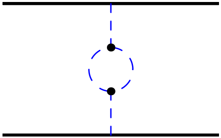
Diagrammatic expansion



Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)



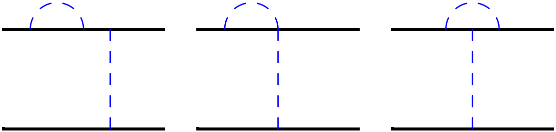
$\sim \frac{\hbar}{mvr}$

- No single-source corrections
- No source-reducible diagrams [Fischler 1977]

Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections



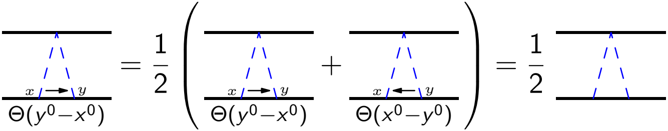
  Absorbed into renormalisation of sources

- No source-reducible diagrams [Fischler 1977]

Potential matching

Diagram selection

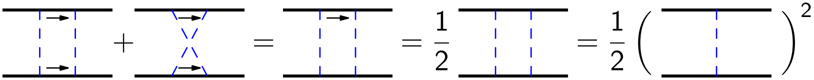
- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]
  Initially *time-ordered* diagrams:



Potential matching

Diagram selection

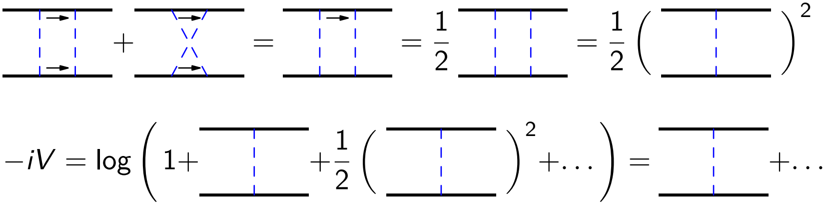
- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]
  Initially *time-ordered* diagrams:



Potential matching

Diagram selection

- No pure graviton loops (quantum corrections)
- No single-source corrections
- No source-reducible diagrams [Fischler 1977]
  Initially *time-ordered* diagrams:



Effective Field Theory Calculations

Known results

Confirmation of previous results:

- 1PN: [1917 (1938)] [Goldberger, Rothstein 2004]
- 2PN: [1969/70] [Gilmore, Ross 2008]
- 3PN: [1997/2001][Foffa, Sturani 2011]
- 4PN:
  - "static" contribution $v = 0$:
    [Foffa, Mastrolia, Sturani, Sturm 2016; Damour, Jaranowski 2017]
  - $v \neq 0$: [2014] [Foffa, Sturani 2019; Foffa, Porto, Rothstein, Sturani 2019]

New:

- **5PN static contribution:**
  [Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla 27 Feb 2019; Blümlein, Maier, Marquard 28 Feb 2019]

Effective Field Theory Calculations

Number of diagrams

	QGRAF	source irred	no source loops	no tadpoles	sym
N	1	1	1	1	1
1PN	2	2	2	2	1
2PN	19	19	19	15	5
3PN	360	276	258	122	8
4PN	10081	5407	4685	1815	50
5PN	332020	128080	101570	27582	154

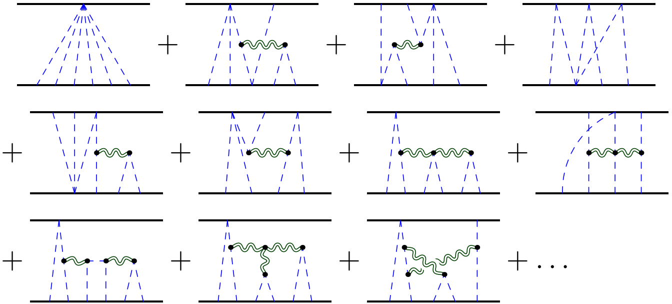
sym = # of identical-value diagrams by symmetry relations.

We do not consider the latter class, i.e. evaluate 27582 diagrams.

Effective Field Theory Calculations

Static 5PN calculation

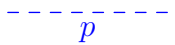
$-iV^{S}_{5\text{PN}} =$



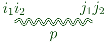
In the static limit there are no contribution form the vector field A_k.

Effective Field Theory Calculations

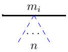
Feynman rules



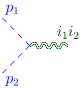
$$= -\frac{i}{2c_d \vec{p}^{\,2}}$$



$$= -\frac{i}{2\vec{p}^{\,2}}\left(\delta_{i_1 j_1}\delta_{i_2 j_2} + \delta_{i_1 j_2}\delta_{i_2 j_1} + (2 - c_d)\delta_{i_1 i_2}\delta_{j_1 j_2}\right)$$

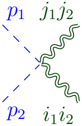


$$= -i\frac{m_i}{m_{\text{Pl}}^n}$$



$$= i\frac{c_d}{2m_{\text{Pl}}}\left(V^{i_1 i_2}_{\phi\phi\sigma} + V^{i_2 i_1}_{\phi\phi\sigma}\right)$$

$$V^{i_1 i_2}_{\phi\phi\sigma} = \vec{p}_1 \cdot \vec{p}_2 \delta^{i_1 i_2} - 2p_1^{i_1} p_2^{i_2}$$

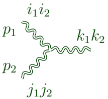


$$= i\frac{c_d}{16m_{\text{Pl}}^2}\left(V^{i_1 i_2, j_1 j_2}_{\phi\phi\sigma\sigma} + V^{i_2 i_1, j_1 j_2}_{\phi\phi\sigma\sigma} + V^{i_1 i_2, j_2 j_1}_{\phi\phi\sigma\sigma} + V^{i_2 i_1, j_2 j_1}_{\phi\phi\sigma\sigma}\right)$$

$$V^{i_1 i_2, j_1 j_2}_{\phi\phi\sigma\sigma} = \vec{p}_1 \cdot \vec{p}_2\left(\delta^{i_1 i_2}\delta^{j_1 j_2} - 2\delta^{i_1 j_1}\delta^{i_2 j_2}\right) - 2\left(p_1^{i_1} p_2^{i_2}\delta^{j_1 j_2} + p_1^{j_1} p_2^{j_2}\delta^{i_1 i_2}\right) + 8\delta^{i_1 j_1} p_1^{i_2} p_2^{j_2}$$

Effective Field Theory Calculations

Feynman rules



$$= \frac{i}{32 m_{\mathrm{Pl}}} \left(\tilde{V}^{i_1 i_2, j_1 j_2, k_1 k_2}_{\sigma\sigma\sigma} + \tilde{V}^{i_2 i_1, j_1 j_2, k_1 k_2}_{\sigma\sigma\sigma} \right)$$

$$\tilde{V}^{i_1 i_2, j_1 j_2, k_1 k_2}_{\sigma\sigma\sigma} = V^{i_1 i_2, j_1 j_2, k_1 k_2}_{\sigma\sigma\sigma} + V^{i_1 i_2, j_2 j_1, k_1 k_2}_{\sigma\sigma\sigma} + V^{i_1 i_2, j_1 j_2, k_2 k_1}_{\sigma\sigma\sigma} + V^{i_1 i_2, j_2 j_1, k_2 k_1}_{\sigma\sigma\sigma}$$

$$\begin{aligned}
V^{i_1 i_2, j_1 j_2, k_1 k_2}_{\sigma\sigma\sigma} = {} & (\vec{p}_1^{\,2} + \vec{p}_1 \cdot \vec{p}_2 + \vec{p}_2^{\,2}) \Big(- \delta^{j_1 j_2} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) \\
& \quad + 2 \left[\delta^{i_1 j_1} \left(4\delta^{i_2 k_1} \delta^{j_2 k_2} - \delta^{i_2 j_2} \delta^{k_1 k_2} \right) - \delta^{i_1 i_2} \delta^{j_1 k_1} \delta^{j_2 k_2} \right] \Big) \\
& + 2 \Big\{ 4 \left(p_1^{k_2} p_2^{i_2} - p_1^{i_2} p_2^{k_2} \right) \delta^{i_1 j_1} \delta^{j_2 k_1} \\
& \quad + 2 \left[\left(p_1^{i_1} + p_2^{i_1} \right) p_2^{i_2} \delta^{j_1 k_1} \delta^{j_2 k_2} - p_1^{k_1} p_2^{k_2} \delta^{i_1 j_1} \delta^{i_2 j_2} \right] \\
& \quad + \delta^{j_1 j_2} \left[p_1^{k_1} p_2^{k_2} \delta^{i_1 i_2} + 2 \left(p_1^{k_2} p_2^{i_2} - p_1^{i_2} p_2^{k_2} \right) \delta^{i_1 k_1} - \left(p_1^{i_1} + p_2^{i_1} \right) p_2^{i_2} \delta^{k_1 k_2} \right] \\
& \quad + p_2^{j_2} \Big(4 p_1^{i_2} \delta^{i_1 k_1} \delta^{j_1 k_2} + p_1^{j_1} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) \\
& \qquad + 2 \left[\delta^{i_1 j_1} \left(p_1^{i_2} \delta^{k_1 k_2} - 2 p_1^{k_2} \delta^{i_2 k_1} \right) - p_1^{k_2} \delta^{i_1 i_2} \delta^{j_1 k_1} \right] \Big) \\
& \quad + p_1^{j_2} \Big(p_1^{j_1} \left(2\delta^{i_1 k_1} \delta^{i_2 k_2} - \delta^{i_1 i_2} \delta^{k_1 k_2} \right) - 4 p_2^{i_2} \delta^{i_1 k_1} \delta^{j_1 k_2} \\
& \qquad + 2 \left[p_2^{k_2} \delta^{i_1 i_2} \delta^{j_1 k_1} + \delta^{i_1 j_1} \left(2 p_2^{k_2} \delta^{i_2 k_1} - p_2^{i_2} \delta^{k_1 k_2} \right) \right] \Big) \Big\}, \quad c_d = 2\frac{d-1}{d-2}, \; m_{\mathrm{Pl}} = \sqrt{32\pi G}
\end{aligned}$$

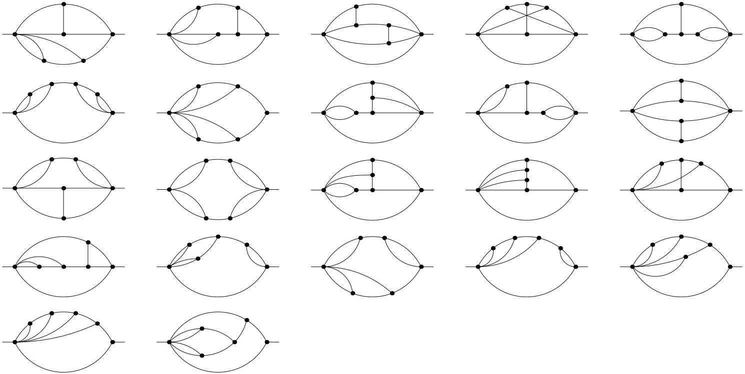
Note errors in the last two Feynman rules in [Foffa, Mastrolia, Sturani, Sturm 2016].

Effective Field Theory Calculations

Diagram families

Massless propagators:

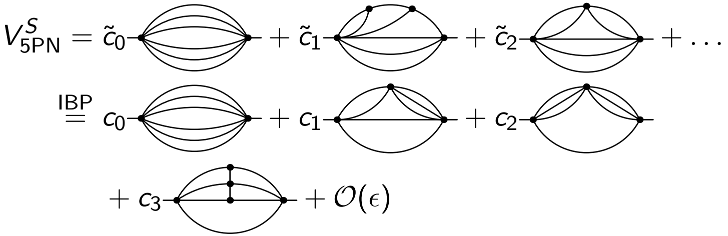
$$P_f(q) = \int \frac{d^d l_1}{\pi^{d/2}} \cdots \frac{d^d l_5}{\pi^{d/2}} \frac{\mathcal{N}(q, l_1, \ldots, l_5)}{\vec{p}_1^{\,2a_1} \cdots \vec{p}_{10}^{\,2a_{10}}}$$



Effective Field Theory Calculations

Reduction to master integrals

Apply integration-by-parts relations (`crusher`): [Marquard & Seidel, unpubl.], [Chetyrkin, Tkachov 1981; Laporta 2000]

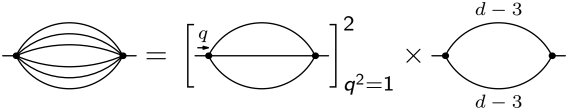


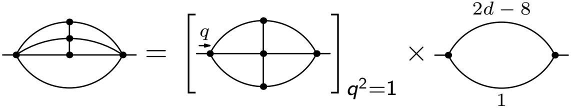
$\tilde{c}_i, c_j$: Laurent series in $\epsilon = \frac{3-d}{2}$, polynomials in m_1, m_2, r^{-1}, G^{-1}

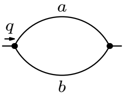
Effective Field Theory Calculations

Calculation of master integrals

Master integrals factorise, e.g.

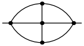






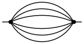
$$= \int \frac{d^d l}{\pi^{d/2}} \frac{1}{((q-l)^2)^a} \frac{1}{(l^2)^b}$$

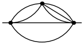
$$= \frac{1}{(q^2)^{a+b-d/2}} \frac{\Gamma\left(\frac{d}{2}-a\right)\Gamma\left(\frac{d}{2}-b\right)\Gamma\left(a+b-\frac{d}{2}\right)}{\Gamma(a)\Gamma(b)\Gamma\left(d-a-b\right)}$$

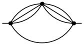
 known from 4PN [Foffa, Mastrolia, Sturani, Sturm 2016; Damour, Jaranowski 2017] up to the power in ε presently needed.

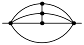
Effective Field Theory Calculations

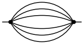
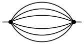
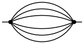
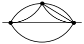
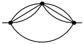
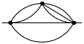
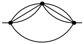
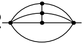
Results for master integrals


$$= e^{5\epsilon\gamma_E}\frac{\Gamma\left(6-\frac{5d}{2}\right)\Gamma^6\left(-1+\frac{d}{2}\right)}{\Gamma(-6+3d)}$$


$$= e^{5\epsilon\gamma_E}\frac{\Gamma\left(7-\frac{5d}{2}\right)\Gamma(3-d)\Gamma\left(2-\frac{d}{2}\right)\Gamma^7\left(-1+\frac{d}{2}\right)\Gamma(5-2d)}{\Gamma\left(5-\frac{3}{2}d\right)\Gamma(-2+d)\Gamma\left(-3+\frac{3}{2}d\right)\Gamma\left(-7+3d\right)}$$


$$= e^{5\epsilon\gamma_E}\frac{\Gamma\left(7-\frac{5d}{2}\right)\Gamma^2(3-d)\Gamma^7\left(-1+\frac{d}{2}\right)\Gamma\left(-6+\frac{5d}{2}\right)}{\Gamma\left(6-2d\right)\Gamma^2\left(-3+\frac{3d}{2}\right)\Gamma\left(-7+3d\right)}$$


$$= 6\pi^{7/2}\left[\frac{2}{\epsilon}-4-4\ln(2)-\left(48+8\ln(2)-4\ln^2(2)-105\zeta_2\right)\epsilon+\mathcal{O}(\epsilon^2)\right]$$

$$V_{5\mathrm{PN}}^{S} \overset{\epsilon=0}{=} \frac{G^6}{r^6\pi^{7/2}}(m_1 m_2)\left\{\frac{15}{32}(m_1^5+m_2^5)\left[\right.\right.$$

$$\left.\right]_{\epsilon^0}+\frac{91}{4}m_1 m_2(m_1^3+m_2^3)\left[\right.$$

$$\left.\right]_{\epsilon^0}$$
$$+m_1^2 m_2^2(m_1+m_2)\left(\left[\frac{293}{4}\right.\right.$$

$$-\frac{45}{16}$$

$$+\frac{45}{32}$$

$$\left.\right]_{\epsilon^0}$$
$$+\left[\frac{519}{16}\right.$$

$$-\frac{627}{32}$$

$$+2$$

$$\left.\left.\left.\right]_{\epsilon^{-1}}\right)\right\}$$

The Static Potential

Result

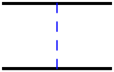
$$
\begin{aligned}
V_{\mathrm{N}}^{S} &= -\frac{G}{r} m_1 m_2 \\
V_{1\mathrm{PN}}^{S} &= \frac{G^2}{2r^2} m_1 m_2 (m_1 + m_2) \\
V_{2\mathrm{PN}}^{S} &= -\frac{G^3}{r^3} m_1 m_2 \left[\frac{1}{2}(m_1^2 + m_2^2) + 3 m_1 m_2 \right] \\
V_{3\mathrm{PN}}^{S} &= \frac{G^4}{r^4} m_1 m_2 \left[\frac{3}{8}\left(m_1^3 + m_2^3\right) + 6 m_1 m_2 (m_1 + m_2) \right] \\
V_{4\mathrm{PN}}^{S} &= -\frac{G^5}{r^5} m_1 m_2 \left[\frac{3}{8}\left(m_1^4 + m_2^4\right) + \frac{31}{3} m_1 m_2 \left(m_1^2 + m_2^2\right) + \frac{141}{4} m_1^2 m_2^2 \right] \\
V_{5\mathrm{PN}}^{S} &= \frac{G^6}{r^6} m_1 m_2 \left[\frac{5}{16}(m_1^5 + m_2^5) + \frac{91}{6} m_1 m_2 (m_1^3 + m_2^3) + \frac{653}{6} m_1^2 m_2^2 (m_1 + m_2) \right]
\end{aligned}
$$

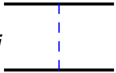
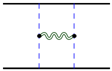
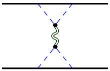
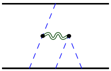
All ζ-values cancel. This is not the case for velocity corrections.

Effective Field Theory Calculations

Factorisation of 5PN static potential [Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla 2019]

Factorisation at only *odd* PN orders, e.g.

$V^S_{3\text{PN}} = \sum_i C_i \Big($  $\Big)^4$

$+ \sum_j C_j$  $\times \Big\{$  ,  ,  $\Big\}$

Similar: $V^S_{5\text{PN}}$ from diagrams with 1–4 loops
The static potential at odd loop number can be obtained from known lower order terms.
This does not apply to the velocity, acceleration etc. terms, however.

Effective Field Theory Calculations

Velocity corrections

Full corrections include velocities and *higher time derivatives*:

$$\begin{aligned}\mathcal{L}_{2PN} = &+\frac{G^3}{r^3}m_1 m_2\left[\frac{1}{2}(m_1^2+m_2^2)+3m_1 m_2\right]\\ &+Gm_1 m_2 r\left[\frac{15}{8}\vec{a}_1\vec{a}_2-\frac{1}{8}(\vec{a}_1\vec{r}\,)(\vec{a}_2\vec{r}\,)\right]\\ &+(\text{terms depending on } \vec{v}_1,\vec{v}_2)\end{aligned}$$

Can be eliminated using

- *Total time derivatives* $\delta\mathcal{L}\propto\frac{d}{dt}F(\vec{r},\vec{v}_1,\vec{v}_2)$
- *Equations of motion* $\delta\mathcal{L}\propto\left(\vec{a}_1+\frac{Gm_2}{r^3}\vec{r}\right)\left(\vec{a}_2-\frac{Gm_1}{r^3}\vec{r}\right)$

$$\begin{aligned}\mathcal{L}_{2PN} = &-\frac{G^3}{r^3}m_1 m_2\left[\frac{1}{4}(m_1^2+m_2^2)+\frac{5}{4}m_1 m_2\right]\\ &+(\text{terms depending on } \vec{v}_1,\vec{v}_2)\end{aligned}$$

Conclusions and Outlook

- The inspiral phase of compact binary systems is well described by *Post-Newtonian (PN) expansion* $v \sim \sqrt{Gm/r} \ll 1$.
- Effective field theory and calculational methods from particle physics are very effective for high PN orders.
- The static gravitational potential is now known at five loops.
- Higher order corrections extend the area perturbatively accessible towards the region where presently only pure numerical methods are applied.
- The next important corrections are the finite velocity terms to the same order.
- Fully resummed velocity expressions would be welcome to have: Post-Minkowskian approach.

Upcoming KMPB Conference:

"From Classical Gravity to Quantum Amplitudes and Back",
Berlin, Nov. 17-20 2019 [org. J. Blümlein and Th. Damour];