Two–Loop Massive Operator Matrix Elements and Heavy Flavor Production in Deep–Inelastic Scattering

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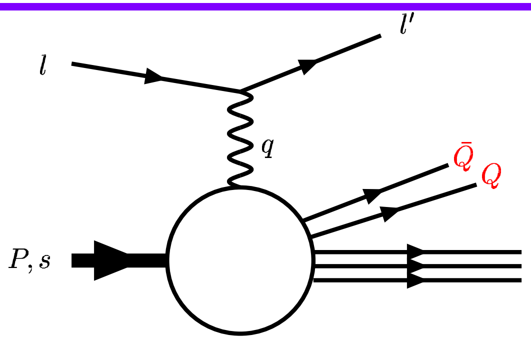
- Introduction
- The Method
- The Calculation
- The Results
- The Comparison with Previous Results
- The Conclusions

Refs.: J. Blümlein, A. De Freitas, W. L. van Neerven and S. Klein, Nucl. Phys. **B 755** (2006) 272.
I. Bierenbaum, J. Blümlein and S. Klein, Nucl. Phys. Proc. Suppl. **160**, 85 (2006); Phys. Lett. **B** (2007) in print, [hep-ph/0702265]; DESY 07–026, [hep-ph/0703285].
J. Blümlein and S. Klein, DESY 07–027.

1. Introduction



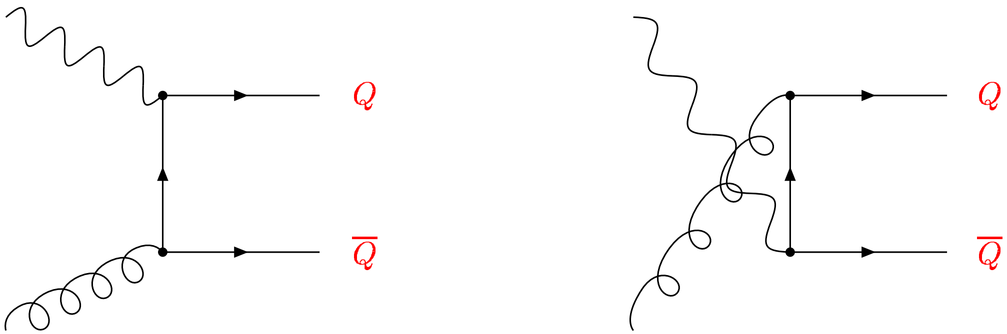
Kinematic variables:

$$Q^2 := -q^2, \qquad \nu := \frac{Pq}{M}, \qquad x := \frac{Q^2}{2M\nu} \ ,$$
$$s^2 = -1, \qquad sP = 0 \ .$$

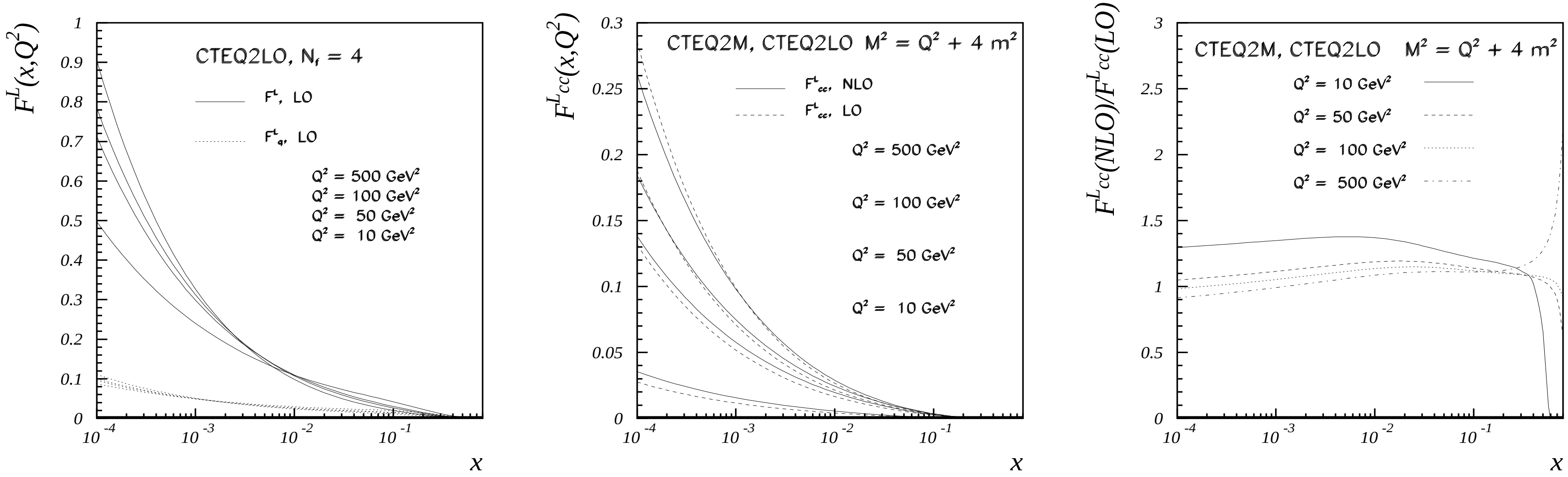
Hadronic Tensor for heavy quark production via single photon exchange:

$$W_{\mu\nu}^{Q\bar{Q}}(q,P,s) = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi)\langle P,s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P,s\rangle_{Q\bar{Q}}$$
$$= \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L^{Q\bar{Q}}(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2^{Q\bar{Q}}(x,Q^2)$$
$$-\frac{M}{2Pq}\varepsilon_{\mu\nu\alpha\beta}q^\alpha\left[s^\beta g_1^{Q\bar{Q}}(x,Q^2) + \left(s^\beta - \frac{sq}{Pq}p^\beta\right) g_2^{Q\bar{Q}}(x,Q^2)\right] \ .$$

LO contribution:



- Heavy Flavour (charm) contributions to DIS structure functions are rather large.



- Need: Increase accuracy of the perturbative description of DIS structure functions.
- $\Longleftrightarrow$ QCD Analysis and Determination of $\Lambda_{\rm QCD}$ from DIS data.
- $\Longleftrightarrow$ Future precise determination of the Gluon and Sea Quark Distributions.

Status of Heavy Flavor Corrections

Unpolarized DIS :

- LO : [Witten, 1976; Babcock & Sivers, 1978; Shifman, Vainshtein, Zakharov 1978; Leveille & Weiler, 1979]
- NLO semi-analytic : [Laenen, Riemersma, Smith, van Neerven, 1993, 1995]
  NLO asymptotic : [Buza, Matiounine, Smith, Migneron, van Neerven, 1996]

Polarized DIS :

- LO : g_1 [Watson, 1982; Glück, Reya, Vogelsang, 1991; Vogelsang, 1991]
- NLO asymprotic : g_1 [Buza, Matiounine, Smith, van Neerven, 1997]
- g_2 : Wandzura–Wilczek Rel. [Blümlein, Ravindran, van Neerven, 2003]

Precise Mellin Space Expressions: [Alekhin, Blümlein, 2003].

2. The Method

- massless RGE and Light–Cone Expansion in Bjørken–Limit $\{Q^2,\ \nu\} \rightarrow \infty$, x fixed: mass factorization between Wilson coefficients and parton densities;
- parton densities are always massless, i.e. their evolution is free of any quark mass effects.
- RGE with a mass : the derivative $m^2\partial/\partial m^2$ acts on the Wilson coefficients only.
  $\Longrightarrow$ Seek all terms, but power corrections.
- For these terms a similar factorization in the limit $Q^2 \gg m_Q^2$ is obtained. The non-power mass corrections are process independent and calculated through partonic operator matrix elements, $\langle i|A_l|j\rangle$. [Likewise, parton densities stem from nucleonic matrix elements.]

$$H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C_{(2,L),k}^{\rm S,NS}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light–Wilson coefficients}}.$$

- holds for polarized and unpolarized case. OMEs obey expansion

$$A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right) = \langle i|O_k^{\rm S,NS}|i\rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{{\rm S,NS},(l)}\left(\frac{m^2}{\mu^2}\right), \qquad i = q, g$$

[Buza, Matiounine, Migneron, Smith, van Neerven, Nucl. Phys. **B 472** (1996) 611;
Buza, Matiounine, Smith, van Neerven, Nucl. Phys. **B485** (1997) 420.]

One–Loop Example

Consider $F_2^{Q\overline{Q}}(x, Q^2)$:

$$H_{2,g}^{(1)}\left(z, \frac{m^2}{Q^2}\right) = 8T_R a_s \left\{ v\left[-\frac{1}{2} + 4z(1-z) + \frac{m^2}{Q^2} z(2z-1)\right] + \left[-\frac{1}{2} + z - z^2 + 2\frac{m^2}{Q^2} z(3z-1) - 4\frac{m^4}{Q^4} z^2\right] \ln\left(\frac{1-v}{1+v}\right)\right\} ,$$

$$\lim_{Q^2 \gg m^2} H_{2,g}^{(1)}\left(z, \frac{m^2}{Q^2}\right) = 4T_R a_s \left\{ [z^2 + (1-z)^2] \ln\left(\frac{Q^2}{m^2} \frac{1-z}{z}\right) + 8z(1-z) - 1 \right\} .$$

$\overline{\rm MS}$ result for $m^2 = 0$:

$$C_{2,g}^{(1)}\left(z, \frac{Q^2}{\mu^2}\right) = 4T_R a_s \left\{ [z^2 + (1-z)^2] \ln\left(\frac{Q^2}{\mu^2} \frac{1-z}{z}\right) + 8z(1-z) - 1 \right\}.$$

Massive operator matrix element:

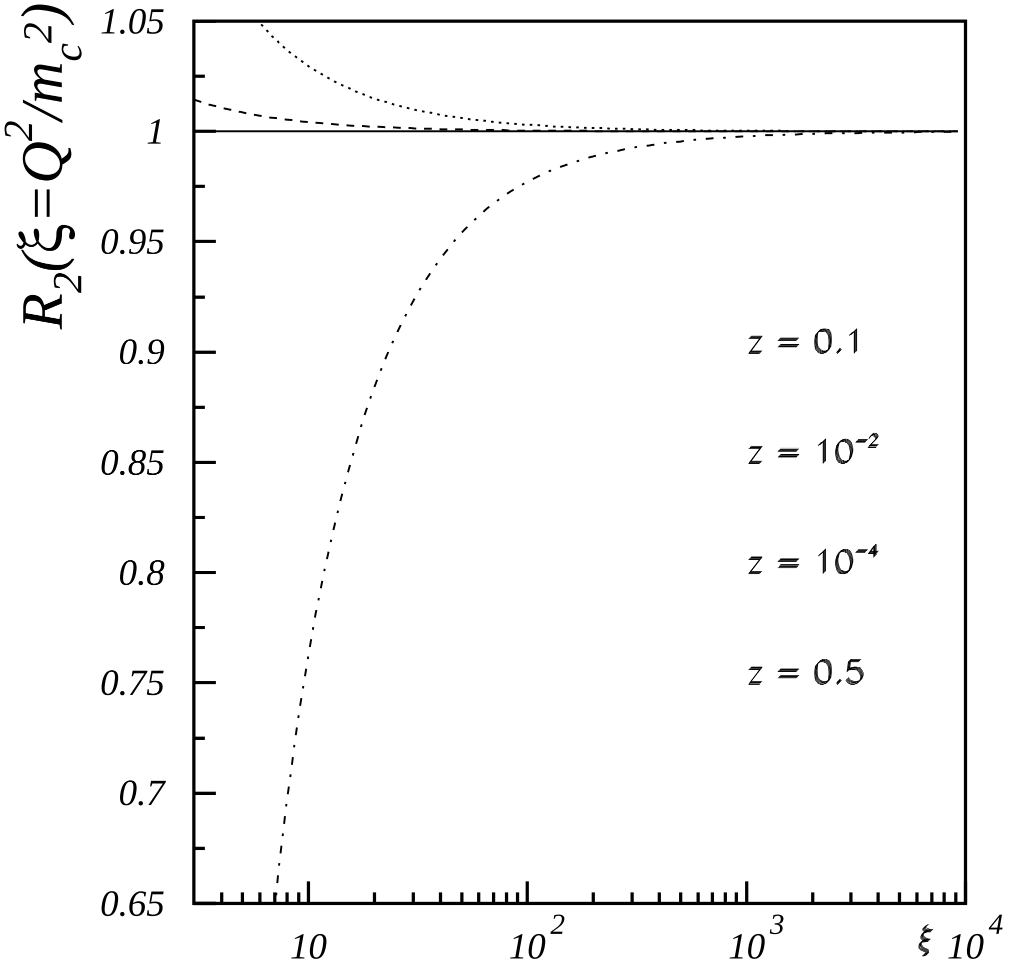
$$A_{Qg}^{(1)}\left(z, \frac{m^2}{\mu^2}\right) = -4T_R a_s [z^2 + (1-z)^2] \ln\left(\frac{m^2}{\mu^2}\right) + a_{Qg}^{(1)} , \quad a_{Qg}^{(1)} = 0 .$$

$$\Longrightarrow \lim_{Q^2 \gg m^2} H_{2,g}^{(1)}\left(z, \frac{m^2}{Q^2}\right) = C_{2,g}^{(1)}\left(z, \frac{Q^2}{\mu^2}\right) + A_{Qg}^{(1)}\left(z, \frac{m^2}{\mu^2}\right) .$$

- Comparison for LO:

$$R_2\left(\xi \equiv \frac{Q^2}{m_c^2}\right) \equiv \frac{H_{2,g}^{(1)}}{H_{2,g,(asym)}^{(1)}} .$$

- Comparison to exact order $O(a_s^2)$ result: asymptotic formulae valid for $Q^2 \geq 20$ $(\mathrm{GeV}/c)^2$ in case of $F_2^{c\bar{c}}(x,Q^2)$ and $Q^2 \geq 1000$ $(\mathrm{GeV}/c)^2$ for $F_L^{c\bar{c}}(x,Q^2)$



Expansion up to $O(\alpha_s^2)$ of unpolarized Heavy Flavor Wilson Coefficient H_2:

$$
\begin{aligned}
H_{2,g}^{S}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s\left[A_{Qg}^{(1)}\left(\frac{m^2}{\mu^2}\right)+\widehat{C}_{2,g}^{(1)}\left(\frac{Q^2}{\mu^2}\right)\right] \\
&+ a_s^2\left[A_{Qg}^{(2)}\left(\frac{m^2}{\mu^2}\right)+A_{Qg}^{(1)}\left(\frac{m^2}{\mu^2}\right)\otimes C_{2,q}^{(1)}\left(\frac{Q^2}{\mu^2}\right)+\widehat{C}_{2,g}^{(2)}\left(\frac{Q^2}{\mu^2}\right)\right], \\
H_{2,q}^{PS}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s^2\left[A_{Qq}^{\mathrm{PS},(2)}\left(\frac{m^2}{\mu^2}\right)+\widehat{C}_{2,q}^{\mathrm{PS},(2)}\left(\frac{Q^2}{\mu^2}\right)\right], \\
H_{2,q}^{\mathrm{NS}}\left(\frac{Q^2}{m^2},\frac{m^2}{\mu^2}\right) &= a_s^2\left[A_{qq,Q}^{\mathrm{NS},(2)}\left(\frac{m^2}{\mu^2}\right)+\widehat{C}_{2,q}^{\mathrm{NS},(2)}\left(\frac{Q^2}{\mu^2}\right)\right].
\end{aligned}
$$

- Polarized and longitudinal Heavy Wilson coefficients obey similar expansion.
- For H_L, $O(a_s^3)$ contributions have been derived recently.

  [J. Blümlein, A. De Freitas, W. van Neerven and S. Klein (2006)].

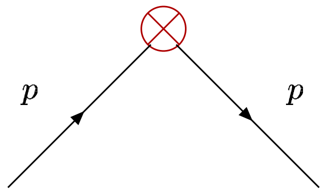
Gluonic Massive Operator Matrix Elements have the same structure in the polarized and unpolarized case. Up to $O(a_s^2)$ they are given by:

$$
\begin{aligned}
A_{Qg}^{(1)} &= -\frac{1}{2}\widehat{P}_{qg}^{(0)}\ln\left(\frac{m^2}{\mu^2}\right) \\
A_{Qg}^{(2)} &= \frac{1}{8}\left\{\widehat{P}_{qg}^{(0)}\otimes\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qg}^{(1)}\ln\left(\frac{m^2}{\mu^2}\right) \\
&\quad + \overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] + a_{Qg}^{(2)} \\
A_{Qq}^{\mathrm{PS},(2)} &= -\frac{1}{8}\widehat{P}_{qg}^{(0)}\otimes P_{gq}^{(0)}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qq}^{\mathrm{PS},(1)}\ln\left(\frac{m^2}{\mu^2}\right) + a_{Qq}^{\mathrm{PS},(2)} + \overline{a}_{Qg}^{(1)}\otimes P_{gq}^{(0)} \\
A_{qq,Q}^{\mathrm{NS},(2)} &= -\frac{\beta_{0,Q}}{4}P_{qq}^{(0)}\ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qq}^{\mathrm{NS},(1)}\ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{\mathrm{NS},(2)} + \frac{1}{4}\beta_{0,Q}\zeta_2 P_{qq}^{(0)}\ .
\end{aligned}
$$

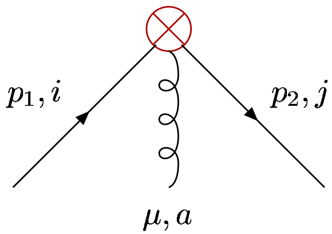
with

$$\widehat{f} = f(N_F+1) - f(N_F)\ .$$

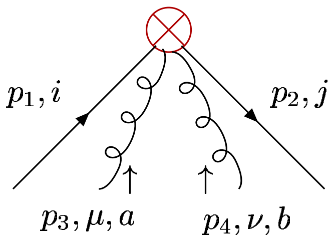
Operator insertions in light-cone expansion:



$$\not{\Delta}\gamma_{\pm}(\Delta\cdot p)^{N-1}~,$$



$$g t^a_{ji} \Delta^{\mu} \not{\Delta} \gamma_{\pm} \sum_{j=0}^{N-2} (\Delta\cdot p_1)^j (\Delta\cdot p_2)^{N-j-2}~,$$

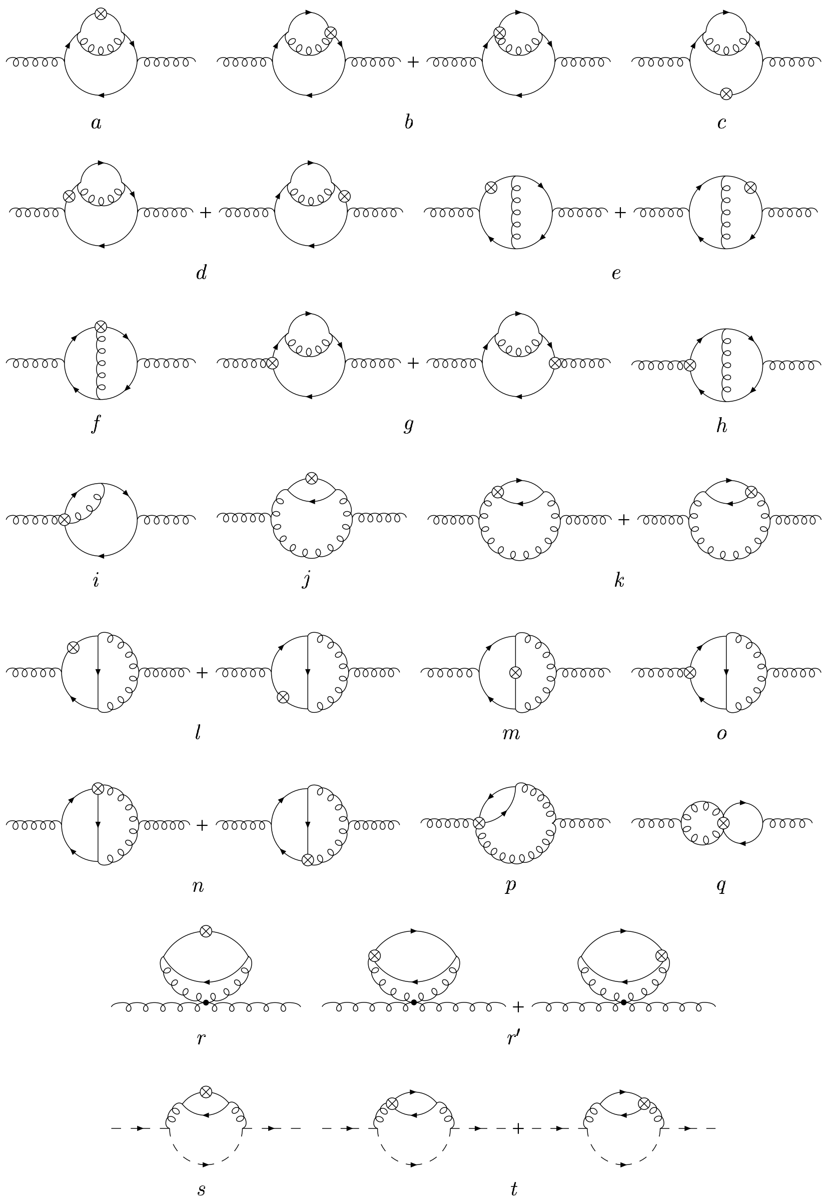


$$g^2 \Delta^{\mu} \Delta^{\nu} \not{\Delta} \gamma_{\pm} \sum_{0\le j<l}^{N-2} \Big[(\Delta p_1)^{N-l-2} (\Delta p_1 + \Delta p_4)^{l-j-1} (\Delta p_2)^j (t^a t^b)_{ji} + (\Delta p_1)^{N-l-2} (\Delta p_1 + \Delta p_3)^{l-j-1} (\Delta p_2)^j (t^b t^a)_{ji} \Big]~,$$

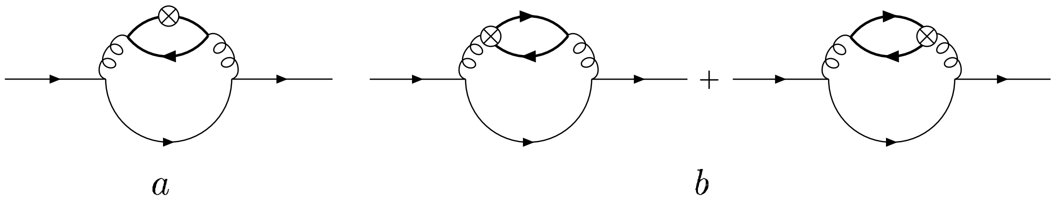
$$\gamma_{+} = 1~, \quad \gamma_{-} = \gamma_5~.$$

Δ: light-like momentum, $\Delta^2 = 0$.

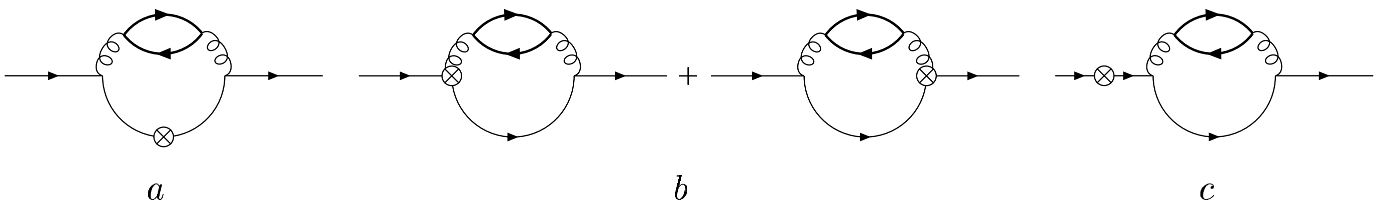
- 20 Diagrams contributing to the gluonic OME $\hat{A}_{Qg}^{(2),S}$ $\Longrightarrow$



Pure–singlet:



Non singlet:



OMEs are obtained by applying projectors to the truncated 2–point Green's functions:

- Unpolarized case, only even moments contribute

$$\hat{A}_{Qg}^{(2)} = \frac{\delta^{ab}}{N_c^2 - 1} \frac{(-g_{\mu\nu})}{D-2} (\Delta\, p)^{-N} G_{Q,\mu\nu}^{ab,(2)} \ .$$

- Polarized case, only odd moments contribute

$$\hat{A}_{Qg}^{(2)} = \frac{\delta^{ab}}{N_c^2 - 1} \frac{\varepsilon^{\mu\nu\lambda\sigma} \Delta_\lambda p_\sigma}{(D-2)(D-3)} (\Delta \cdot p)^{-N-1} G_{Q,\mu\nu}^{ab,(2)} \ .$$

- γ_5 was treated in the 't Hooft–Veltman–Scheme:

$$\not\Delta \gamma_5 = \frac{i}{6} \varepsilon_{\mu\nu\rho\sigma} \Delta^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \ .$$

  Finite renormalizations required to re-install WT- and ST-identities.

- Mellin–Transform in the polarized and unpolarized case are defined as

$$\begin{aligned} \mathbf{M}[A_{kl}^{unpol}](N) &= \frac{1+(-1)^N}{2} \int_0^1 dx x^{N-1} A_{kl}^{unpol}(x) \ , \\ \mathbf{M}[A_{kl}^{pol}](N) &= \frac{1-(-1)^N}{2} \int_0^1 dx x^{N-1} A_{kl}^{pol}(x) \ . \end{aligned}$$

3. The Calculation

- Evaluation in Mellin space. Calculation partially automatized in computer programs. For polarized case only few minor changes had to be implemented.
- use of Mellin-Barnes integrals for numerical checks and some analytic results

$$\frac{1}{(A+B)^\nu} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\sigma A^\sigma B^{-\nu-\sigma}\frac{\Gamma(-\sigma)\Gamma(\nu+\sigma)}{\Gamma(\nu)}$$

- use of hypergeometric functions for general analytic results

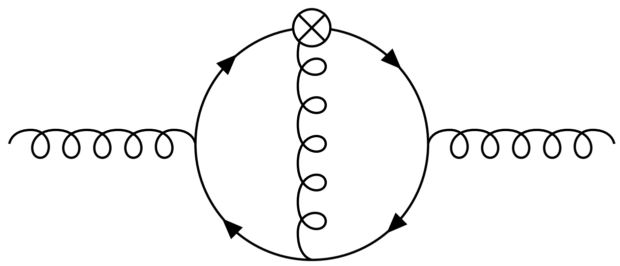
$$_PF_Q\left[\begin{array}{c}(a_1)...(a_P)\\(b_1)...(b_Q)\end{array};z\right] = \sum_{i=0}^{\infty}\frac{(a_1)_i...(a_P)_i}{(b_1)_i...(b_Q)_i}\frac{z^i}{(1)_i}~, \qquad (c)_n = \frac{\Gamma(c+n)}{\Gamma(c)}~.$$

- Analytic results for general value of Mellin N is obtained in terms of harmonic sums [Blümlein and Kurth, Phys. Rev. **D60** (1999) 014018; Vermaseren, Int. J. Mod. Phys. **A14** (1999) 2037.]

$$S_{a_1,\ldots,a_m}(N) = \sum_{n_1=1}^{N}\sum_{n_2=1}^{n_1}\ldots\sum_{n_m=1}^{n_{m-1}}\frac{(\mathrm{sign}(a_1))^{n_1}}{n_1^{|a_1|}}\frac{(\mathrm{sign}(a_2))^{n_2}}{n_2^{|a_2|}}\ldots\frac{(\mathrm{sign}(a_m))^{n_m}}{n_m^{|a_m|}}~,$$
$$N \in \mathbb{N},\ \forall\, l,\ a_l \in \mathbb{Z}\setminus 0~.$$

- use of algebraic relations to simplify expressions [Blümlein, Comput. Phys. Commun. **159** (2004) 19.]

Consider e.g scalar Integral of Diagram



$$
\begin{aligned}
I &= \frac{S_\varepsilon^2 (\Delta p)^{N-2}}{(4\pi)^4 (m^2)^{1-\varepsilon}} \exp\left\{\sum_{l=2}^{\infty} \frac{\zeta_l}{l}\varepsilon^l\right\} \frac{2\pi}{N \sin(\frac{\pi}{2}\varepsilon)} \sum_{j=1}^{N} \left\{\binom{N}{j}(-1)^j + \delta_{j,N}\right\} \\
&\times \left\{\frac{\Gamma(j)\Gamma(j+1-\frac{\varepsilon}{2})}{\Gamma(j+2-\varepsilon)\Gamma(j+1+\frac{\varepsilon}{2})} - \frac{B(1-\frac{\varepsilon}{2}, 1+j)}{j}\, {}_3F_2\left[1-\varepsilon, \frac{\varepsilon}{2}, j+1; 1, j+2-\frac{\varepsilon}{2}; 1\right]\right\} \\
&= \frac{S_\varepsilon^2 (\Delta p)^{N-2}}{(4\pi)^4 (m^2)^{1-\varepsilon}} \left\{\frac{4}{N}\left[S_2(N) - \frac{S_1(N)}{N}\right] + \frac{\varepsilon}{N}\left[-2S_{2,1}(N) + 2S_3(N) + \frac{4N+1}{N} S_2(N) \right.\right. \\
&\left.\left. - \frac{S_1^2(N)}{N} - \frac{4}{N} S_1(N)\right]\right\} + O(\varepsilon^2)\ .
\end{aligned}
$$

- Method allows for feasible computation of higher orders in ε and automatized check for fixed values of N.
- For genuine scalar 2–Loop Integrals see [Bierenbaum, Blümlein and S. Klein, (2007).]

Unpolarized case, Singlet

$$
\begin{aligned}
a_{Qg}^{(2)}(N) = 4C_F T_R \Biggl\{ & \frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[-\frac{1}{3}S_1^3(N-1)+\frac{4}{3}S_3(N-1)-S_1(N-1)S_2(N-1) \\
& -2\zeta_2 S_1(N-1)\Biggr] + \frac{N^4+16N^3+15N^2-8N-4}{N^2(N+1)^2(N+2)}S_2(N-1) + \frac{3N^4+2N^3+3N^2-4N-4}{2N^2(N+1)^2(N+2)}\zeta_2 \\
& + \frac{2}{N(N+1)}S_1^2(N-1) + \frac{N^4-N^3-16N^2+2N+4}{N^2(N+1)^2(N+2)}S_1(N-1) + \frac{P_1(N)}{2N^4(N+1)^4(N+2)}\Biggr\} \\
+ 4C_A T_R \Biggl\{ & \frac{N^2+N+2}{N(N+1)(N+2)}\Biggl[4\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N+1)+\frac{1}{3}S_1^3(N)+3S_2(N)S_1(N) \\
& + \frac{8}{3}S_3(N)+\beta''(N+1)-4\beta'(N+1)S_1(N)-4\beta(N+1)\zeta_2+\zeta_3\Biggr] - \frac{N^3+8N^2+11N+2}{N(N+1)^2(N+2)^2}S_1^2(N) \\
& -2\frac{N^4-2N^3+5N^2+2N+2}{(N-1)N^2(N+1)^2(N+2)}\zeta_2 - \frac{7N^5+21N^4+13N^3+21N^2+18N+16}{(N-1)N^2(N+1)^2(N+2)^2}S_2(N) \\
& - \frac{N^6+8N^5+23N^4+54N^3+94N^2+72N+8}{N(N+1)^3(N+2)^3}S_1(N) - 4\frac{N^2-N-4}{(N+1)^2(N+2)^2}\beta'(N+1) \\
& + \frac{P_2(N)}{(N-1)N^4(N+1)^4(N+2)^4}\Biggr\}.
\end{aligned}
$$

I. Bierenbaum, J. Blümlein and S. Klein, DESY 07–26, [hep-ph/0703285].

Polarized case, Singlet

$$
\begin{aligned}
a_{Qg}^{(2)} =& C_F T_R \Biggl\{ 4\frac{N-1}{3N(N+1)}\Bigl(-4S_3(N)+S_1^3(N)+3S_1(N)S_2(N)+6S_1(N)\zeta_2\Bigr) \\
& -4\frac{N^4+17N^3+43N^2+33N+2}{N^2(N+1)^2(N+2)}S_2(N)-4\frac{3N^2+3N-2}{N^2(N+1)(N+2)}S_1^2(N) \\
& -2\frac{(N-1)(3N^2+3N+2)}{N^2(N+1)^2}\zeta_2-4\frac{N^3-2N^2-22N-36}{N^2(N+1)(N+2)}S_1(N)-\frac{2P_3(N)}{N^4(N+1)^4(N+2)}\Biggr\} \\
& +C_A T_R\Biggl\{4\frac{N-1}{3N(N+1)}\Bigl(12\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N+1)+3\beta''(N+1)-8S_3(N)-S_1^3(N) \\
& -9S_1(N)S_2(N)-12S_1(N)\beta'(N+1)-12\beta(N+1)\zeta_2-3\zeta_3\Bigr)-16\frac{N-1}{N(N+1)^2}\beta'(N+1) \\
& +4\frac{N^2+4N+5}{N(N+1)^2(N+2)}S_1^2(N)+4\frac{7N^3+24N^2+15N-16}{N^2(N+1)^2(N+2)}S_2(N)+8\frac{(N-1)(N+2)}{N^2(N+1)^2}\zeta_2 \\
& +4\frac{N^4+4N^3-N^2-10N+2}{N(N+1)^3(N+2)}S_1(N)-\frac{4P_4(N)}{N^4(N+1)^4(N+2)}\Biggr\}\,.
\end{aligned}
$$

J. Blümlein and S. Klein, DESY 07–027

Unpolarized case

$$a_{Qq}^{\mathrm{PS},(2)} = C_F T_R \left\{ -4 \frac{(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} \left(2S_2(N) + \zeta_2\right) + \frac{4P_5(N)}{(N-1)N^4(N+1)^4(N+2)^3} \right\},$$

$$a_{qq,Q}^{\mathrm{NS,unpol(pol)},(2)} = C_F T_R \left\{ -\frac{8}{3} S_3(N) - \frac{8}{3} \zeta_2 S_1(N) + \frac{40}{9} S_2(N) + 2 \frac{3N^2+3N+2}{3N(N+1)} \zeta_2 - \frac{224}{27} S_1(N) \right.$$
$$\left. + \frac{219N^6 + 657N^5 + 1193N^4 + 763N^3 - 40N^2 - 48N + 72}{54N^3(N+1)^3} \right\},$$

$$\begin{aligned}
P_1(N) &= 12N^8 + 54N^7 + 136N^6 + 218N^5 + 221N^4 + 110N^3 - 3N^2 - 24N - 4\ , \\
P_2(N) &= 2N^{12} + 20N^{11} + 86N^{10} + 192N^9 + 199N^8 - N^7 - 297N^6 - 495N^5 \\
&\quad -514N^4 - 488N^3 - 416N^2 - 176N - 32\ , \\
P_3(N) &= 12N^8 + 52N^7 + 60N^6 - 25N^4 - 2N^3 + 3N^2 + 8N + 4\ , \\
P_4(N) &= 2N^8 + 10N^7 + 22N^6 + 36N^5 + 29N^4 + 4N^3 + 33N^2 + 12N + 4\ , \\
P_5(N) &= N^{10} + 8N^9 + 29N^8 + 49N^7 - 11N^6 - 131N^5 - 161N^4 \\
&\quad -160N^3 - 168N^2 - 80N - 16\ .
\end{aligned}$$

- Structure of expression is given by

$$\begin{aligned}
\beta(N+1) &= (-1)^N[S_{-1}(N) + \ln(2)]\ , \\
\beta^{(k)}(N+1) &= \Gamma(k+1)(-1)^{N+k}[S_{-k-1}(N) + (1-2^{-k})\zeta_{k+1}]\ ,\ k \geq 2\ , \\
\mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N+1) - \zeta_2\beta(N+1) &= (-1)^{N+1}[S_{-2,1}(N) + \frac{5}{8}\zeta_3]
\end{aligned}$$

- $\Longrightarrow$ harmonic sums with index $\{-1\}$ cancel (holds even for each diagram)
  [cf. Blümlein, Nucl. Phys. (Proc. Suppl.) **135** (2004) 225; Blümlein and Ravindran, Nucl. Phys. **B716** (2005) 128; Nucl. Phys. **B749** (2006) 1; Blümlein and Moch, in preparation].

5. Comparison

- First Calculation to $O(\alpha_S^2)$: van Neerven et. al. 1996, 1997.
  - IBP method
  - direct integration of individual Feynman-parameters, resulting in combinations of Nielsen integrals with partly complicated arguments, e.g.:

$$\int_0^1 dy \frac{1-x}{1-y(1-x)} \ln(1-y) \ln(1+y\frac{1-x}{x}) = \frac{1}{2} \ln^3(x) + 2\, S_{1,2}(1-x) - 3\,\mathrm{Li}_3(-x) - \zeta_2 \ln(x)$$
$$+ \ln(x)\,\mathrm{Li}_2(-x) - \frac{5}{2}\zeta_3 + 2 \ln(1+x)\,\mathrm{Li}_2(-x) + \zeta_2 \ln(1+x) + \ln(x) \ln^2(1+x) - \frac{1}{2} \ln^2(x) \ln(1+x)$$
$$+ 2\, S_{1,2}(-x) - 2\,\mathrm{Li}_3(1-x) + 2 \ln(x)\,\mathrm{Li}_2(1-x) - 2\,\mathrm{Li}_3(-\frac{1-x}{1+x}) + 2\,\mathrm{Li}_3(\frac{1-x}{1+x})$$

  - in z–space: unpolarized result consists of 48 functions, polarized one of 24.

- Our approach: calculate finite or infinite sums.

- Only 6 basic functions, 5 of which are related algebraically

$$\{S_1,\ S_2,\ S_3,\ S_{-2}, S_{-3}\}, \qquad S_{-2,1}$$

$\Longrightarrow$ 2 basic objects.

- agreement of our result with van Neerven et. al.

5. Comparison

- The heavy flavor components structure functions F_2 and F_L in the asymptotic limit $Q^2 \gg m^2$ are given in Mellin–Space by

$$F_2^{Q\overline{Q}}(N,Q^2) = \sum_{k=1}^{n_f} e_k^2 [f_{k-\bar{k}}(N,\mu^2) H_{2,q}^{\rm NS}\left(N,\frac{Q^2}{m^2},\frac{Q^2}{\mu^2}\right)]$$
$$+ e_Q^2 [\Sigma(N,\mu^2) H_{2,q}^{\rm PS}\left(N,\frac{Q^2}{m^2},\frac{Q^2}{\mu^2}\right) + G(N,\mu^2) H_{2,g}^{\rm S}\left(N,\frac{Q^2}{m^2},\frac{Q^2}{\mu^2}\right)] \ .$$

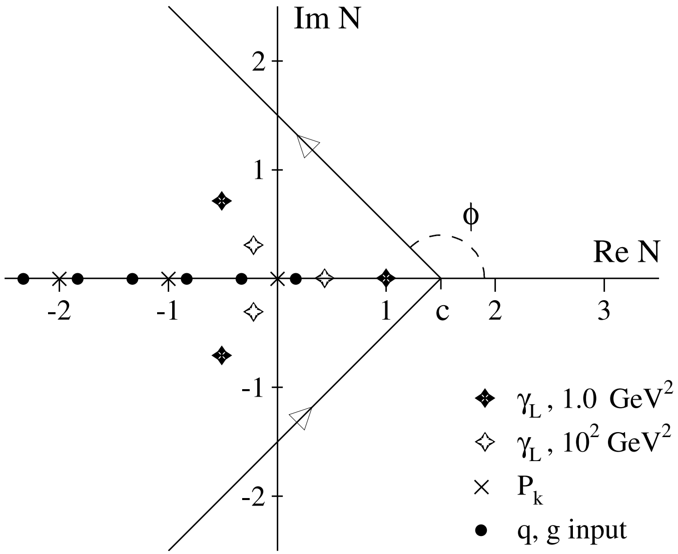
- Light quark densities are defined by

$$f_{k-\bar{k}}(N,\mu^2) = f_k(N,\mu^2) - f_{\bar{k}}(N,\mu^2) \ ,$$
$$\Sigma(N,\mu^2) = \sum_{k=1}^{n_f} f_{k+\bar{k}}(N,\mu^2) \ .$$

- Mellin–Space representation allows for fast numerical analyses using analytical continuations of Mellin–Transforms.
  [cf. Blümlein, Comput. Phys. Commun. **133** (2000) 76; Alekhin and Blümlein, Phys. Lett. **B594** (2004) 299; Blümlein and Moch, Phys. Lett. **B614** (2005) 53.]

Mellin-Inversion: from N– to z–space

Inversion from Mellin-space to z-space: [Blümlein, ANCONT]



Continuation of harmonic sums:

$S_1(N) = \Psi(N+1) + \gamma_E,$

etc.

$$xF_2^{Q\bar{Q}}(x, Q^2) = \int_0^\infty dz\, \mathrm{Im}\left[\, e^{i\Phi} x^{-c(z)}\, F_2^{Q\bar{Q}}(c(z), Q^2)\,\right],$$

$$c(z) = c_0 + z e^{i\Phi}$$

6. Conclusions

Calculation of quark-mass effects in QCD Wilson-coefficients in the asymptotic regime $Q^2 \gg m^2$:

- Calculation in Mellin-space
- Use of Mellin-Barnes integrals (easy numeric check) and generalized hypergeometric functions
- The results are obtained in terms of nested harmonic sums.

$\rightarrow$ Mellin-space representation is essential to achieve the obtained simplification $\rightarrow$ algebraic & structural relations of harmonic sums.

- Calculation of the constant term of the massive Operator Matrix Elements $\rightarrow$ full agreement with results of van Neerven et al.