Radiative Corrections to Production of Scalar and Vector Leptoquarks in e^+e^- Annihilation

Hawaii, April 26 – 30 1993

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1 Classification of Leptoquarks

- B and L conserving
- family–diagonal
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant couplings

BUCHMÜLLER, RÜCKL, WYLER;
J.B., RÜCKL
Phys. Lett. B, May 93.

$$\mathcal{L} = \mathcal{L}^f_{|F|=2} + \mathcal{L}^f_{F=0} + \mathcal{L}^{\gamma,Z,g}$$

$$\mathcal{L}^{\gamma,Z,g} = \sum_{scalars}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - M^2\Phi^\dagger\Phi\right] + \sum_{vectors}\left[-\frac{1}{2}G^\dagger_{\mu\nu}G^{\mu\nu} + M^2\Phi^{\mu\dagger}\Phi_\mu\right]$$

$$D_\mu = \partial_\mu - ieQ^\gamma A_\mu - ieQ^Z Z_\mu - ig_s\frac{\lambda_a}{2}\mathcal{A}^a_\mu$$

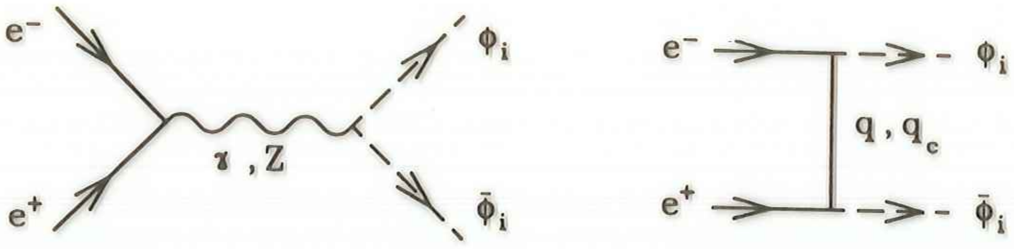
$$\begin{aligned}\mathcal{L}^f_{F=0} &= (h_{2L}\bar{u}_R l_L + h_{2R}\bar{q}_L i\tau_2 e_R)R_2 + \tilde{h}_{2L}\bar{d}_R l_L \tilde{R}_2 \\ &+ (h_{1L}\bar{q}_L\gamma^\mu l_L + h_{1R}\bar{d}_R\gamma^\mu e_R)U_{1\mu} \\ &+ \tilde{h}_{1R}\bar{u}_R\gamma^\mu e_R\tilde{U}_{1\mu} + h_{3L}\bar{q}_L\vec{\tau}\gamma^\mu l_L\vec{U}_{3\mu} + h.c.\end{aligned}$$

$$\begin{aligned}\mathcal{L}^f_{|F|=2} &= (g_{1L}\bar{q}^c_L i\tau_2 l_L + g_{1R}\bar{u}^c_R e_R)S_1 \\ &+ \tilde{g}_{1R}\bar{d}^c_R e_R\tilde{S}_1 + g_{3L}\bar{q}^c_L i\tau_2\vec{\tau} l_L\vec{S}_3 \\ &+ (g_{2L}\bar{d}^c_R\gamma^\mu l_L + g_{2R}\bar{q}^c_L\gamma^\mu e_R)V_{2\mu} \\ &+ \tilde{g}_{2L}\bar{u}^c_R\gamma^\mu l_L\tilde{V}_{2\mu} + h.c.,\end{aligned}$$

leptoquark (Φ)	spin	F	colour	T_3	Q_{em}	$\lambda_L(lq)$	$\lambda_R(lq)$	$\lambda_L(\nu q)$
S_1	0	-2	$\bar{3}$	0	1/3	g_{1L}	g_{1R}	$-g_{1L}$
$\tilde{S}_1$	0	-2	$\bar{3}$	0	4/3	0	$\tilde{g}_{1R}$	0
$\vec{S}_3$	0	-2	$\bar{3}$	+1	4/3	$-\sqrt{2}g_{3L}$	0	0
				0	1/3	$-g_{3L}$	0	$-g_{3L}$
				-1	$-2/3$	0	0	$\sqrt{2}g_{3L}$
R_2	0	0	3	1/2	5/3	h_{2L}	h_{2R}	0
				$-1/2$	2/3	0	$-h_{2R}$	h_{2L}
$\tilde{R}_2$	0	0	3	1/2	2/3	$\tilde{h}_{2L}$	0	0
				$-1/2$	$-1/3$	0	0	$\tilde{h}_{2L}$
$V_{2\mu}$	1	-2	$\bar{3}$	1/2	4/3	g_{2L}	g_{2R}	0
				$-1/2$	1/3	0	g_{2R}	g_{2L}
$\tilde{V}_{2\mu}$	1	-2	$\bar{3}$	1/2	1/3	$\tilde{g}_{2L}$	0	0
				$-1/2$	$-2/3$	0	0	$\tilde{g}_{2L}$
$U_{1\mu}$	1	0	3	0	2/3	h_{1L}	h_{1R}	h_{1L}
$\tilde{U}_{1\mu}$	1	0	3	0	5/3	0	$\tilde{h}_{1R}$	0
$\vec{U}_{3\mu}$	1	0	3	+1	5/3	$\sqrt{2}h_{3L}$	0	0
				0	2/3	$-h_{3L}$	0	h_{3L}
				-1	$-1/3$	0	0	$\sqrt{2}h_{3L}$

18 States

2 The Born Cross Section



J.B., RÜCKL;
S: ZERWAS
et al.

$$\frac{d\sigma_{scalar}}{d\cos\theta} = \frac{3\pi\alpha^2}{8s}\beta^3\sin^2\theta\sum_{a=L,R}\left\{|\kappa_a(s)|^2 + \left(\frac{\lambda_a}{e}\right)^2\frac{4Re[\kappa_a(s)]}{t(\beta,\cos\theta)} + \left(\frac{\lambda_a}{e}\right)^4\frac{4}{t^2(\beta,\cos\theta)}\right\}$$

$$\frac{d\sigma_{vector}}{d\cos\theta} = \frac{3\pi\alpha^2}{8M_\Phi^2}\beta\sum_{a=L,R}\left\{|\kappa_a(s)|^2\tilde{F}_1(\theta,\beta) + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]\tilde{F}_2(\theta,\beta) + \left(\frac{\lambda_a}{e}\right)^4\tilde{F}_3(\theta,\beta)\right\}$$

$$\kappa_a(s) = \sum_{V=\gamma,Z} Q_a^V(e)\frac{s}{s - M_V^2 + iM_V\Gamma_V}Q^V(\Phi)$$

$$\tilde{F}_1(\theta,\beta) = \beta^2\left[1 + \frac{1}{4}(1-3\beta^2)\sin^2\theta\right]$$

$$\tilde{F}_2(\theta,\beta) = 2\left[1 - \frac{1-\beta^2}{t(\beta,\cos\theta)}\right](1-\beta^2) + 4\beta^2 - \beta^2\left[1 - 2\frac{1-\beta^2}{t(\beta,\cos\theta)}\right]\sin^2\theta$$

$$\tilde{F}_3(\theta,\beta) = 4 + \frac{\beta^2}{4}\left\{(1-\beta^2)\left[\frac{4}{t(\beta,\cos\theta)}\right]^2 + \frac{s}{M_\Phi^2}\right\}\sin^2\theta$$

$$\sigma_{scalar}(s) = \frac{\pi\alpha^2\beta^3}{2s}\sum_{a=L,R}\left\{|\kappa_a(s)|^2 + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]F_1(\beta) + \left(\frac{\lambda_a}{e}\right)^4 F_2(\beta)\right\}$$

$$\sigma_{vector}(s) = \frac{\pi\alpha^2\beta}{2M_\Phi^2}\sum_{a=L,R}\left\{|\kappa_a(s)|^2\tilde{F}_1(\beta) + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]\tilde{F}_2(\beta) + \left(\frac{\lambda_a}{e}\right)^4 \tilde{F}_3(\beta)\right\}$$

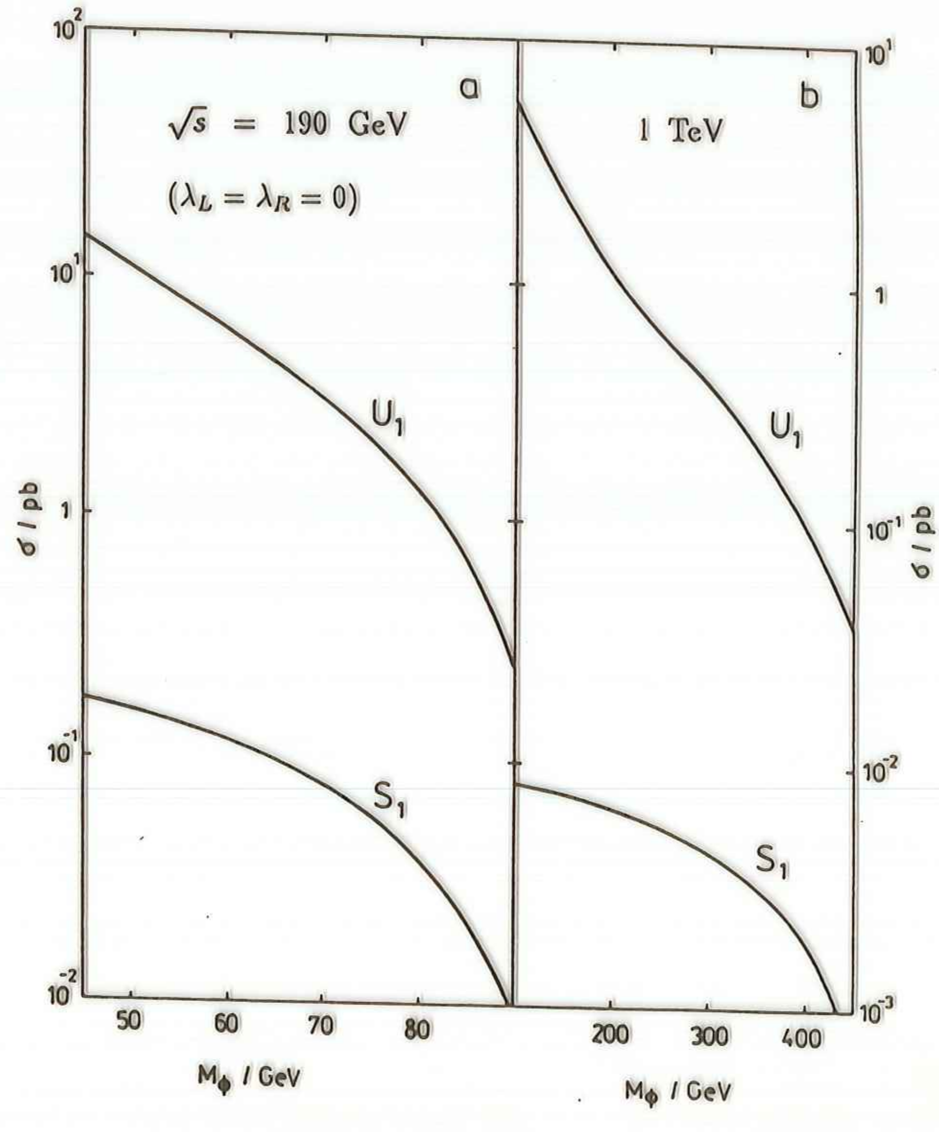
$$F_1(\beta) = \frac{3}{2}\left(\frac{1+\beta^2}{\beta^2} - \frac{(1-\beta^2)^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right)$$

$$F_2(\beta) = 3\left(-\frac{1}{\beta^2} + \frac{1+\beta^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right)$$

$$\tilde{F}_1(\beta) = \beta^2\left(\frac{7-3\beta^2}{4}\right)$$

$$\tilde{F}_2(\beta) = \frac{15}{4} + 2\beta^2 - \frac{3}{4}\beta^4 - \frac{3}{8\beta}(1-\beta^2)^2(5-\beta^2)\ln\frac{1+\beta}{1-\beta}$$

$$\tilde{F}_3(\beta) = 3(1+\beta^2) + \frac{\beta^2}{4}\frac{s}{M_\Phi^2} + \frac{3}{2\beta}(1-\beta^4)\ln\frac{1+\beta}{1-\beta}.$$



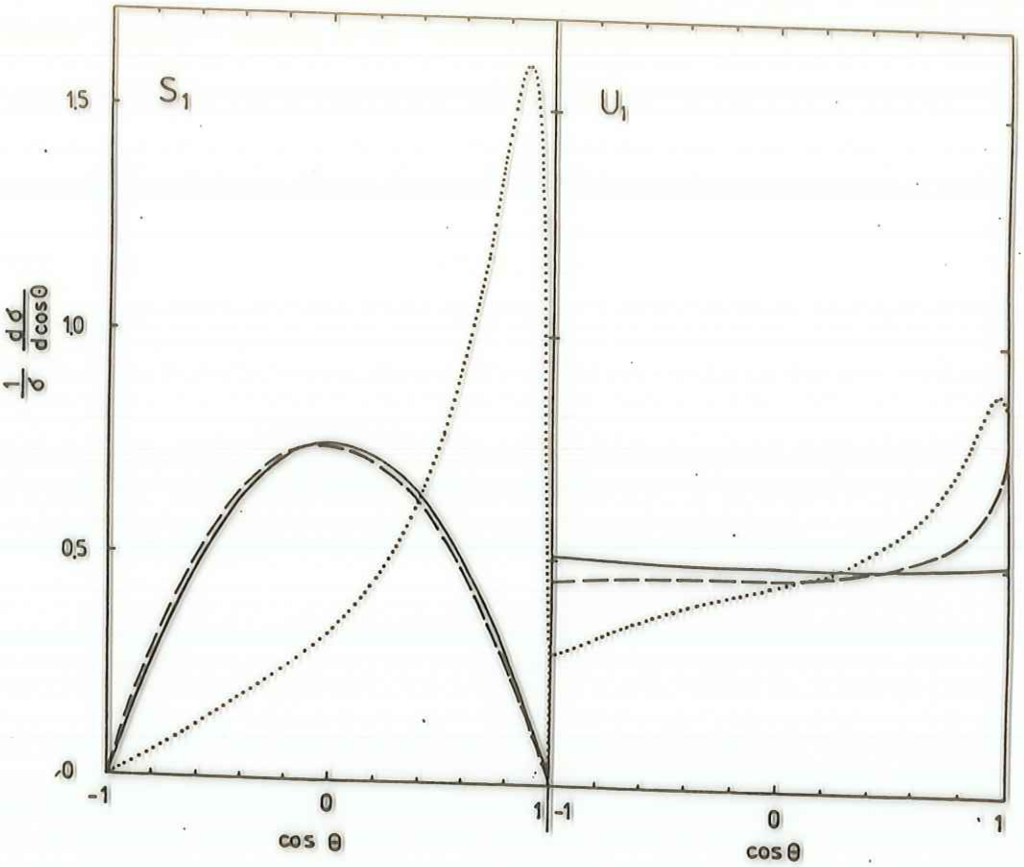


Figure 4: Angular distributions of the scalar leptoquark S_1 and the vector leptoquark U_1 at $\sqrt{s}$ = 1 TeV and for M_Φ = 400 GeV: $\lambda_L = \lambda_R = 0$ (solid); $\lambda_L/e = 0.3$, $\lambda_R = 0$ (dashed); $\lambda_R/e = 1$, $\lambda_L = 0$ (dotted).

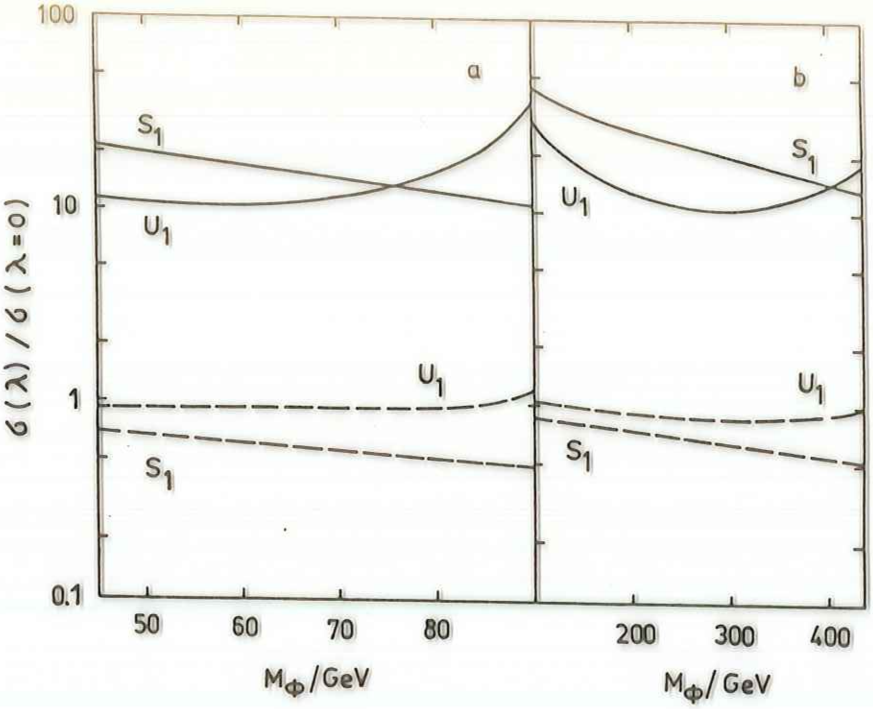


Figure 3: Effect of right-handed fermion couplings, $\lambda_R/e = 0.3$ (dashed) and 1 (solid), on the production cross sections for scalar (S_1) and vector (U_1) leptoquarks at $\sqrt{s}$ = 190 GeV (a) and 1 TeV (b).

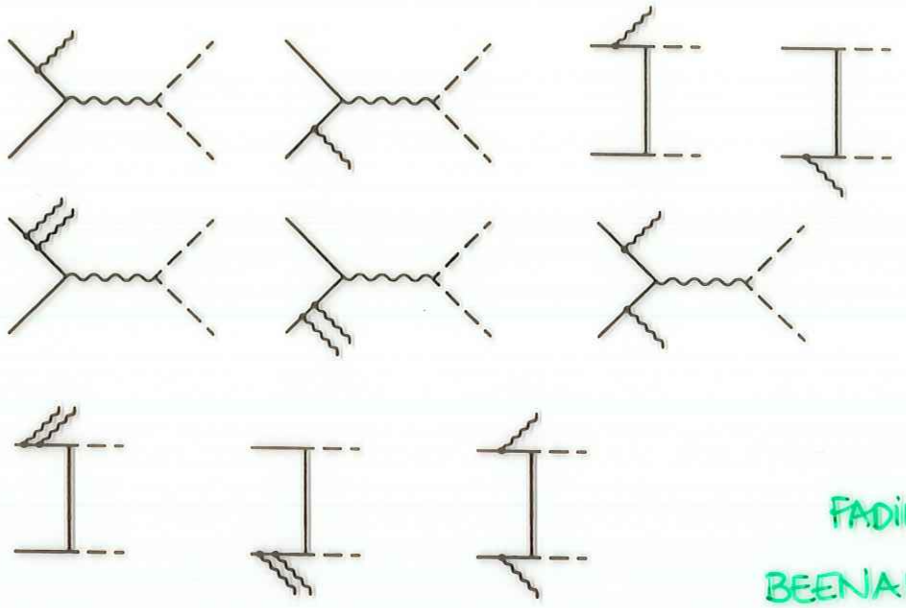
	scalars		vectors		Luminosity
	min	max	min	max	
$\sqrt{s} = 190\,\mathrm{GeV}$ $M_\Phi = 80\,\mathrm{GeV}$	$S_1,\, S_3^{1/3}$ 0.040	$S_3^{4/3}$ 1.18	$U_1,\, U_3^{2/3}$ 1.366	$U_3^{5/3}$ 12.08	200 pb^{-1}
$\sqrt{s} = 1\,\mathrm{TeV}$ $M_\Phi = 430\,\mathrm{GeV}$	$S_1,\, S_3^{1/3}$ 0.001	$S_3^{4/3}$ 0.029	$U_1,\, U_3^{2/3}$ 0.060	$U_3^{5/3}$ 0.368	$10 fb^{-1}$

Table : Maximum and minimum cross sections in pb assuming $\lambda_{L,R} = 0$. The cross sections for the remaining leptoquark states in table 1 ly in between these values. The masses correspond roughly to $\beta = 0.5$.

3 QED Corrections

3.1 Initial State Radiation

i)



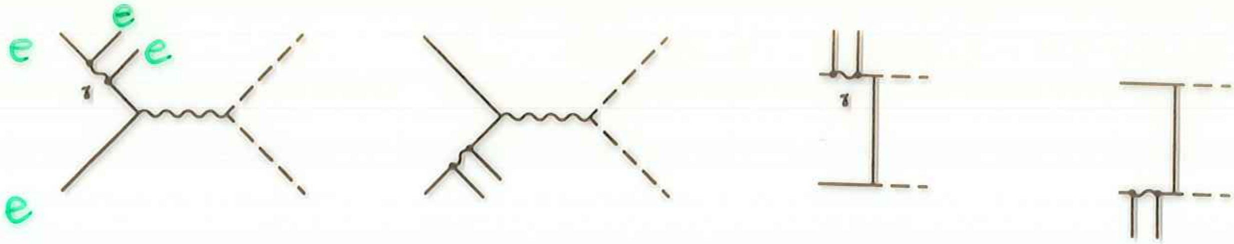
FADIN, KURAEV;
BEENAKKER, BERENDS, VAN NEERVEN;
BERENDS et al.

$$\Gamma_{ij}(z, L_m) = \delta_{ij}\delta(1-z) + \frac{\alpha}{2\pi} P_{ij}^{(0)}(z) L_m + \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 \left[\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(z) L_m^2 + P_{ij}^{(1)}(z) L_m\right] + \text{h.o.}$$

$$P_{ee}^{(0)}(z) = \delta(1-z)\left[\frac{3}{2} + 2\ln\Delta\right] + \theta(1-\Delta-z)\frac{1+z^2}{1-z}$$

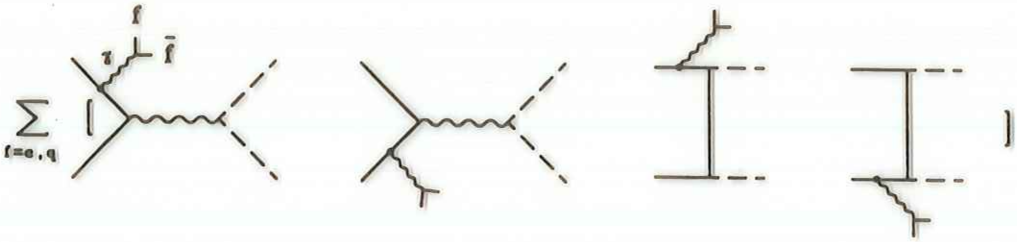
$$\frac{1}{2}\left[P_{ee}^{(0)} \otimes P_{ee}^{(0)}\right](z) = \delta(1-z)\left[2\ln^2\Delta + 3\ln\Delta + \frac{9}{8} - 2\zeta(2)\right] + \theta(1-\Delta-z)\left\{\frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z)\right\}$$

ii)



$$\frac{1}{2}\left[P^{(0)}_{e\gamma}\otimes P^{(0)}_{\gamma e}\right](z) \equiv P^{(1)}_{e\gamma e}(z) = (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3)$$

iii)



$$P^{(1)}_{ff}(z) = N_c(f)e_f^2\frac{1}{3}P^{(0)}_{ee}(z)\theta\left(1-z-\frac{4m_f}{\sqrt{s}}\right)$$

$$\alpha(s) = \frac{\alpha}{1-\frac{\alpha}{3\pi}\sum_f e_f^2 N_c(f)\ln\left(\frac{s}{m_f^2}\right)}$$

m_u	=	62	MeV	m_d	=	83	MeV	m_u	=	215	MeV
m_c	=	1500	MeV	m_b	=	4500	MeV				

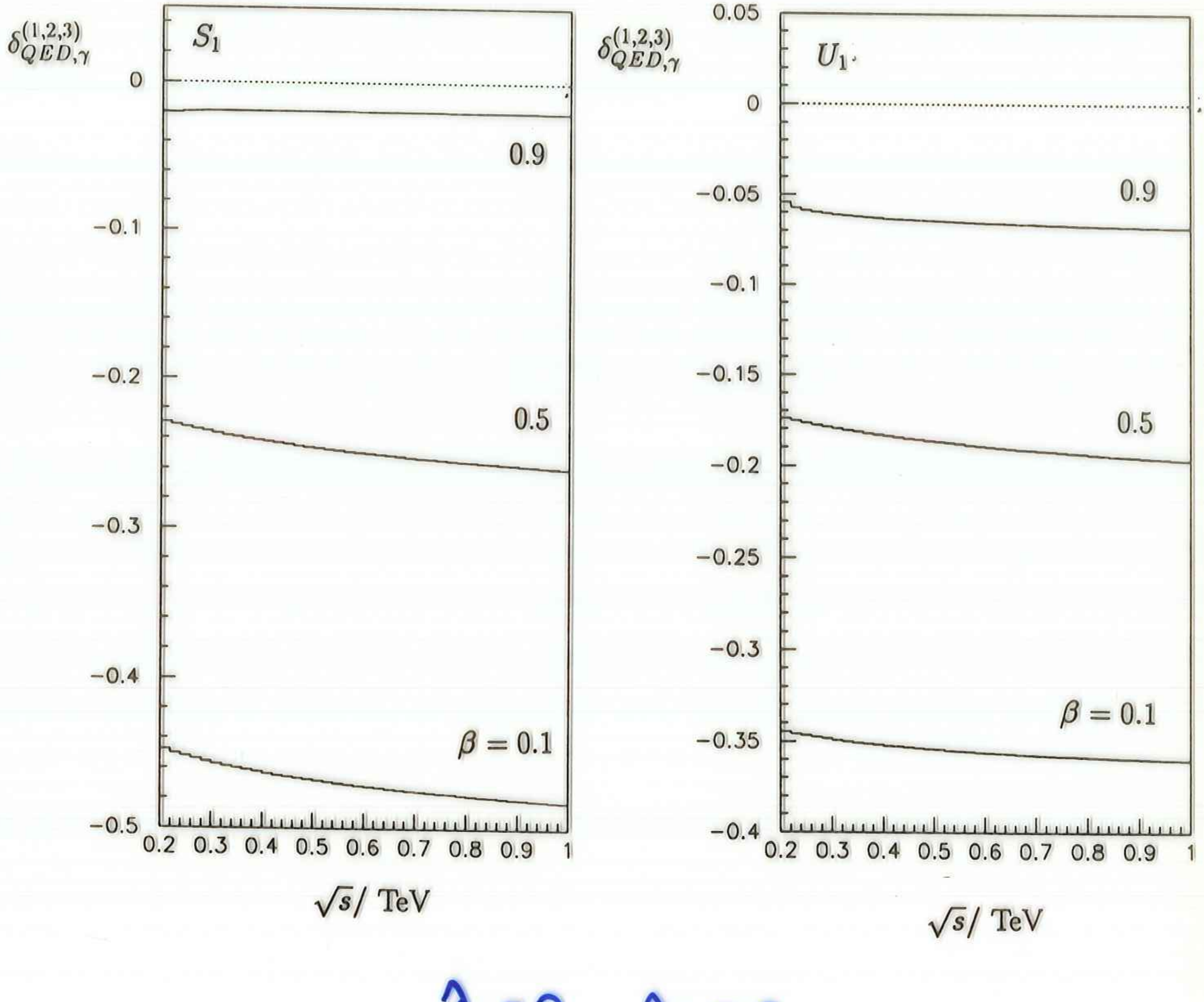
F. JEGERLEHNER

iv)

$$P^{(3)}_{soft}(z) = b(1-z)^{b-1}(1+\delta_1+\delta_2) - \frac{b(1+\delta_1)+b^2\ln(1-z)}{1-z}$$

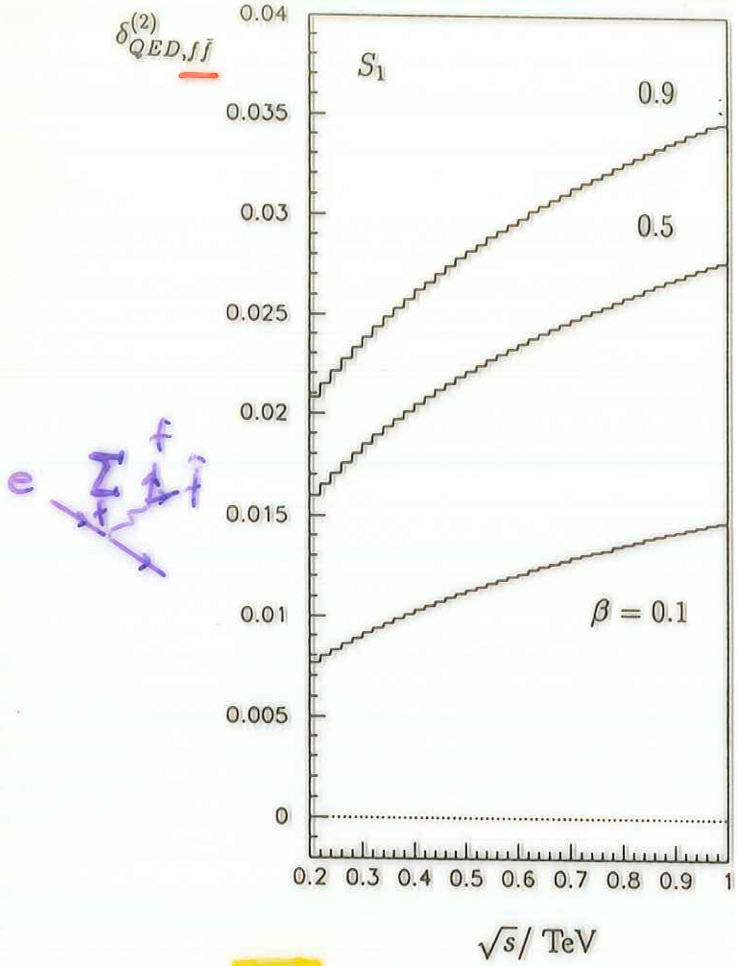
$b = (2\alpha/\pi)(L_m - 1)$, $\delta_1 = (3\alpha/2\pi)L_m$ and $\delta_2 = (\alpha/\pi)^2[9/8 - 2\zeta(2)]L_m^2$

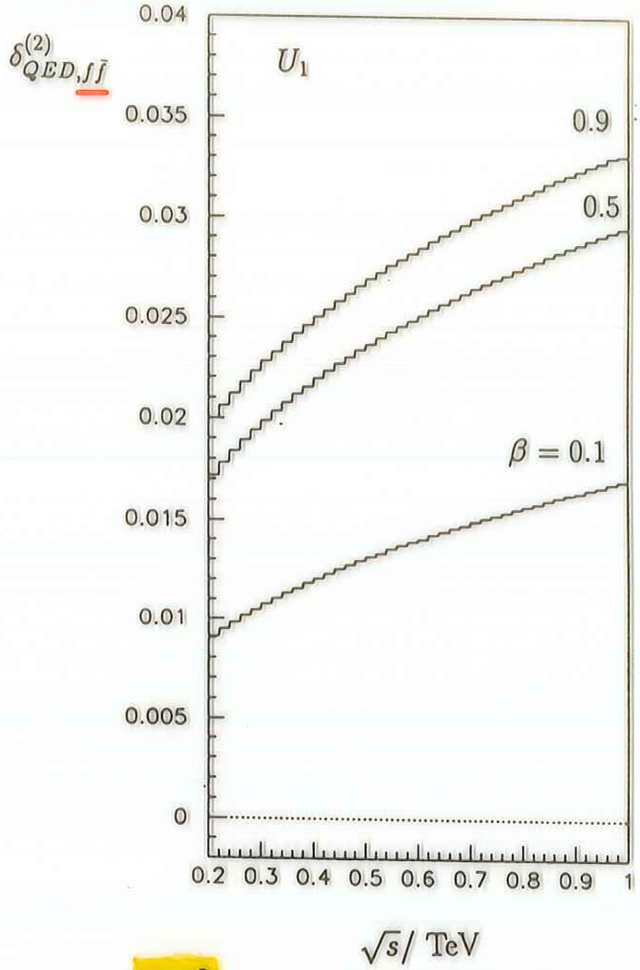
i) Bremsstrahlung: $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$, soft exponentiation

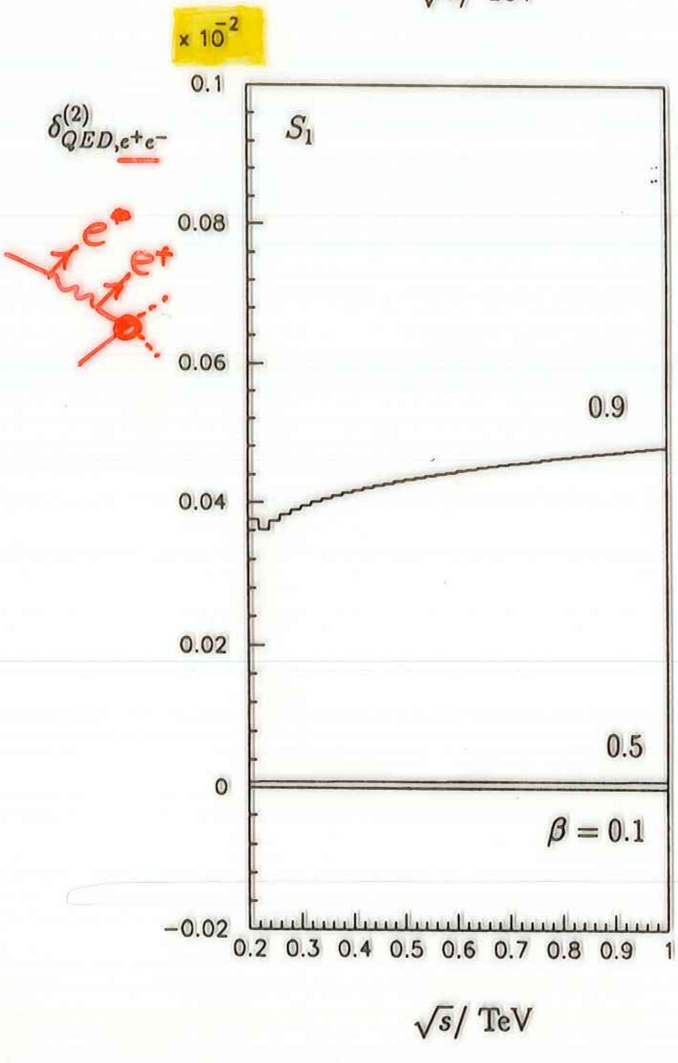


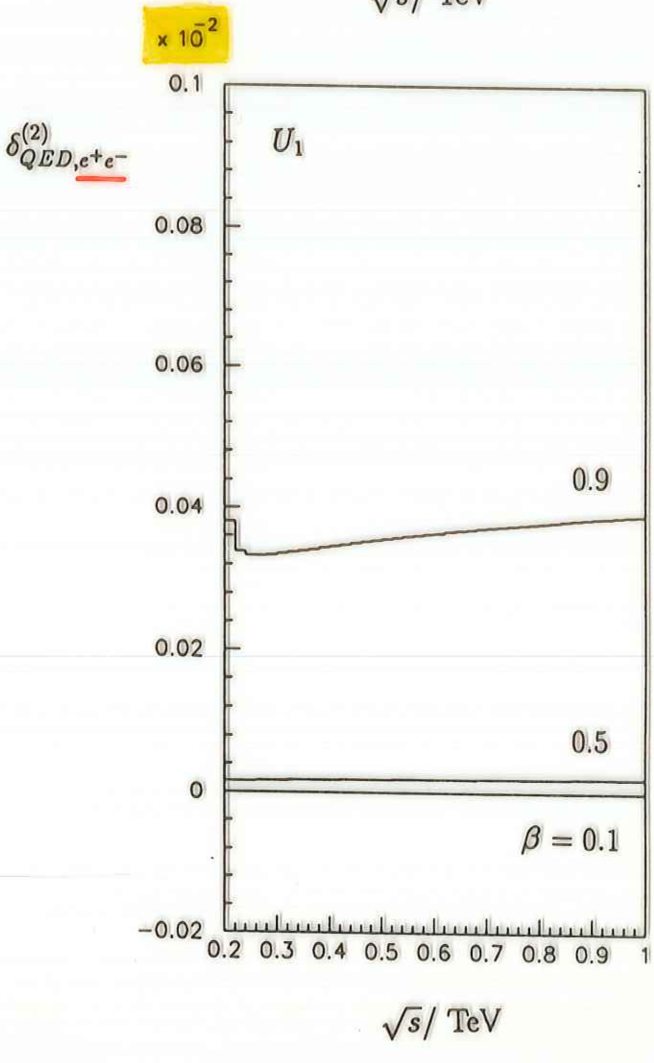
$\lambda_L = 0$, $\lambda_R = e$

iii) $\mathcal{O}(\alpha^2)$ e^+e^- and $f\bar{f}$ pair creation









3.2 Beamstrahlung

$$\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dx} = \frac{x^a(1-x)^b}{B(a+1,b+1)}$$

à la PYTHIA

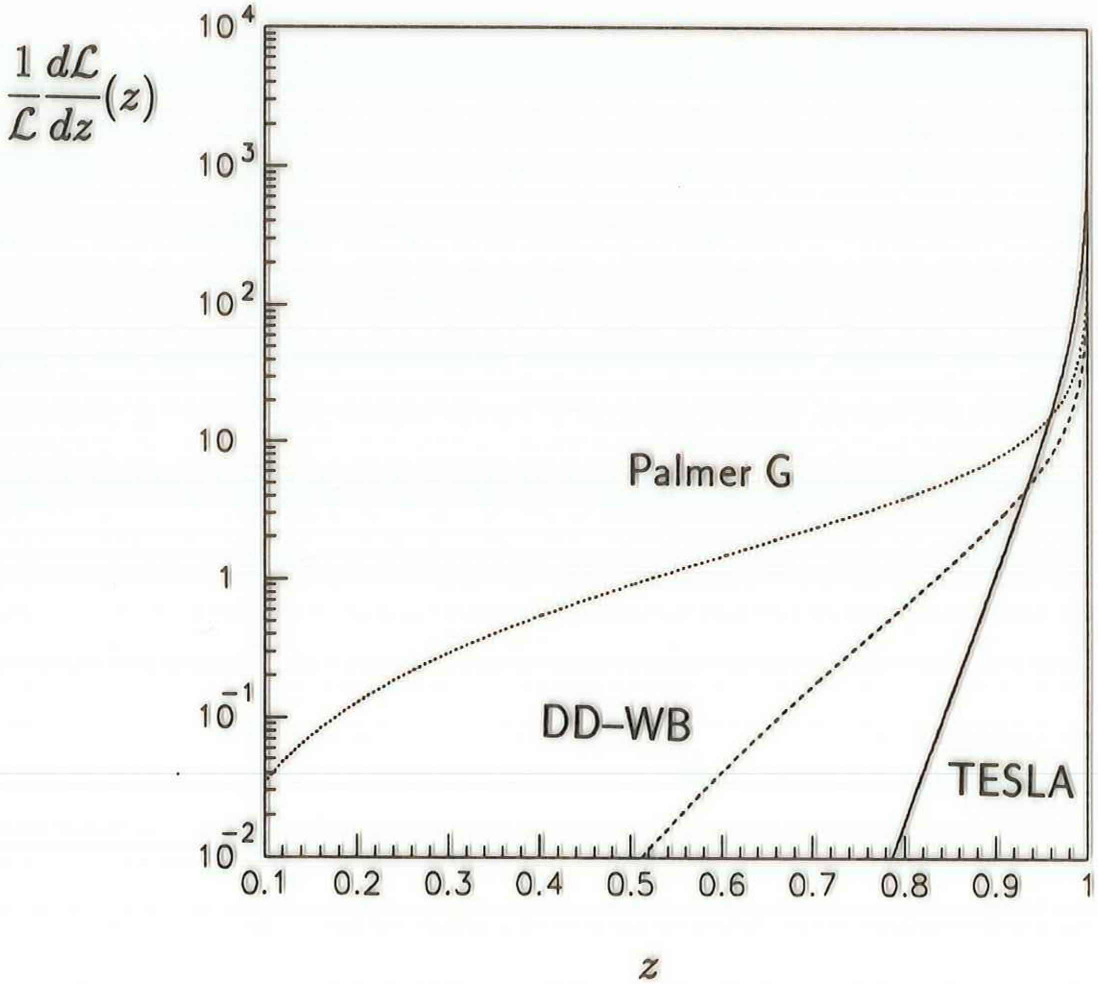
Accelerator	a	b
Palmer G	1.8	-0.67
DESY/Darmstadt WB	8.0	-0.67
TESLA	30.0	-0.85

Table 1: Fitted parameters a and b in (35) [14]

- BARKLOW, CHEN KOZANECKI

PARAM.: SCHREIBER

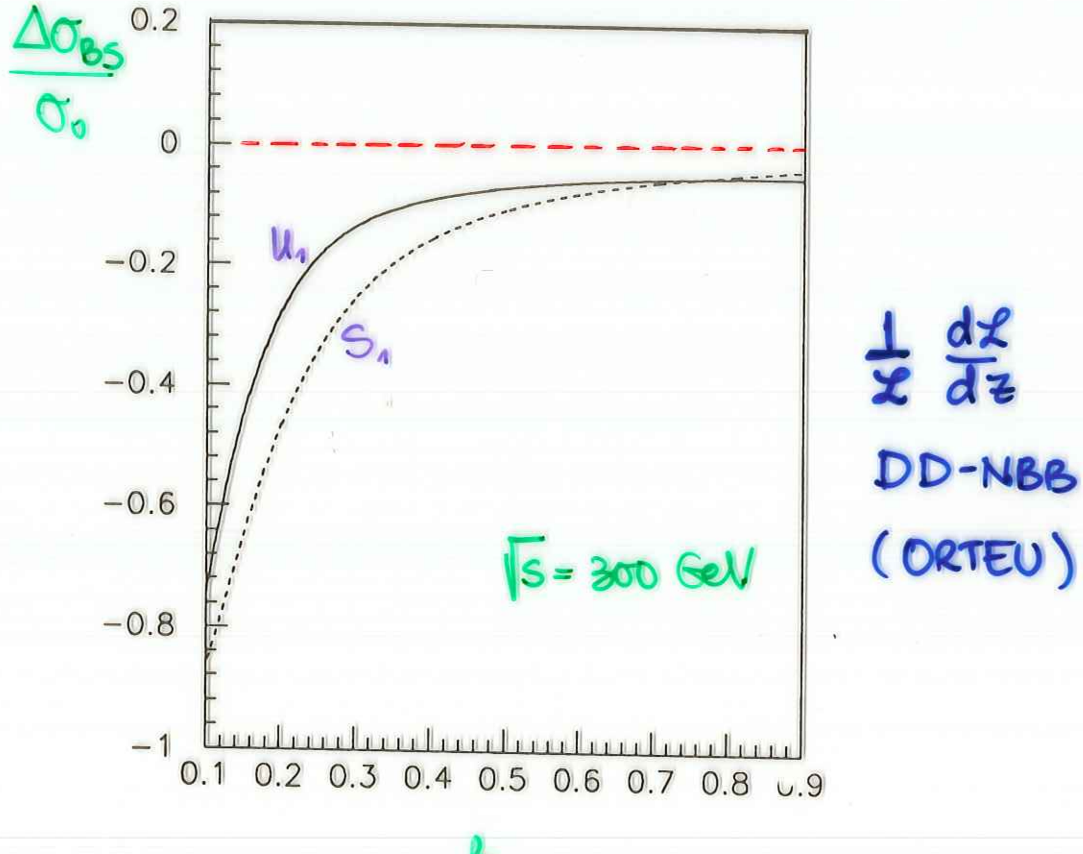
$$\Delta\sigma_{BS}(s) = \int_0^1 dz \frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dz}(z)\sigma^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma^{(0)}(s)$$



THRESH. RANGE: $\beta \sim 0.1$ $\Delta\sigma_{BS} \sim -0.77$ (U1) DD.NB

$\Delta\sigma_{BS} \sim -0.87$ (S1)

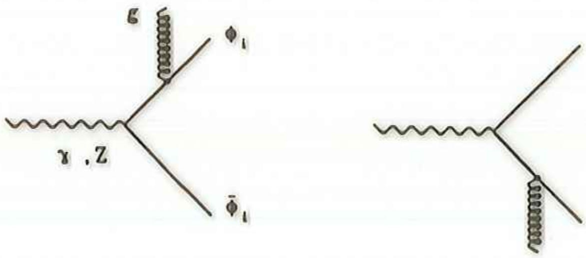
(FURTHER CONFIG. TO BE INVESTIGATED.)

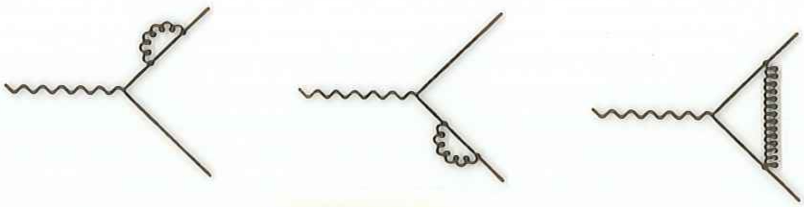


$\frac{\Delta\sigma_{BS}}{\sigma_0}$
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
u_1
s_1
$\sqrt{s} = 300$ GeV
β
$\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dz}$
DD-NBB
(ORTEU)

4 QCD Corrections

$\lambda_L, \lambda_R \ll 1$





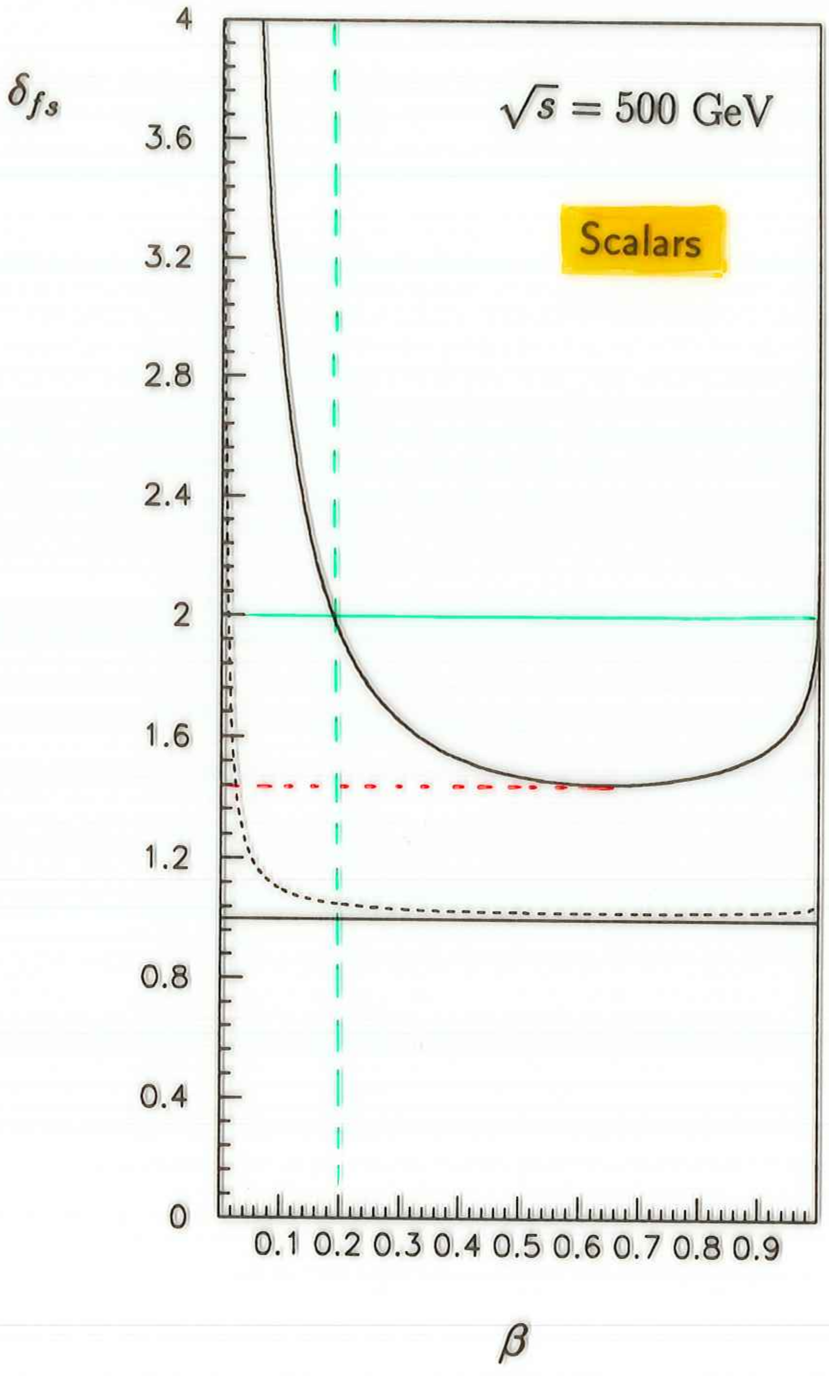
Scalars

$$\mathcal{F}_s(\beta) = \frac{1+\beta^2}{\beta}\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta} - 2\ln\beta\ln\frac{1+\beta}{1-\beta}\right]$$
$$- 3\ln\left(\frac{4}{1-\beta^2}\right) - 4\ln\beta + \frac{1}{\beta^3}\left[\frac{5(1+\beta^2)^2}{4} - 2\right]\ln\frac{1+\beta}{1-\beta} + \frac{3}{2}\frac{1+\beta^2}{\beta^2}$$

(J. SCHWINGER)

$$\sigma^{(1,QCD)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{4\alpha_s}{3\pi}\mathcal{F}_s(\beta)\right\}$$

$$\sigma^{(1,QED,fs)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{\alpha}{\pi}\mathcal{F}_s(\beta)\right\}$$

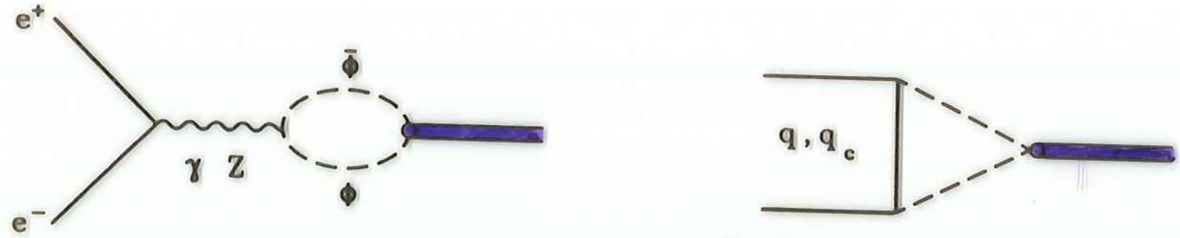


δ_{fs}
$\sqrt{s} = 500$ GeV
Scalars
4
3.6
3.2
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
β

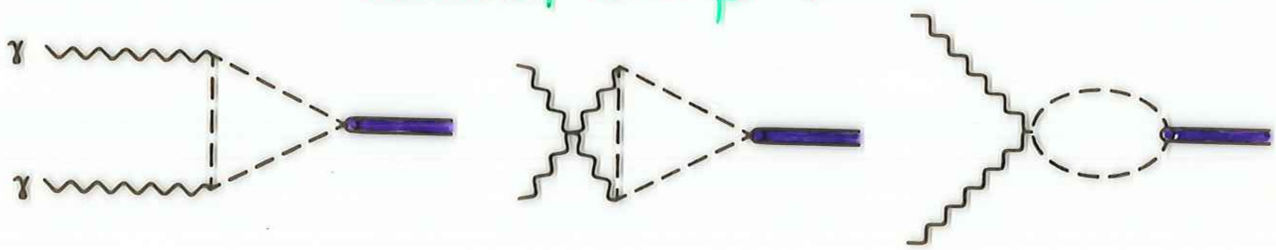
Formation of Bound States

"Leptoquarkonia"

$\beta \ll 1$



However, $\sigma \sim \beta^3$!



SCALARS: → SQUARKONIA

KÜHN, ZERWAS

⋮

$\Gamma_0 = 0.4\,\text{GeV}\, f_{S,V} \left(\frac{\lambda}{e}\right)^2 \frac{M}{200\,\text{GeV}}$;

MASS ⟷ LIFETIME BOUND STATE FORMATION?

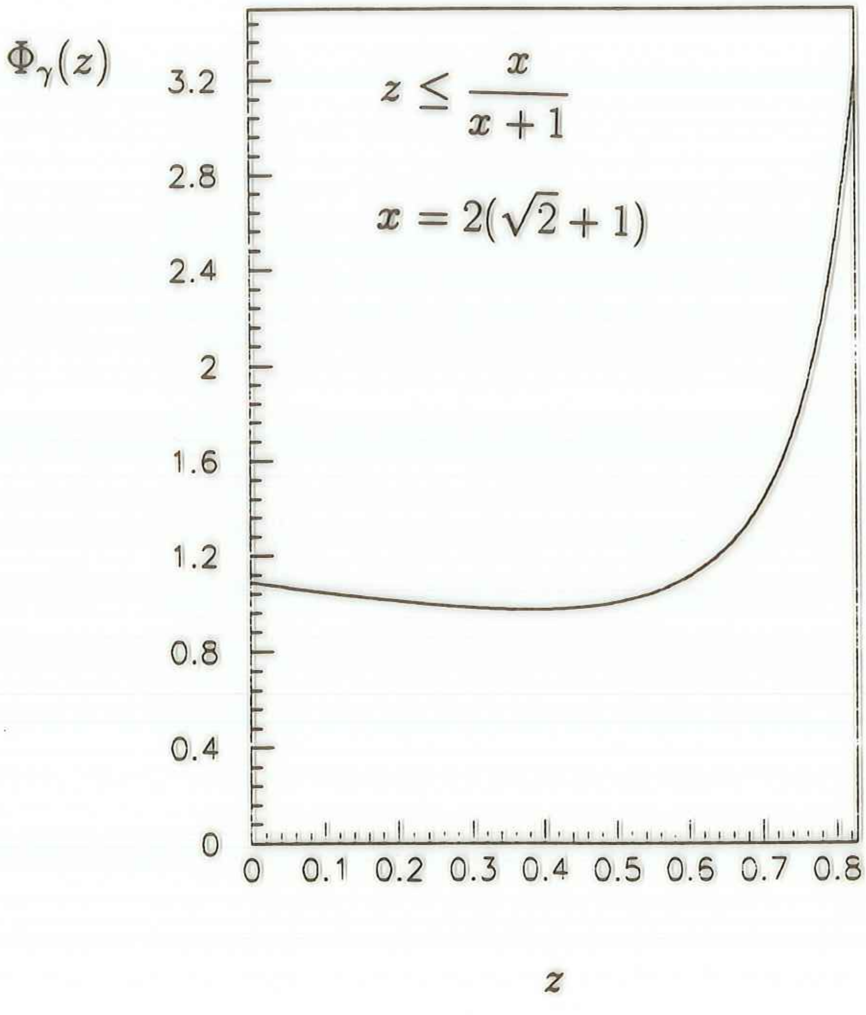
DIFFICULT TO OBSERVE THE RESONANCE, $\sigma_{S,V} \sim \beta^3$ for $\beta \ll 1$.

(ENERGYSPREAD!)

5 $\gamma\gamma$ Fusion

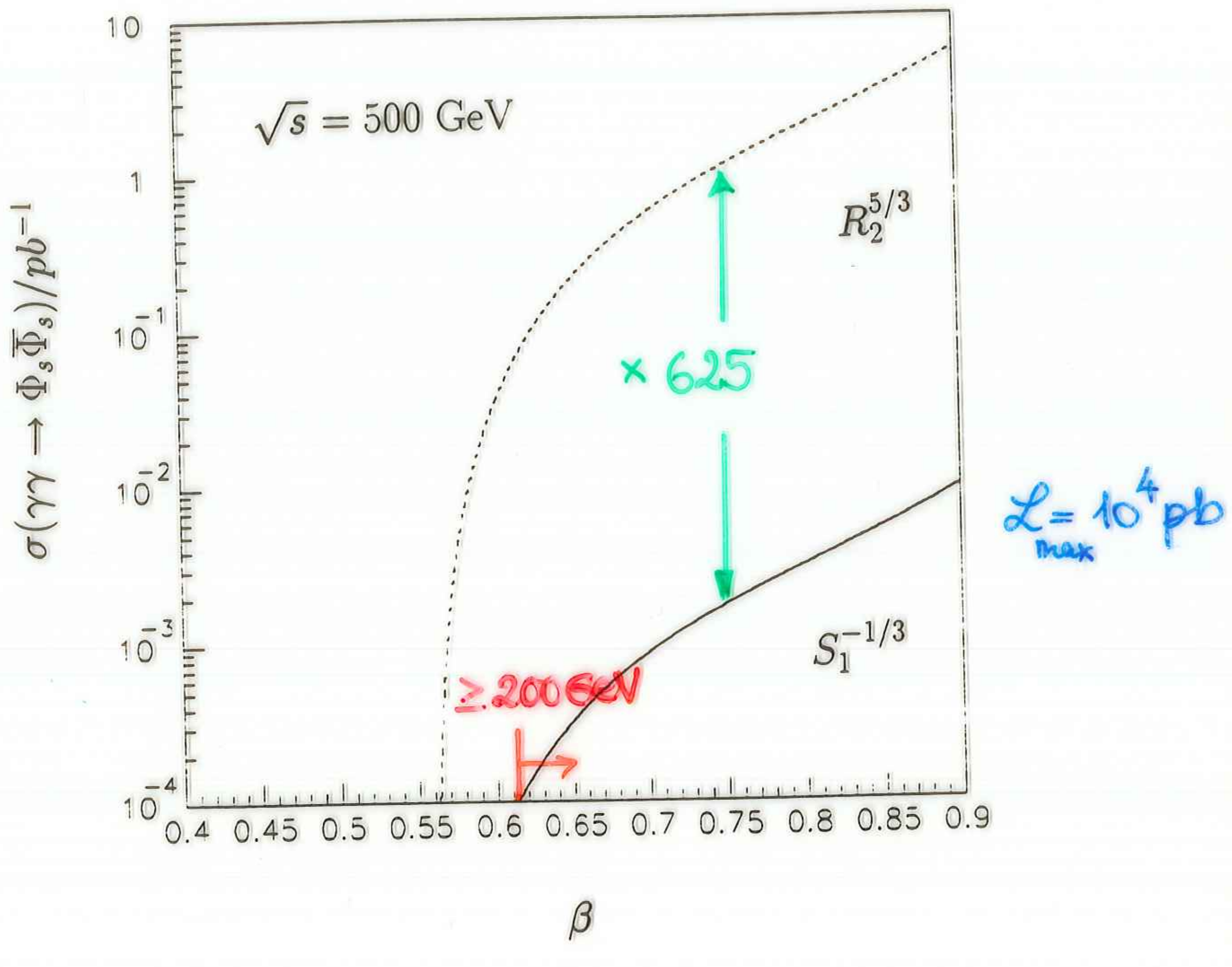
$$\Phi_\gamma(z) = \frac{1}{N(x)}\left[1 - z + \frac{1}{1-z} - \frac{4z}{x(1-z)} + \frac{4z^2}{x^2(1-z)^2}\right]$$

$$N(x) = \frac{16 + 32x + 18x^2 + x^3}{2x(1+x)^2} + \frac{x^2 - 4x - 8}{x^2}\ln(1+x)$$



$$\sigma_{scalar}(s) = \frac{\pi\alpha^2}{s} Q_\Phi^4 \left\{ 2(2-\beta^2)\beta - (1-\beta^4)\ln\left|\frac{1+\beta}{1-\beta}\right| \right\}$$

$$\sigma = \int_0^{z_{max}} dz_1 \int_0^{z_{max}} dz_2 \Phi_\gamma(z_1)\Phi_\gamma(z_2)\hat{\sigma}(\hat{s})\theta\left(\hat{s} - 4M_\Phi^2\right)$$



6 Summary

- QED & QCD CORRECTIONS TO LEPTOQUARK PAIR PRODUCTION HAVE BEEN CALCULATED NUMERICALLY.

QED ISR: $\beta = 0.1$

$\sqrt{s} = 500$ GeV $\quad -47\%$ (incl.: $+8\%$ α^2 already) $\quad \gamma, \gamma\gamma \ldots$

S_1

-35%

$\mathcal{U}_1$

$+3\%$ $\quad e^+e^-$-pairs

QCD: $\lambda_{L,e} \ll e$

$\delta \geq 42\%$ $\quad ; \beta \lesssim 0.2 \longrightarrow \delta \geq 100\%$

↳ BOUND STATE FORMATION.

QED: FSR $\quad \delta \gtrsim 3\%$ $\quad \beta = 0.1, \ \delta \sim 8\%$

- LARGE CORRECTIONS DUE TO BEAMSTRAHLUNG

$\longrightarrow$ NBB REQUIRED;

- $\gamma\gamma$ FUSION: FAVOURS $\frac{5}{3}$ LQ's $\quad \sigma \sim Q_{LQ}^4$

$\Phi_S = R_2^{5/2}$ e.g.

$(\sigma_i / \sigma_j) = (Q_i / Q_j)^4 \lesssim 625$

RANGE: $\beta \ll 1$ SUPPRESSED DUE TO $\Phi_\gamma(z)$.