Polarized Nucleons: What is the nucleon's spin made off ?

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DESY

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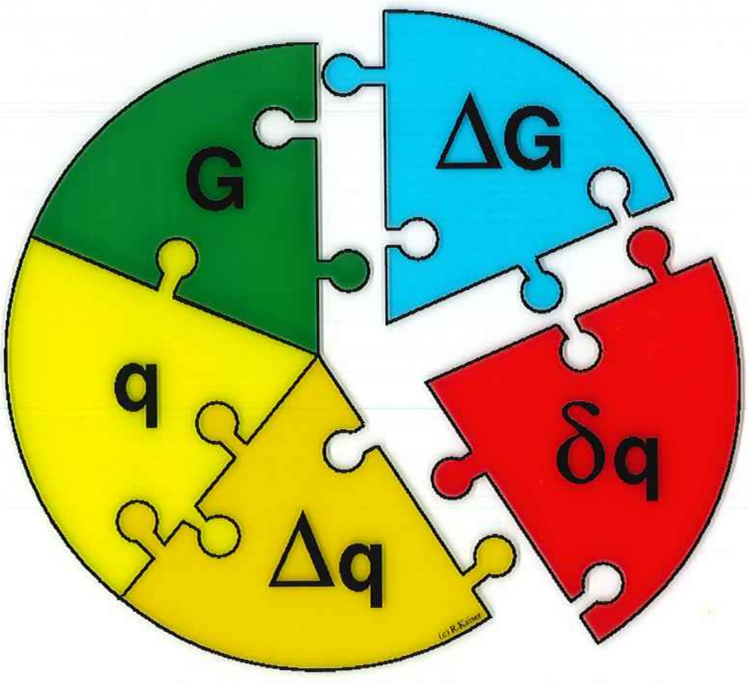
Introduction

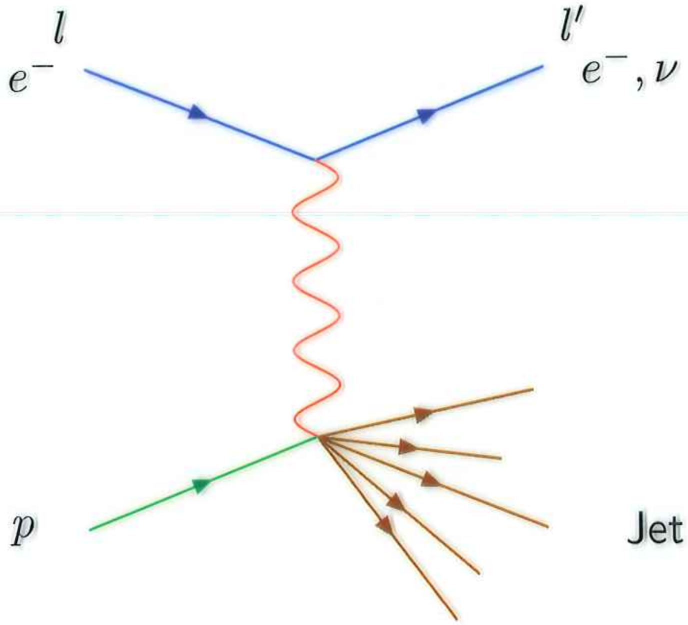
Why and for Which Purpose do we study Polarized Deep Inelastic Scattering ?

- Study of short distance structure of nucleon spin
- Test of perturbative QCD in spin sector: Λ_{QCD}
- Test of fundamental and less fundamental sum rules
- Does QCD describe polarized nucleons non-perturbatively? (QCD Moments vs. Lattice Moments)

Solve the Spin Puzzle!



Deeply Inelastic Scattering



space-like process :

$$q^2 = (l - l')^2 = -Q^2 < 0$$
$$W^2 = (p + q)^2 \geq M_p^2$$

$$x = \frac{Q^2}{2p.q}, \qquad y = \frac{p.q}{p.l}$$

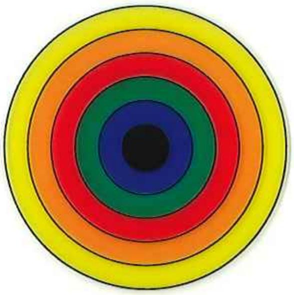
$$0 \leq x, y \leq 1$$

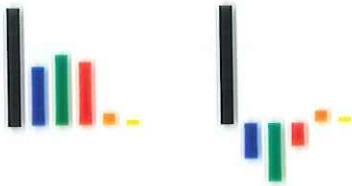
The Resolution of the Nucleon Microscope

$$\Delta x \sim \frac{1}{|Q|} = \frac{1}{\sqrt{-q^2}}$$

Examples :

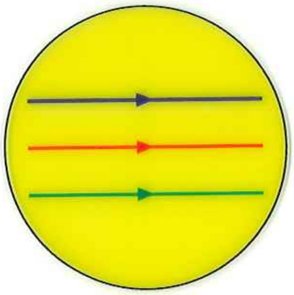
$Q^2 \sim 0.5 \cdot M_p^2$





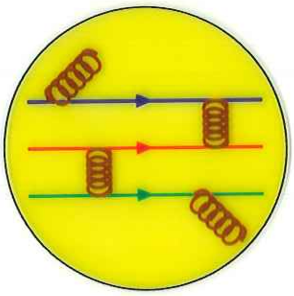
Charge Distribution

$Q^2 \sim 3 \cdot M_p^2$



Scaling

$Q^2 \sim 10...500 \cdot M_p^2$

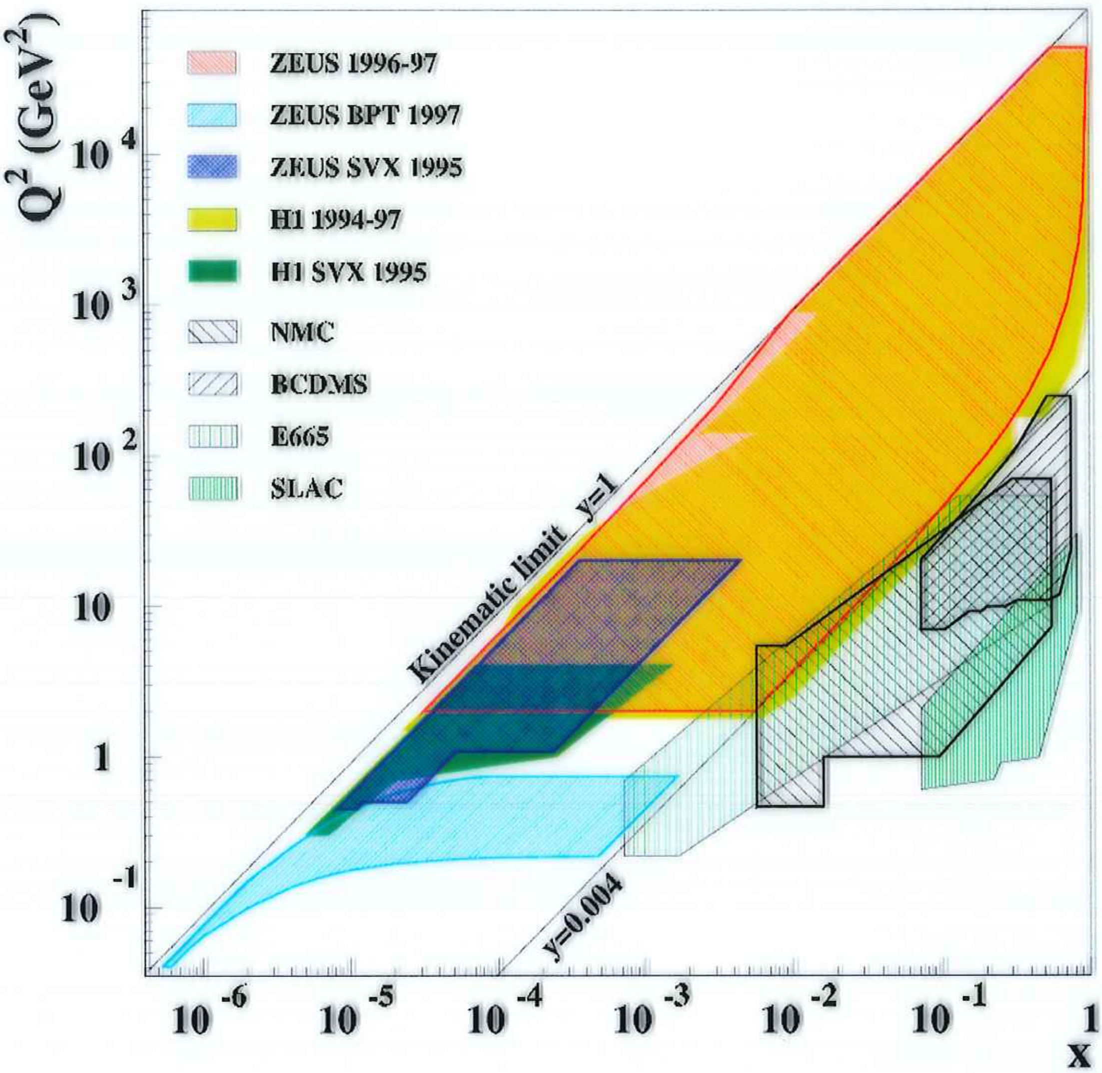


Violation of Scaling

If there are New Compositeness Scales, one may find them in the Future.

$$Q^2 > 10^4\ \mathrm{GeV}^2, \qquad 1\ \mathrm{GeV}^2 \sim M_p^2$$

Kinematic Domain



When is a Parton ?

S. DRELL: **Infinite Momentum Frame:** P - large

$$\tau_{\rm int} \ll \tau_{\rm life}$$

$$\tau_{\rm int} \sim \frac{1}{q_0} = \frac{4Px}{Q^2(1-x)}$$

$$\tau_{\rm life} \sim \frac{1}{\sum_i E_i - E} = \frac{2P}{\sum_i (k_{\perp i}^2 + M_i^2)/x_i - M^2} \simeq \frac{2Px(1-x)}{k_\perp^2}$$

$$\frac{\tau_{\rm int}}{\tau_{\rm life}} = \frac{2k_\perp^2}{Q^2(1-x)^2}$$

Stay away from $x \rightarrow 0$, since xP becomes too small.

Stay away from $x \rightarrow 1$.

$$Q^2 \gg k_\perp^2.$$

7. Polarized Nucleons

HOW IS THE NUCLEON SPIN DISTRIBUTED OVER THE PARTONS?

$$S_n = \frac{1}{2}\left[\Delta(u+\bar{u}) + \Delta(d+\bar{d}) + \Delta(s+\bar{s})\right] + \Delta G + L_q + L_g$$

$$S_n = \frac{1}{2}$$

$$\begin{aligned} \Delta\Sigma &= 0.138 \pm 0.082, \quad (0.150 \pm 0.061) \\ \Delta G &= 1.026 \pm 0.554, \quad (0.931 \pm 0.679) \end{aligned}$$

EMC, 1987: THE NUCLEON SPIN IS NOT THE SUM OF THE LIGHT QUARK SPINS.

MEASURE:

POLARIZED PARTON DENSITIES: $\Delta q_i, \Delta G$

HOW CAN ONE ACCESS THE PARTON ANGULAR MOMENTUM ?

POLARIZED HEAVY FLAVOR CONTRIBUTIONS.

- POLARIZED STRUCTURE FUNCTIONS CONTAIN ALSO TWIST 3 CONTRIBUTIONS.

HOW TO UNFOLD THESE TERMS ?

Motivation

- A number of QCD analyses for polarized data performed so far :
  - T.Gehrmann and W.J.Stirling (GS), Phys.Rev.**D53**(1996)6100.
  - G.Altarelli et al. (ABFR), Nucl.Phys.**B496**(1997)337.
  - Y.Goto et al. (AAC), Phys.Rev.**D62**(2000)034017; London ν–Factory Conference to appear. HIRAI, KUMANO, SAITO et al.
  - M.Glück et al. (GRSV), Phys.Rev.**D63**(2001)094005.
  - E.Leader et al. (LSS), Eur.Phys.J.**C23**(2002)479.
  - J. Blümlein and H. Böttcher, Nucl. Phys. **B636** (2002) 225

  - E154 Collaboration, Phys.Lett.**B405**(1997)180.
  - SMC Collaboration, Phys.Rev.**D58**(1998)112002.

  However, no reliable parametrization of the error bands for the polarized parton densities are given. (most of the cases.)

- We aim at parametrizations of polarized densities and their fully correlated 1σ error bands which are directly applicable to determine 'experimental' errors for other polarized observables.

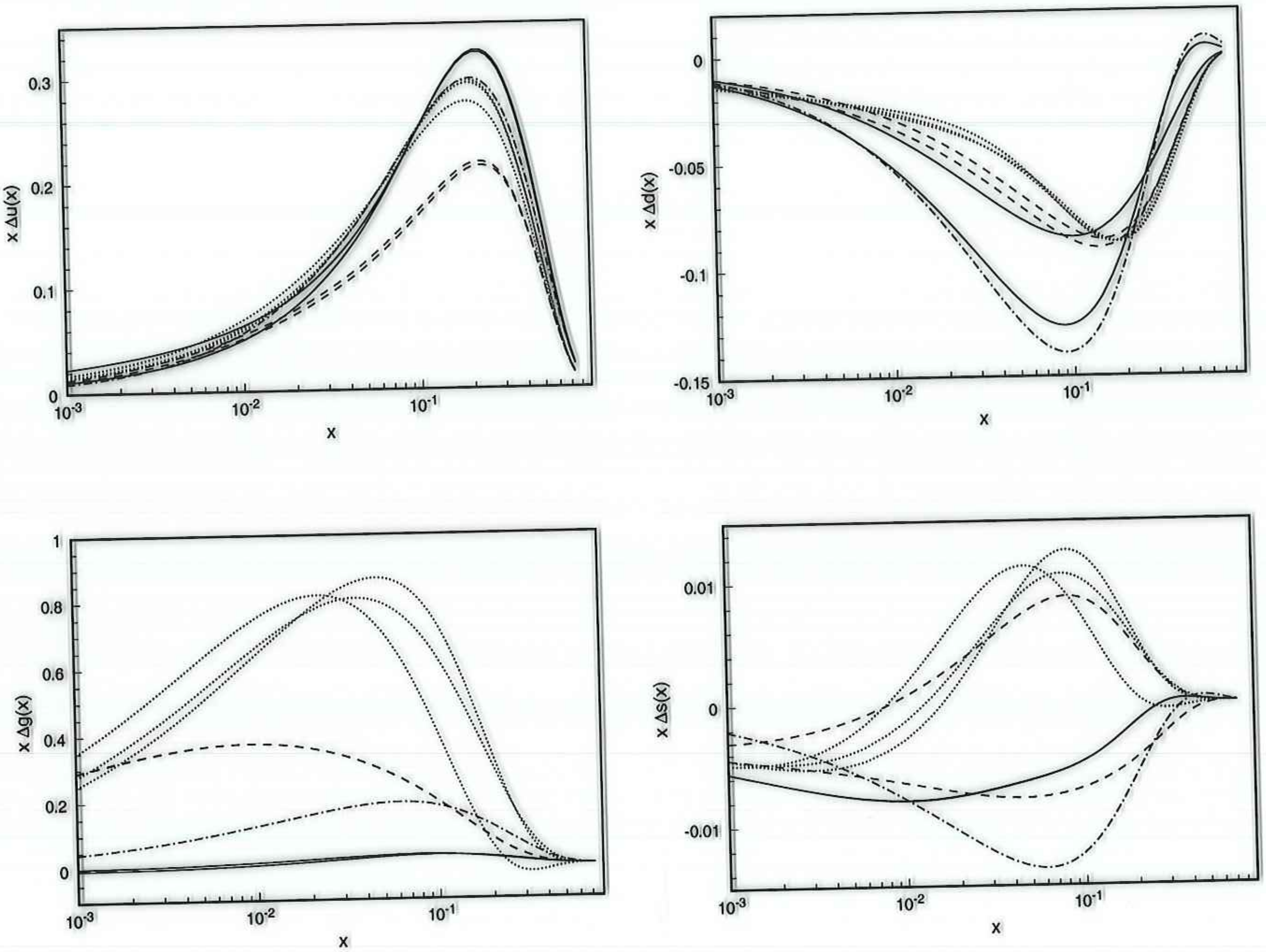
- Such an analysis has a value of its own within the framework of spin physics in order to understand the spin puzzle.

- Comparison of QCD analysis results with results from recent lattice simulations concerning both QCD parameters and low order moments.

Comparison Polarized Parton Densities

- Compilation by G. A. Ladinsky: (at $Q = 15\ GeV$)

  Ref.: Proc. of the Workshop on Prospects of SPIN PHYSICS at HERA p.285, DESY 95–200, MSU–51120,hep-ph/9601287.



⇒ solid lines: N–94 sets 1 and 2

Ref.: Nadolsky, Z.Phys. C63 (1994) 601.

⇒ dashed–dotted line: BBS–95

Ref.: Brodsky, Burkardt and Schmidt, Nucl.Phys. B441 (1995) 197.

⇒ dotted lines: GS–95 sets A, B, and C

Ref.: Gehrmann and Stirling, Z.Phys. C65 (1995) 461.

⇒ dashed lines: GRV–95 standard and valence scenarios

Ref.: Glück, Reya, and Vogelsang, Phys.Lett. B359 (1995) 201.

Evolution in MELLIN space

- The polarized structure function $g_1(x, Q^2)$ represented in terms of a MELLIN convolution of polarized parton densities Δf_j and Wilson coefficients ΔC_j:

$$g_1(x,Q^2) = \frac{1}{2}\sum_{j=1}^{N_f} e_j^2 \int_x^1 \frac{dz}{z}\Bigg[\frac{1}{N_f}\Delta\Sigma\left(\frac{x}{z},\mu_f^2\right)\Delta C_q^S\left(z,\frac{Q^2}{\mu_f^2}\right)$$
$$+\Delta G\left(\frac{x}{z},\mu_f^2\right)\Delta C_G\left(z,\frac{Q^2}{\mu_f^2}\right)$$
$$+\Delta q_j^{NS}\left(\frac{x}{z},\mu_f^2\right)\Delta C_q^{NS}\left(z,\frac{Q^2}{\mu_f^2}\right)\Bigg],$$

with the singlet density $\Delta\Sigma$

$$\Delta\Sigma\left(z,\mu_f^2\right) = \sum_{j=1}^{N_f}\left[\Delta q_j\left(z,\mu_f^2\right)+\Delta\bar{q}_j\left(z,\mu_f^2\right)\right],$$

the gluon density ΔG,
the non–singlet density Δq_j^{NS}

$$\Delta q_j^{NS}\left(z,\mu_f^2\right) = \Delta q_j\left(z,\mu_f^2\right)+\Delta\bar{q}_j\left(z,\mu_f^2\right) - \frac{1}{N_f}\Delta\Sigma\left(z,\mu_f^2\right),$$

and the factorization scale μ_f.

- The above quantities also depend on the renormalization scale μ_r of the strong coupling constant $a_s(\mu_r^2) = g_s^2(\mu_r^2)/(16\pi^2)$. The observable $g_1(x, Q^2)$ is independent of the choice of both scales.

Evolution in MELLIN space (cont'd)

- The evolution equations are given by

$$\frac{\partial \Delta q_i^{\rm NS}(x,Q^2)}{\partial \log Q^2} = \Delta P^-_{NS}(x,a_s) \otimes \Delta q_i^{\rm NS}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2} \begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix} = \Delta \boldsymbol{P}(x,a_s) \otimes \begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix}$$

with

$$\Delta P^-_{\rm NS}(x,a_s) = a_s \Delta P^{(0)}_{\rm NS}(x) + a_s^2 \Delta P^{-(1)}_{\rm NS}(x) + \mathcal{O}(a_s^3)$$

$$\Delta \boldsymbol{P}(x,a_s) \equiv \begin{pmatrix} \Delta P_{qq}(x,Q^2) & \Delta P_{qg}(x,Q^2) \\ \Delta P_{gq}(x,Q^2) & \Delta P_{gg}(x,Q^2) \end{pmatrix} = a_s \Delta \boldsymbol{P}^{(0)}(x) + a_s^2 \Delta \boldsymbol{P}^{(1)}(x) + \mathcal{O}(a_s^3)$$

and $\otimes$ the MELLIN convolution

$$[A \otimes B](x) = \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2)$$

- The polarized Wilson coefficient functions $\Delta C_i(x, \alpha_s(Q^2))$ and the polarized splitting functions $\Delta P_{ij}(x, \alpha_s(Q^2))$ are known in the $\overline{MS}$ scheme up to NLO. [W.L. van Neerven and E.B. Zijlstra, Nucl. Phys. B417 (1994) 61, R. Mertig and W.L. van Neerven, Z. Phys. C70 (1996) 637, W. Vogelsang, Phys. Rev. D54 (1996) 2023]

$\Longrightarrow$ A complete NLO QCD Analysis possible.

Evolution in MELLIN space (cont'd)

- $a_s(\mu_r)$ is obtained as the solution of

$$\mu_r^2 \frac{da_s(\mu_r^2)}{d\mu_r^2} = -\beta_0 a_s^2(\mu_r^2) - \beta_1 a_s^3(\mu_r^2) + \mathcal{O}(a_s^4),$$

where the coefficients of the β–function are given by (in the $\overline{\mathrm{MS}}$ scheme)

$$\begin{aligned} \beta_0 &= \frac{11}{3} C_A - \frac{4}{3} T_F N_f, \\ \beta_1 &= \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F N_f - 4 C_F T_F N_f, \end{aligned}$$

and

$$C_A = 3, \quad T_F = 1/2, \quad C_F = 4/3.$$

- $\Lambda_{QCD}^{\overline{\mathrm{MS}}}$ is given by:

$$\begin{aligned} \Lambda_{\mathrm{QCD}}^{\overline{\mathrm{MS}}} &= \mu_r \exp\left\{ -\frac{1}{2} \left[\frac{1}{\beta_0 a_s(\mu_r^2)} \right.\right. \\ &\qquad \left.\left. - \frac{\beta_1}{\beta_0^2} \log\left(\frac{1}{\beta_0 a_s(\mu_r^2)} + \frac{\beta_1}{\beta_0} \right) \right] \right\}. \end{aligned}$$

$\Longrightarrow$ We extract $\Lambda_{QCD}^{(4)}$ from the data and choose $N_f = 4$ whereas the polarized structure function $g_1(x, Q^2)$ is presented using only the three light flavors.

Evolution in MELLIN space (cont'd)

- The evolution equations are solved analytically in MELLIN–N space:

→ A MELLIN–transformation is performed

$$\mathbf{M}[f](N) = \int_0^1 dx x^{N-1} f(x), \quad N \in \mathbf{N} \,,$$

which turns the MELLIN convolution $\otimes$ into an ordinary product.

- The non–singlet solution:

$$\begin{aligned}\Delta q^{\rm NS}(N, a_s) &= \left(\frac{a_s}{a_0}\right)^{-P_{\rm NS}^{(0)}/\beta_0} \left[1 - \frac{1}{\beta_0}(a_s - a_0)\right. \\ &\quad \times \left.\left(P_{\rm NS}^{-(1)} - \frac{\beta_1}{\beta_0} P_{\rm NS}^{(0)}\right)\right] \Delta q^{\rm NS}(N, a_0)\end{aligned}$$

and the singlet solution:

$$\begin{aligned}\begin{pmatrix} \Delta\Sigma(N, a_s) \\ \Delta G(N, a_s) \end{pmatrix} &= [\mathbf{1} + a_s \boldsymbol{U}_1(N)] \, \boldsymbol{L}(N, a_s, a_0) \, [\mathbf{1} - a_0 \boldsymbol{U}_1(N)] \\ &\quad \times \begin{pmatrix} \Delta\Sigma(N, a_0) \\ \Delta G(N, a_0) \end{pmatrix} \,,\end{aligned}$$

where $a_s = a_s(Q^2)$ and $a_0 = a_s(Q_0^2)$.

⟹ The input and the evolution parts factorize.

[W.Furmanski and R.Petronzio, Z.Phys.**C11**(1982)293, M.Glück, E.Reya, and A.Vogt, Z.Phys.**C48**(1990)471, J.Blümlein and A.Vogt, Phys.Rev.**D58**(1998)014020.]

Evolution in MELLIN space (cont'd)

- The Leading Order singlet evolution matrix is given by

$$\boldsymbol{L}(a_s, a_0, N) = \boldsymbol{e}_-(N)\left(\frac{a_s}{a_0}\right)^{-r_-(N)} + \boldsymbol{e}_+(N)\left(\frac{a_s}{a_0}\right)^{-r_+(N)}$$

with the eigenvalues

$$r_\pm = \frac{1}{\beta_0}\left[tr(\boldsymbol{P}^{(0)}) \pm \sqrt{tr(\boldsymbol{P}^{(0)})^2 - det_2(\boldsymbol{P}^{(0)})}\right]$$

and the eigenvectors

$$\boldsymbol{e}_\pm = \frac{\boldsymbol{P}^{(0)}/\beta_0 - r_\mp \boldsymbol{1}}{r_\pm - r_\mp}\,.$$

- The Next–to–Leading Order singlet solution is obtained from the LO singlet solution through the matrix $\boldsymbol{U}_1(N)$

$$\begin{aligned}\boldsymbol{U}_1(N) \quad &= \quad -\boldsymbol{e}_-\boldsymbol{R}_1\boldsymbol{e}_- - \boldsymbol{e}_+\boldsymbol{R}_1\boldsymbol{e}_+ + \frac{\boldsymbol{e}_+\boldsymbol{R}_1\boldsymbol{e}_-}{r_- - r_+ - 1} \\ &\quad + \frac{\boldsymbol{e}_-\boldsymbol{R}_1\boldsymbol{e}_+}{r_+ - r_- - 1}\end{aligned}$$

with

$$\boldsymbol{R}_1 = [\boldsymbol{P}^{(1)} - (\beta_1/\beta_0)\boldsymbol{P}^{(0)}]/\beta_0.$$

Evolution in MELLIN space (cont'd)

- The input densities

$$\Delta\Sigma(N, a_0),\ \Delta G(N, a_0),\ \text{and}\ \Delta q_i^{NS}(N, a_0)$$

are evolved to the scale Q^2, respectively to the coupling $\alpha_s(Q^2)$. An inverse MELLIN–transformation to x–space is then performed by a contour integral in the complex plane around all singularities:

$$\Delta f(x) = \frac{1}{\pi} \int_0^\infty dz\ \text{Im}\left[\exp(i\phi) x^{-c(z)} \Delta f[c(z)]\right].$$

(Path: $c(z) = c_1 + \rho[\cos(\phi) + i\sin(\phi)]$, with $c_1 = 1.1$, $\rho \geq 0$, and $\phi = \frac{3}{4}\pi$.)

- The function $\Delta f(x)$ finally depends on the parameters of the parton distributions chosen at the input scale Q_0^2 and on Λ_{QCD}. These parameters are determined by the fit to the data.

Parametrization

- General choice for the parametrization of the polarized parton distributions at Q_0^2:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})$$

- Normalization:

$$A_i^{-1} = \left(1 + \gamma_i \frac{a_i}{a_i + b_i + 1}\right) \frac{\Gamma(a_i)\Gamma(b_i+1)}{\Gamma(a_i + b_i + 1)} + \rho_i \frac{\Gamma(a_i + 0.5)\Gamma(b_i + 1)}{\Gamma(a_i + b_i + 1.5)}$$

such that $\int_0^1 dx \Delta q_i(x, Q_0^2) = \eta_i$

are the first moment of $\Delta q_i(x, Q_0^2)$.

- The polarized parton distributions to be fitted are:

$$\Delta u_v \ , \ \Delta d_v \ , \ \Delta \bar{q} \ , \ \Delta G \ ,$$

where the index v denotes the $valence$ quark.

Note : $\Delta q + \Delta \bar{q} = \Delta q_v + 2\Delta \bar{q}$.

Choice of Parameters

- Parameters which have been fixed since the data do not constrain those parameters well enough:
  - For Δu_v and Δd_v: γ

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x)$$

  - For $\Delta\bar{q}$ and ΔG: b

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}$$

- Relations adopted between the parameters a_i and b_i for $\Delta\bar{q}$ and ΔG:

$$a_G = a_{\bar{q}} + C, \quad with \quad 0.5 < C < 1.0.$$

$$\left(\frac{b_{\bar{q}}}{b_G}\right)^{pol} = \left(\frac{b_{\bar{q}}}{b_G}\right)^{unpol}$$

$\Longrightarrow$ Essential to respect Positivity for $\Delta\bar{q}$ and ΔG.

- No Positivity constraint assumed for Δu_v and Δd_v.

$\Longrightarrow$ Finally 7 parameters are left free to be determined in the fit. In addition Λ_{QCD} is fitted. $\rightarrow (7+1)$

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})$$

The World Data

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	x–range	Q^2–range $[GeV^2]$	number of data points	
			g_1/F_1 or A_1	g_1
E143(p)	0.027 – 0.749	1.17 – 9.52	82	28
HERMES(p)	0.028 – 0.660	1.13 – 7.46	39	39
E155(p)	0.015 – 0.750	1.22 – 34.72	24	24
SMC(p)	0.005 – 0.480	1.30 – 58.0	59	12
EMC(p)	0.015 – 0.466	3.50 – 29.5	10	10
proton			214	113
E143(d)	0.027 – 0.749	1.17 – 9.52	82	28
E155(d)	0.015 – 0.750	1.22 – 34.79	24	24
SMC(d)	0.005 – 0.479	1.30 – 54.8	65	12
deuteron			171	64
E142(n)	0.035 – 0.466	1.10 – 5.50	30	8
HERMES(n)	0.033 – 0.464	1.22 – 5.25	9	9
E154(n)	0.017 – 0.564	1.20 – 15.0	11	17
neutron			50	34
total			435	211

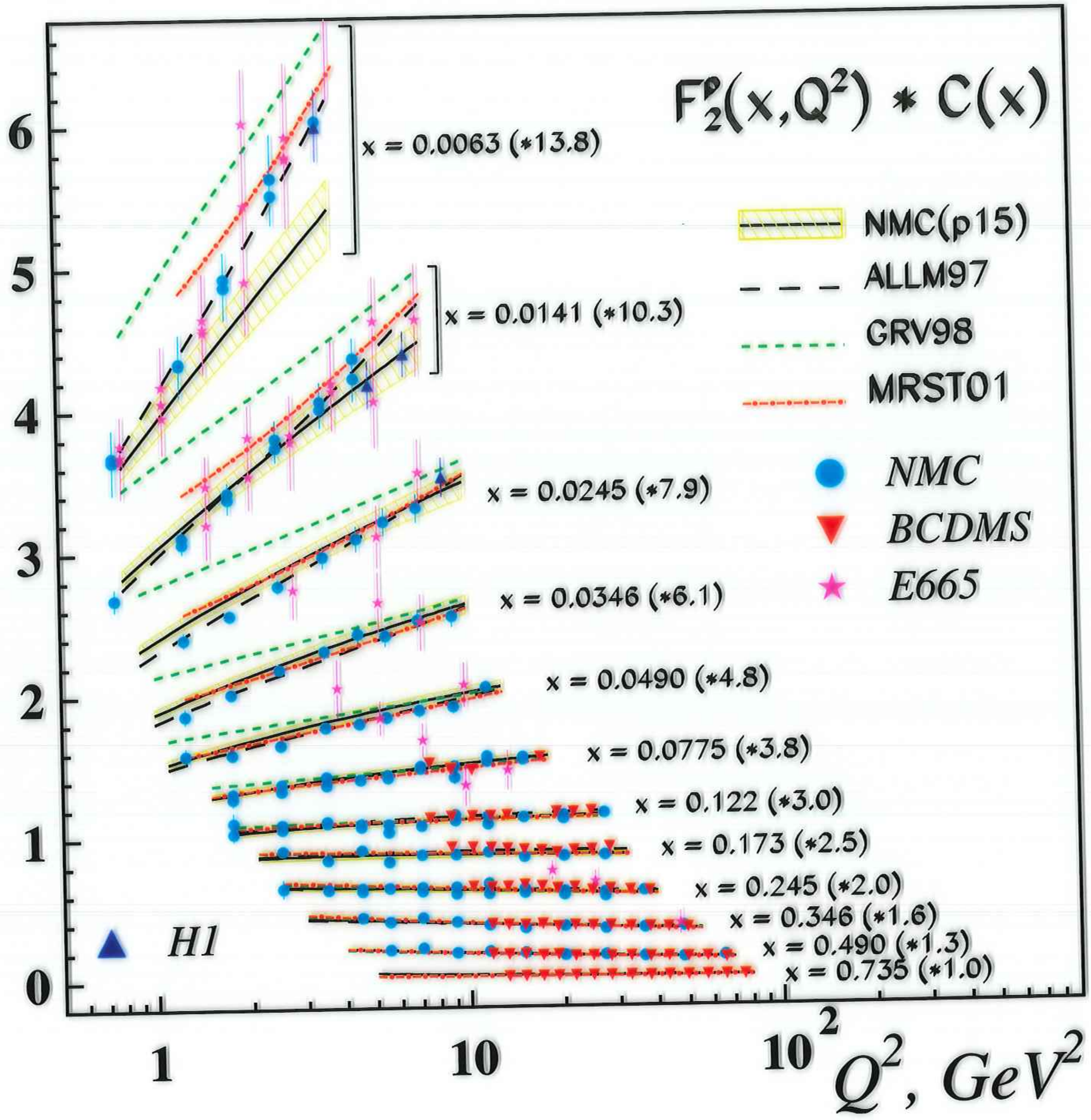
$$g_1/F_1 \approx \frac{1}{(1+\gamma^2)} A_1\,, \quad where \quad \gamma^2 = Q^2/\nu^2$$

$$F_1 = \frac{(1+\gamma^2)}{2x(1+R)} F_2$$

F_2–Parametrization: NMC, M. Arneodo et al., Phys. Lett. **B364** (1995) 107.

R–Parametrization: SLAC, L. Withlow et al., Phys. Lett. **B250** (1990) 193.

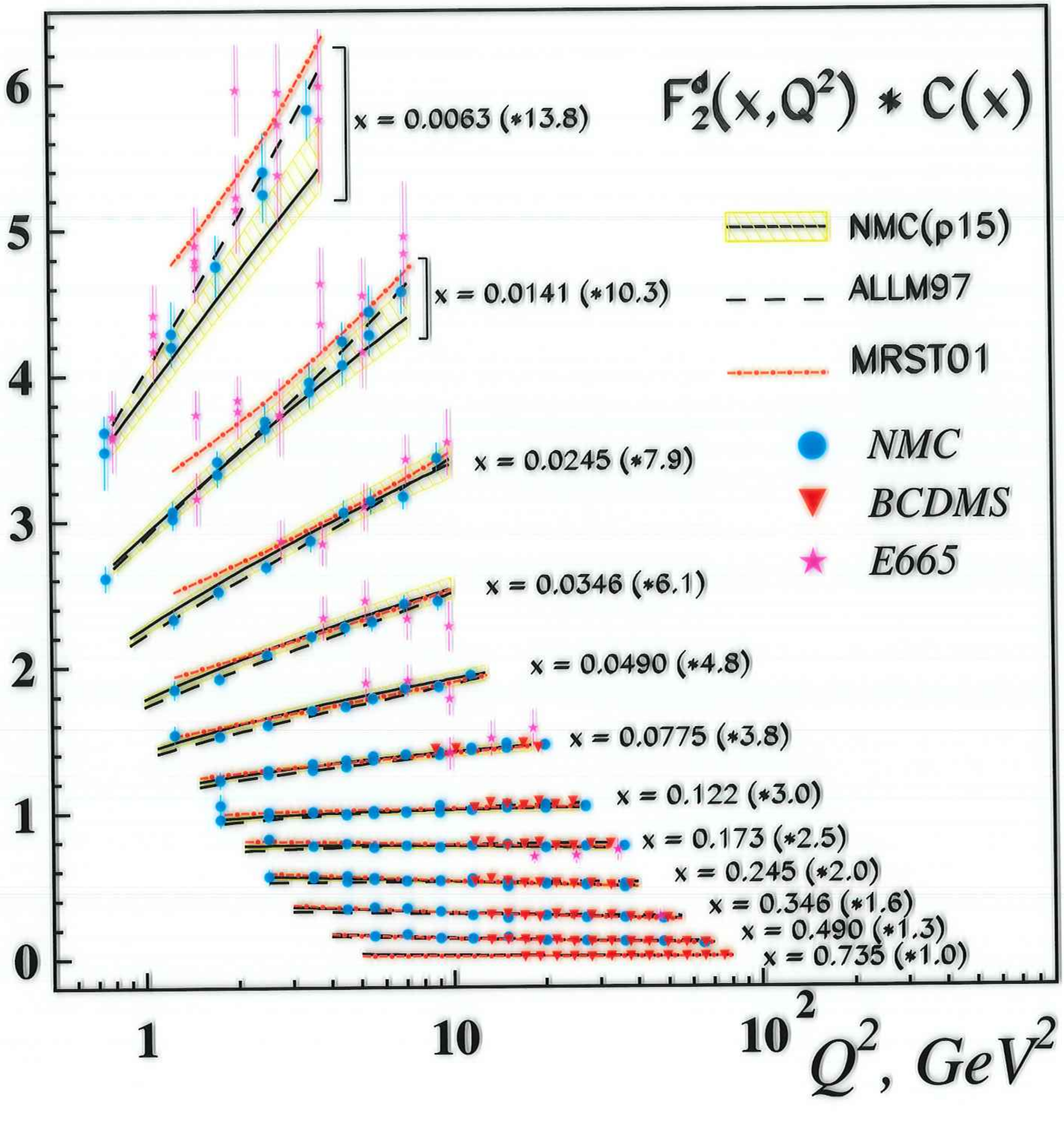
F_2^p – Comparison Data / Parameterization



⇒ Question: Which Parameterization to be used?

⇒ Answer: ALLM97

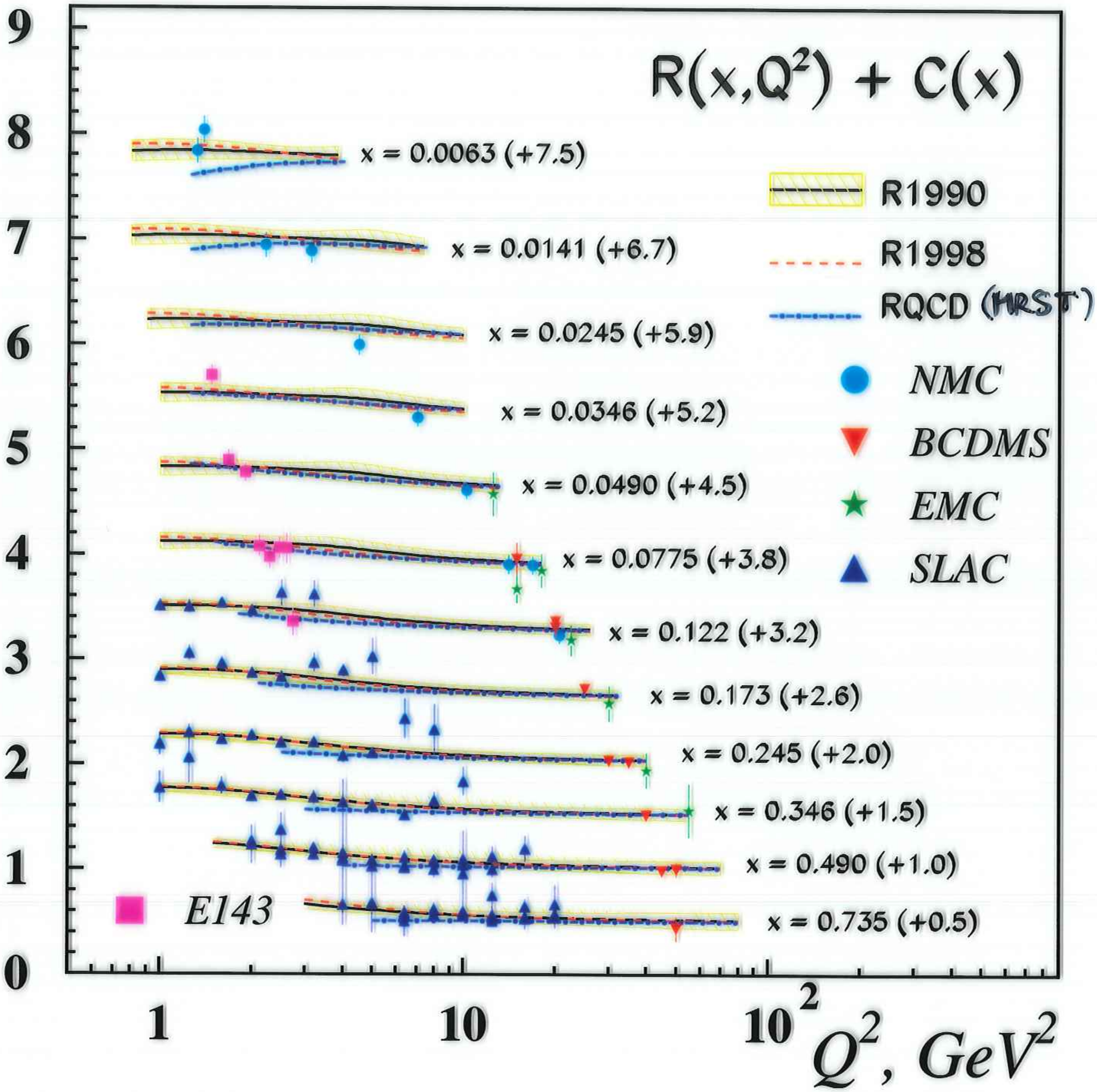
F_2^d – Comparison Data / Parameterization



⇒ Question: Which Parameterization to be used?

⇒ Answer: ALLM97

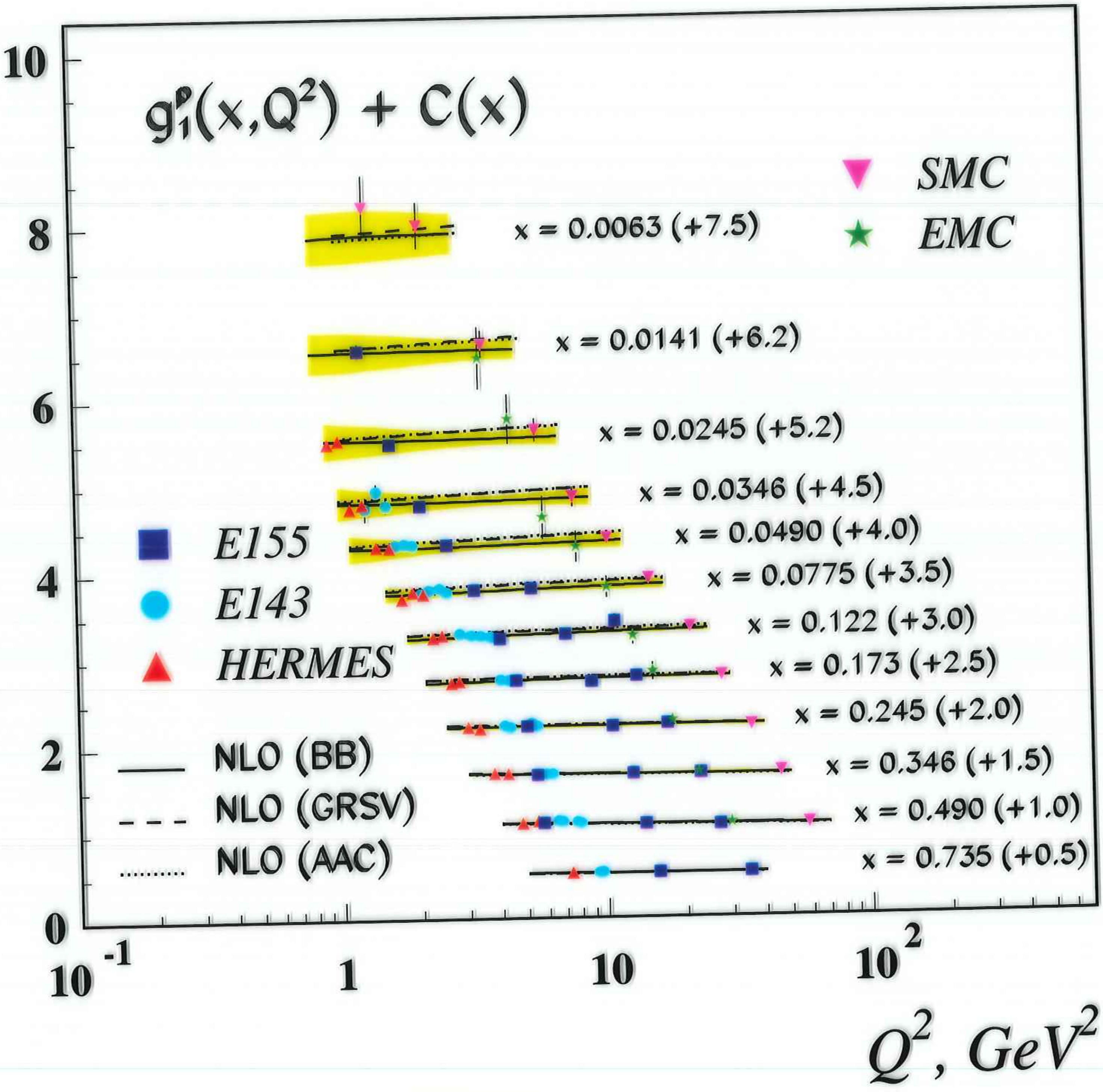
R – Comparison Data / Parameterization



⇒ Question: Which Parameterization to be used?

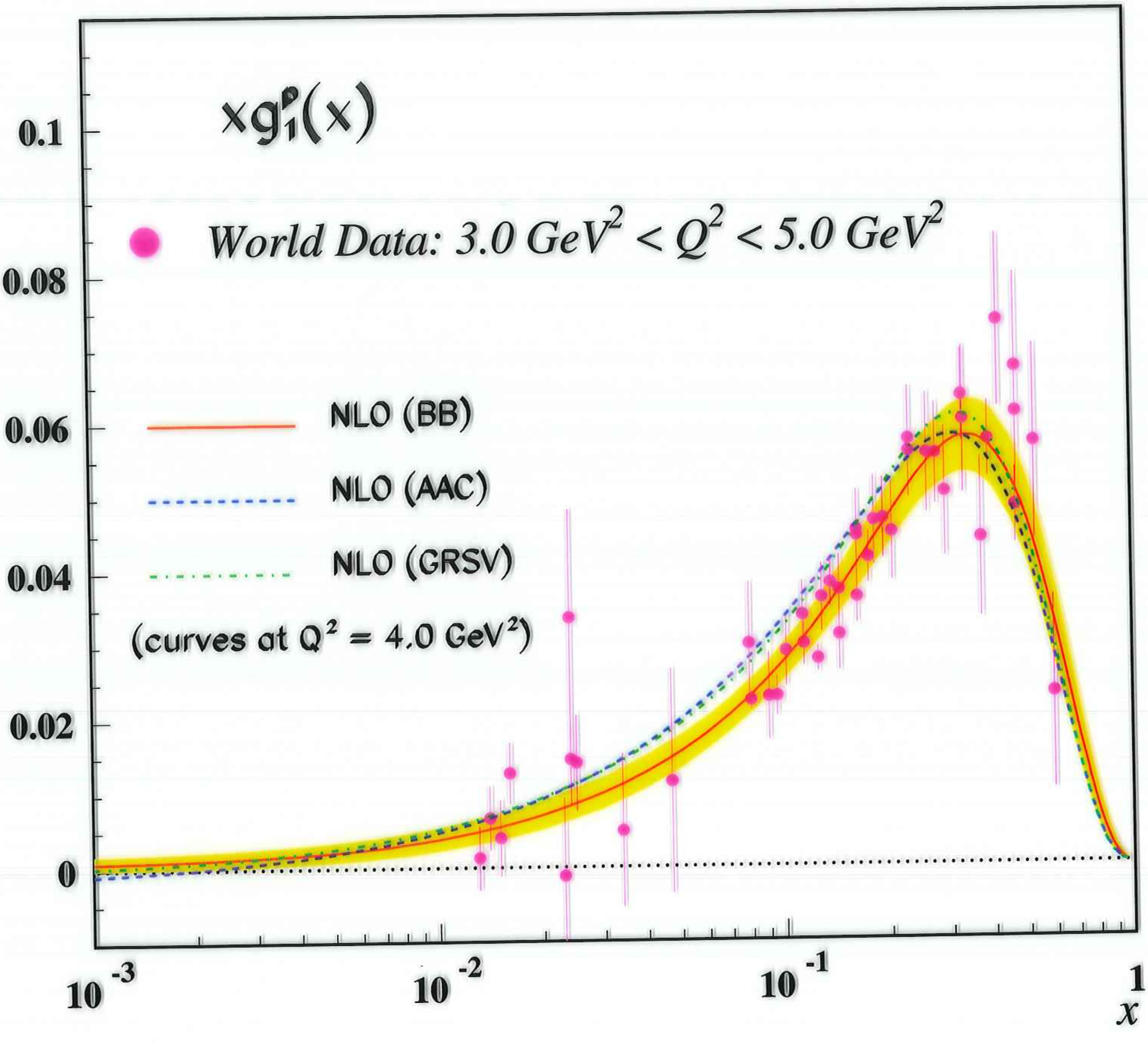
⇒ Answer: R1998

$g_1^p(x)$ versus Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$xg_1^p(x)$ from Measured Asymmetry Data



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

What about the Errors?

⇒ Problem: Systematic errors are known to be partly correlated which would lead to an overestimation of the errors when added in quadrature with the statistical ones.

- Statistical Errors:
  To treat all data sets on the same footing statistical errors are taken only. Accept only fits with a Positive Definite Covariance Matrix.

  ⟹ Calculate the Fully Correlated 1σ Error Bands by Gaussian error propagation.

- Systematic Uncertainties:
  Allow for a Relative Normalization Shift between the different data sets within the normalization uncertainties quoted by the experiments (fitted and then fixed).

$$\chi^2 = \sum_{i=1}^{n^{exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{(N_i g_{1,j}^{data} - g_{1,j}^{theor})^2}{(\Delta g_{1,j}^{data})^2} \right]$$

⟹ Thereby accounting for the main systematic uncertainties (luminosity and beam and target polarization).

Gaussian Error Propagation

In the treatment used in our analysis the evolved polarized parton densities are linear functions of the input densities for all parameters, except Λ_{QCD}.

Let $f(x, Q^2; a_i|_{i=1}^k)$ be the evolved density at Q^2 depending on the fitted parameters $a_i|_{i=1}^k$ at the input scale Q_0^2. Then its fully correlated error Δf as given by Gaussian error propagation is

$$
\begin{aligned}
\Delta f(x, Q^2) \; &= \; \left[\sum_{i=1}^{k} \left(\frac{\partial f}{\partial a_i} \right)^2 C(a_i, a_i) + \right. \\
&\left. \sum_{i \neq j=1}^{k} \left(\frac{\partial f}{\partial a_i} \frac{\partial f}{\partial a_j} \right) C(a_i, a_j) \right]^{\frac{1}{2}} .
\end{aligned}
$$

$C(a_i, a_j)$ are the elements of the covariance matrix determined in the QCD analysis at the input scale Q_0^2.

$\Longrightarrow$ All what is needed are the gradients $\partial f / \partial a_i$ w.r.t. the parameters a_i. They can be calculated analytically at the input scale Q_0^2. Their value at Q^2 is then given by evolution.

Error Propagation in MELLIN-N space

The general form of the derivative of the MELLIN moment $\mathbf{M}[f(a)](N)$ w.r.t. parameter a for complex values of N is

$$\frac{\partial \mathbf{M}[f(a)](N)}{\partial a} = F(a) \times \mathbf{M}[f(a)](N),$$

- For Δu_v and Δd_v:

$$\begin{aligned} F(a_i) &= \psi(N-1+a_i) - \psi(N+a_i+b_i) + \\ &\frac{\gamma_i(b_i+1)}{(N+a_i+b_i)(N+a_i+b_i+\gamma_i(N-1+a_i))} \\ &- \psi(a_i) + \psi(a_i+b_i+1) \\ &- \frac{\gamma_i(b_i+1)}{(a_i+b_i+1)(a_i+b_i+1+\gamma_i a_i)}, \end{aligned}$$

$$\begin{aligned} F(b_i) &= \psi(b_i+1) - \psi(N+a_i+b_i) - \\ &\frac{\gamma_i(N-1+a_i)}{(N+a_i+b_i)(N+a_i+b_i+\gamma_i(N-1+a_i))} \\ &- \psi(b_i+1) + \psi(a_i+b_i+1) \\ &+ \frac{\gamma_i a_i}{(a_i+b_i+1)(a_i+b_i+1+\gamma_i a_i)} \end{aligned}$$

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x)$$

Error Propagation in MELLIN-N space (cont'd)

- For $\Delta\bar{q}$ and ΔG:

$$F(\eta_i) = \frac{1}{\eta_i},$$
$$F(a_i) = \psi(N-1+a_i) - \psi(N+a_i+b_i) - \psi(a_i) + \psi(a_i+b_i+1).$$

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}$$

with $\psi(z) = d/dz(log\Gamma(z))$ the EULER ψ–function.

$\Longrightarrow$ The gradients evolved in MELLIN-N space are then transformed back to x-space and can be used according to the error propagation equation.

- When fitting Λ_{QCD} its gradient has to be determined numerically due to non–linear and iterative aspects in the calculation of $\alpha_s(Q^2, \Lambda_{QCD})$:

$$\frac{\partial f(x, Q^2, \Lambda)}{\partial \Lambda} = \frac{f(x, Q^2, \Lambda+\delta) - f(x, Q^2, \Lambda-\delta)}{2\delta}$$

with $\delta \sim 10\ MeV$.

Parameter Values at $Q_0^2 = 4.0\, GeV^2$

7+1 Parameter Fit based on the Asymmetry Data:

	Scenario 1			
	LO		NLO	
	value	error	value	error
$\Lambda_{QCD}^{(4)}, MeV$	203	120	235	53
η_{uv}	0.926	fixed	0.926	fixed
a_{uv}	0.197	0.013	0.294	0.035
b_{uv}	2.403	0.107	3.167	0.212
$\gamma_{uv}(*)$	21.34	fixed	27.22	fixed
η_{dv}	-0.341	fixed	-0.341	fixed
a_{dv}	0.190	0.049	0.254	0.111
b_{dv}	3.240	0.884	3.420	1.332
$\gamma_{dv}(*)$	30.80	fixed	19.06	fixed
η_{sea}	-0.353	0.054	-0.447	0.082
a_{sea}	0.367	0.048	0.424	0.062
$b_{sea}(*)$	8.51	fixed	8.93	fixed
η_G	1.281	0.816	1.026	0.554
a_G	$a_{sea} + 0.9$		$a_{sea} + 1.0$	
$b_G(*)$	5.91	fixed	5.51	fixed
χ^2 / NDF	1.02		0.90	

⇒ The parameters marked by (*) have been fitted first and then fixed since the present data do not constrain their values well enough.

⇒ Scenario 2 : $a_G = a_{sea} + 0.6\ (LO)$

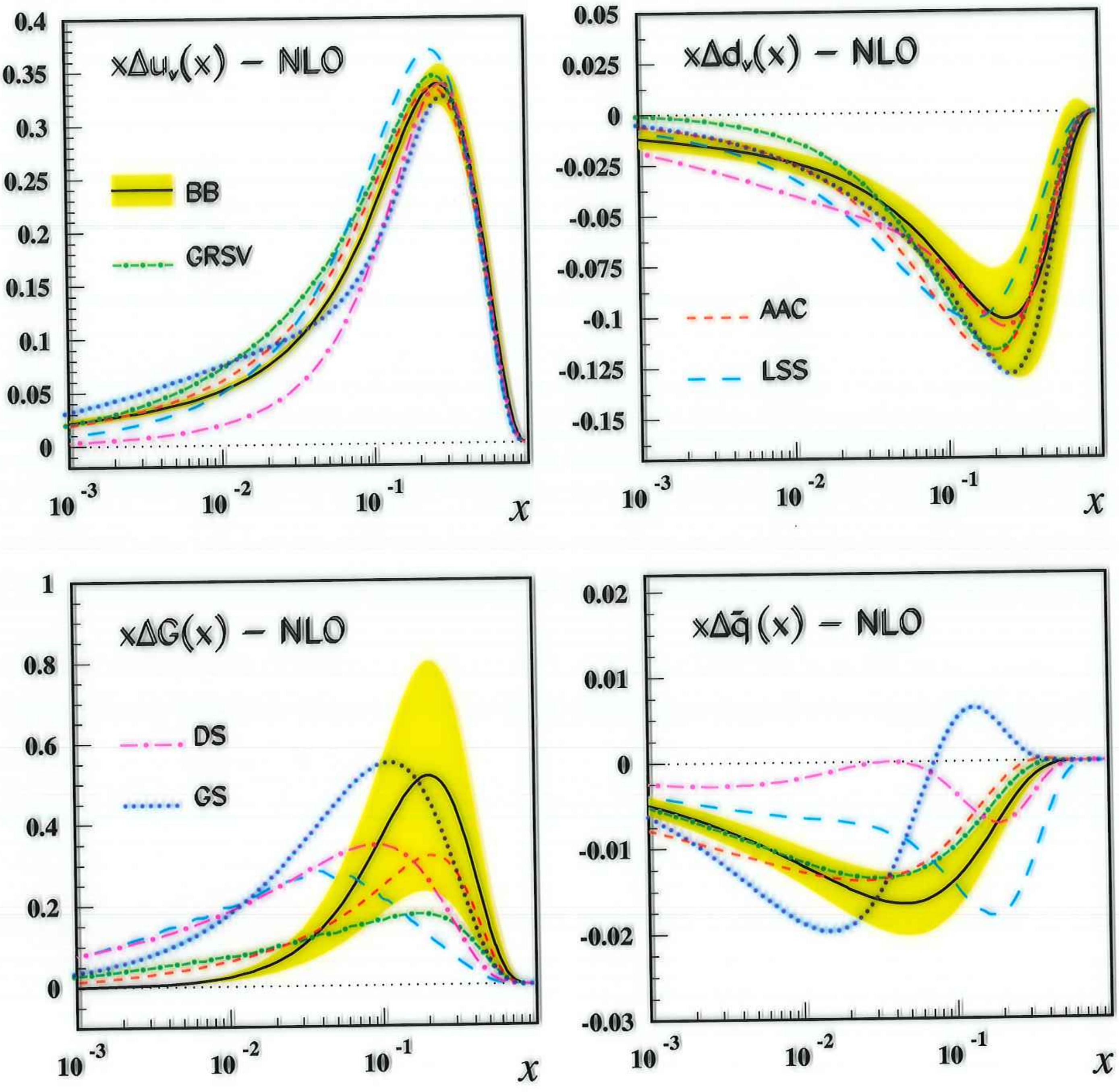
$a_G = a_{sea} + 0.5\ (NLO)$

Covariance Matrices at $Q_0^2 = 4.0\ GeV^2$ – 7 + 1 Parameter Fit - Scenario 1

LO								
	$\Lambda_{QCD}^{(4)}$	a_{uv}	b_{uv}	a_{dv}	b_{dv}	η_{sea}	a_{sea}	η_G
$\Lambda_{QCD}^{(4)}$	1.43E-2							
a_{uv}	-2.05E-5	1.80E-4						
b_{uv}	-9.07E-5	3.91E-4	1.15E-2					
a_{dv}	1.10E-4	1.03E-5	-2.40E-3	2.43E-3				
b_{dv}	-4.65E-5	-7.92E-3	-6.86E-3	5.48E-3	7.82E-01			
η_{sea}	1.02E-4	-4.46E-4	-2.84E-3	9.85E-4	2.82E-2	2.94E-3		
a_{sea}	-4.31E-5	1.58E-4	1.33E-3	-5.96E-4	-9.32E-3	-2.58E-4	2.29E-3	
η_G	-1.03E-3	2.02E-3	1.58E-2	-2.78E-3	-1.61E-1	-1.59E-2	9.56E-3	6.65E-1

NLO								
	$\Lambda_{QCD}^{(4)}$	a_{uv}	b_{uv}	a_{dv}	b_{dv}	η_{sea}	a_{sea}	η_G
$\Lambda_{QCD}^{(4)}$	2.81E-3							
a_{uv}	2.71E-5	1.22E-3						
b_{uv}	-1.30E-4	5.10E-3	4.50E-2					
a_{dv}	-3.35E-4	-5.17E-4	-3.23E-3	1.23E-2				
b_{dv}	-6.22E-4	-1.27E-2	4.65E-2	8.29E-2	1.78E-0			
η_{sea}	-5.30E-5	-2.13E-3	-1.12E-2	5.19E-3	4.74E-2	6.77E-3		
a_{sea}	-4.85E-6	9.07E-4	4.49E-3	-3.78E-3	-2.98E-2	-2.39E-3	3.82E-3	
η_G	4.03E-4	1.41E-2	6.71E-2	-3.07E-2	-2.22E-1	-3.78E-2	1.90E-2	3.07E-1

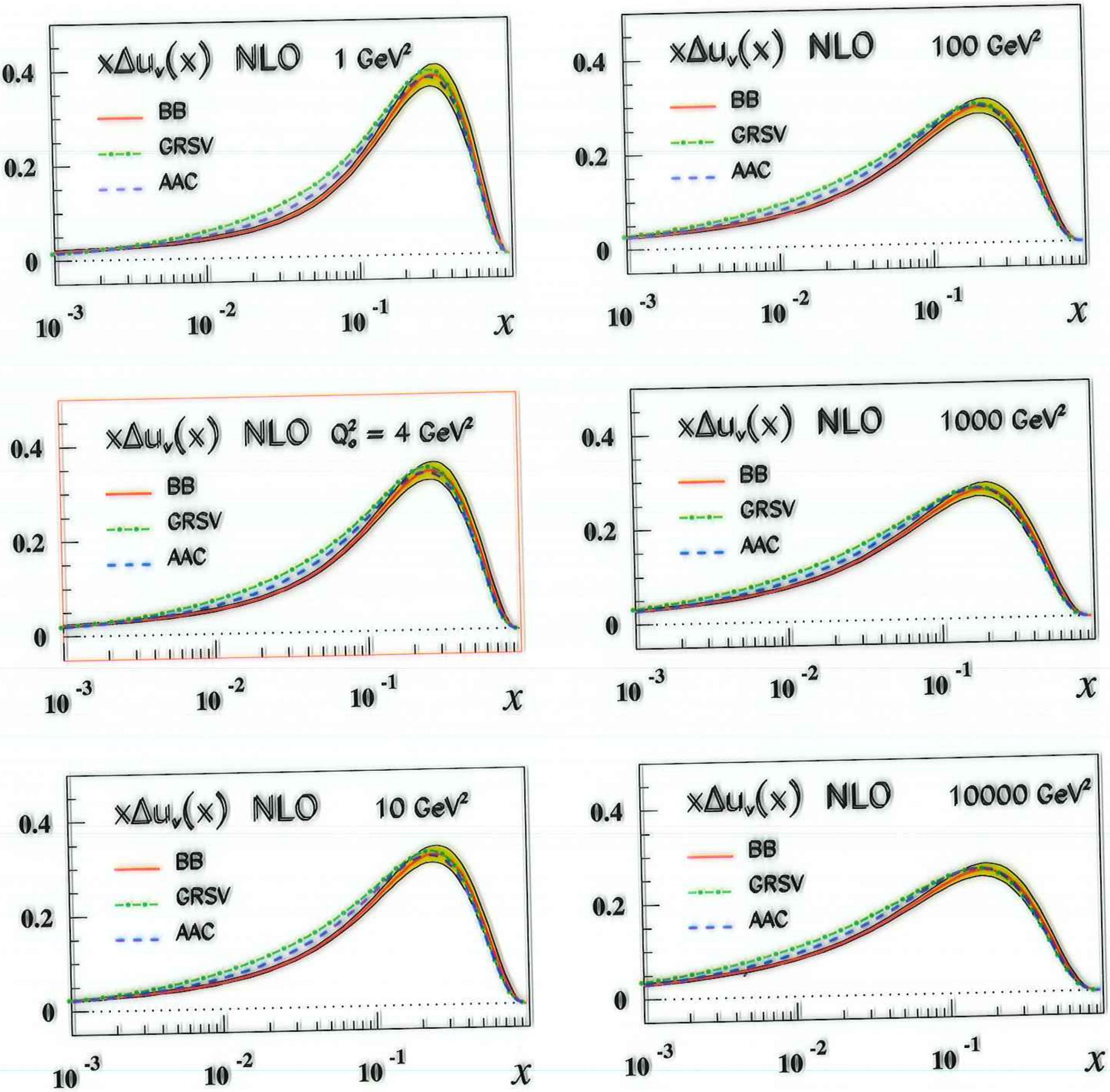
Pol. Parton Densities at $Q^2 = 4.0\ \mathrm{GeV}^2$



⇒ Yellow band: Fully correlated 1σ statistical error band from the BB analysis.

Evolution of Polarized Parton Densities

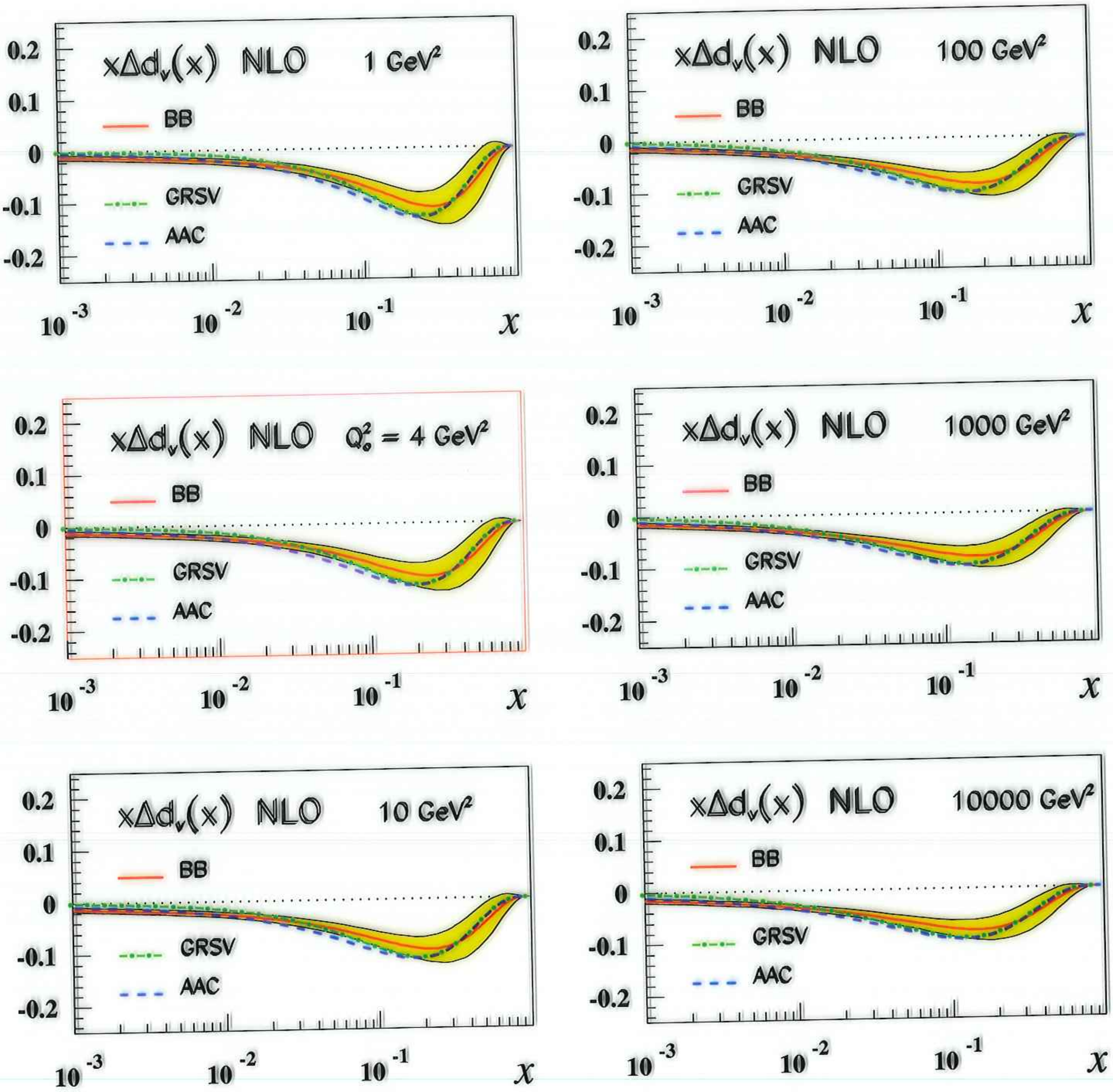
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of Polarized Parton Densities

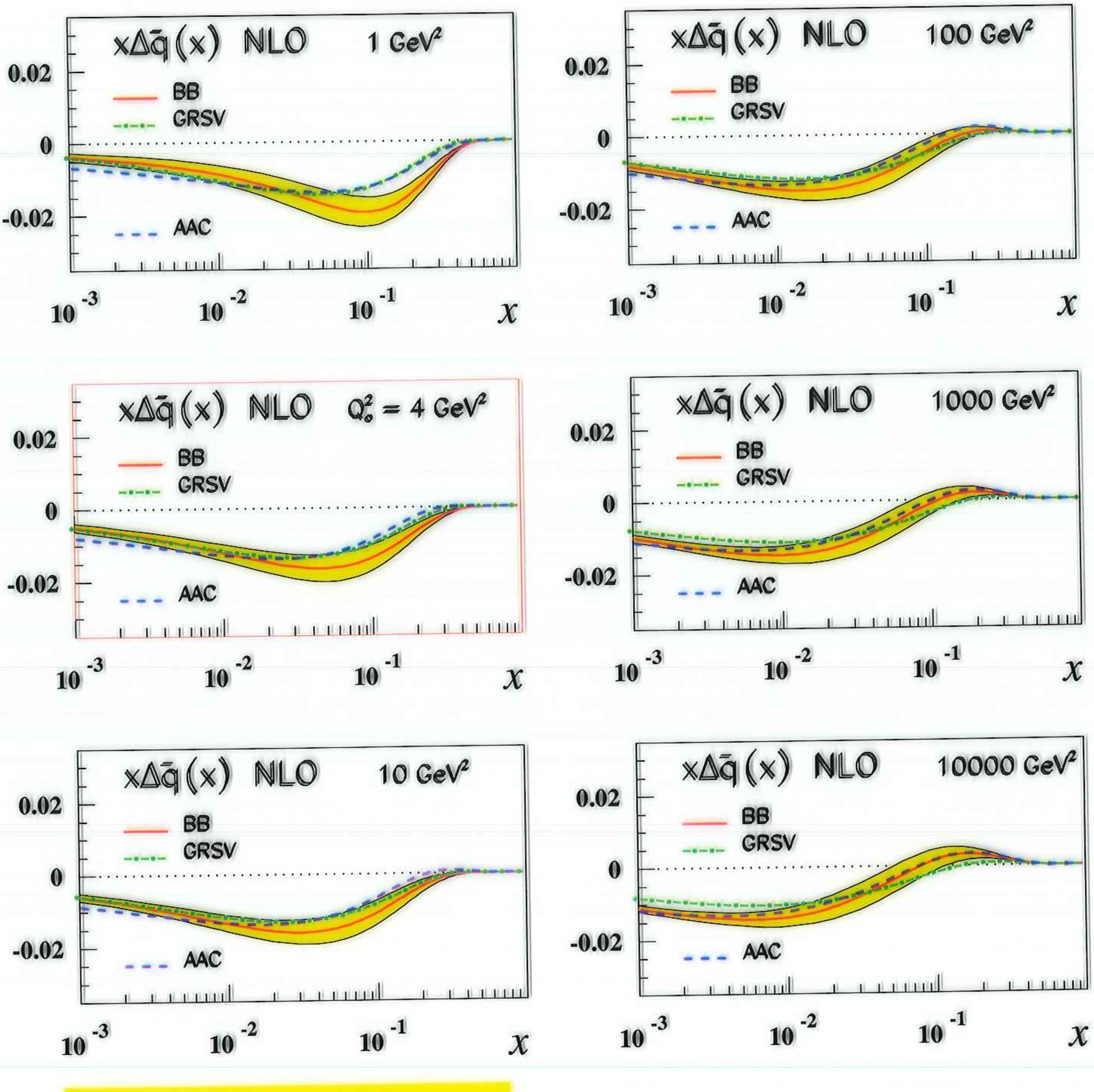
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of Polarized Parton Densities

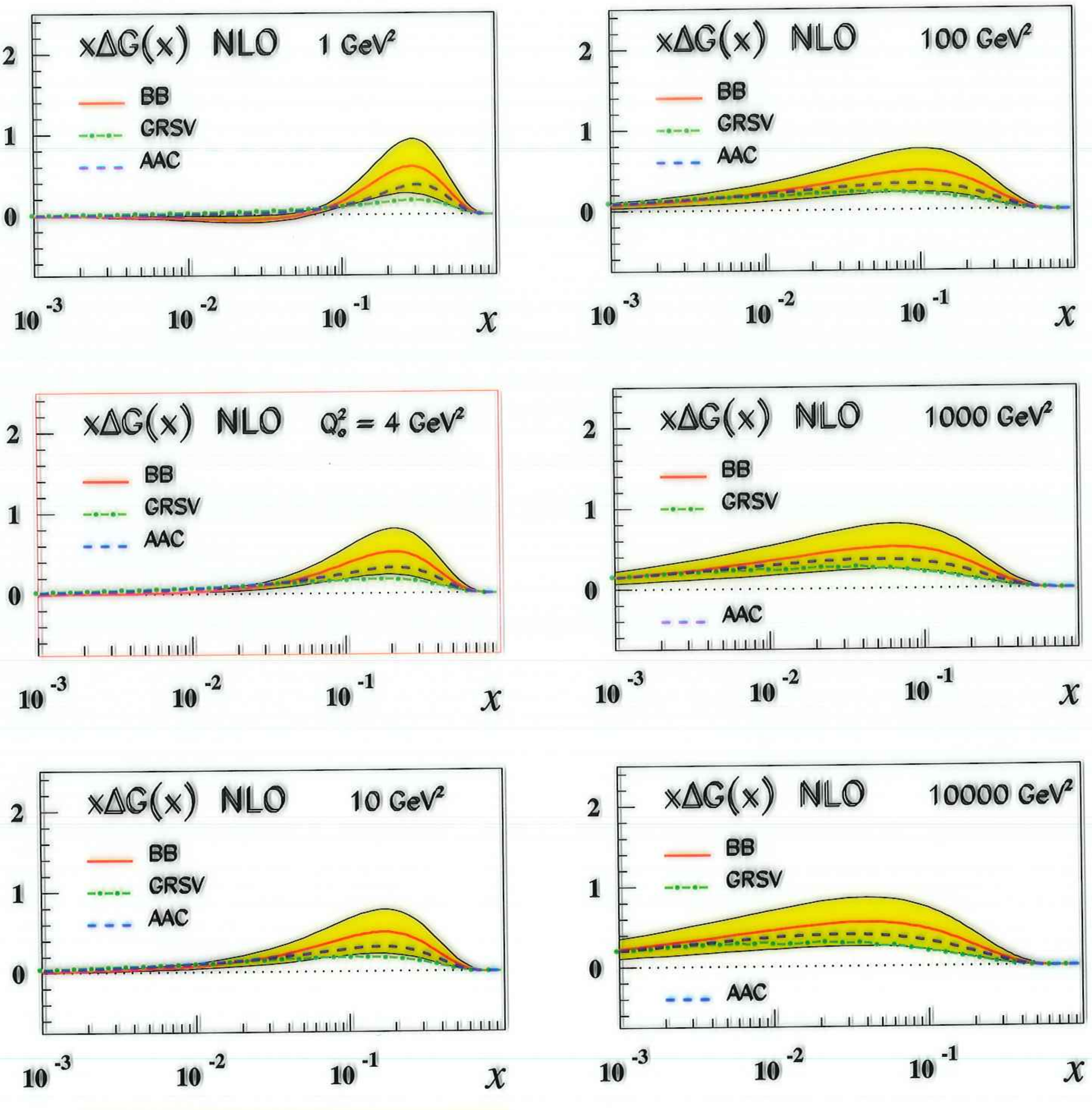
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

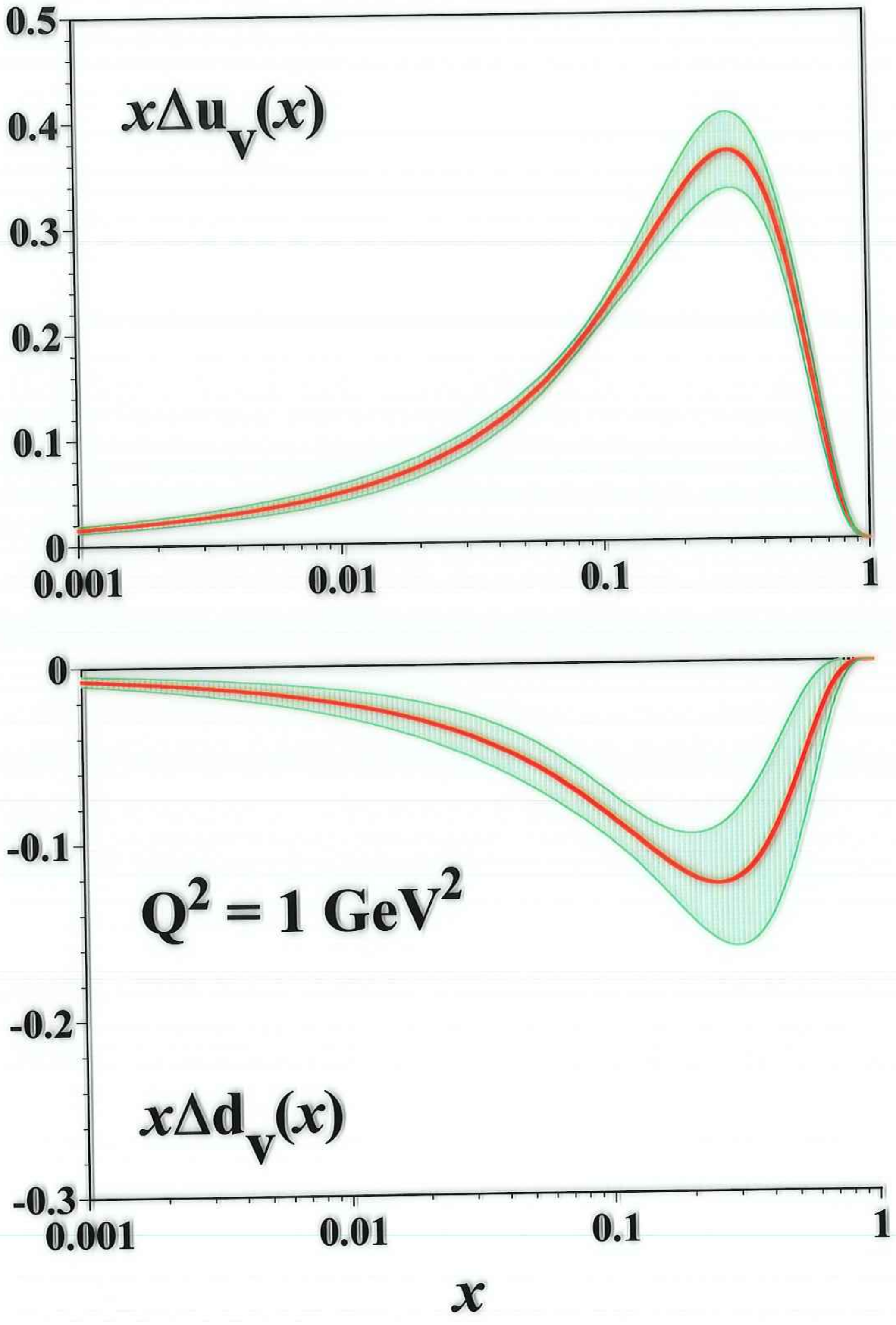
Evolution of Polarized Parton Densities

- 7+1 Parameter Fit based on the Asymmetry Data:



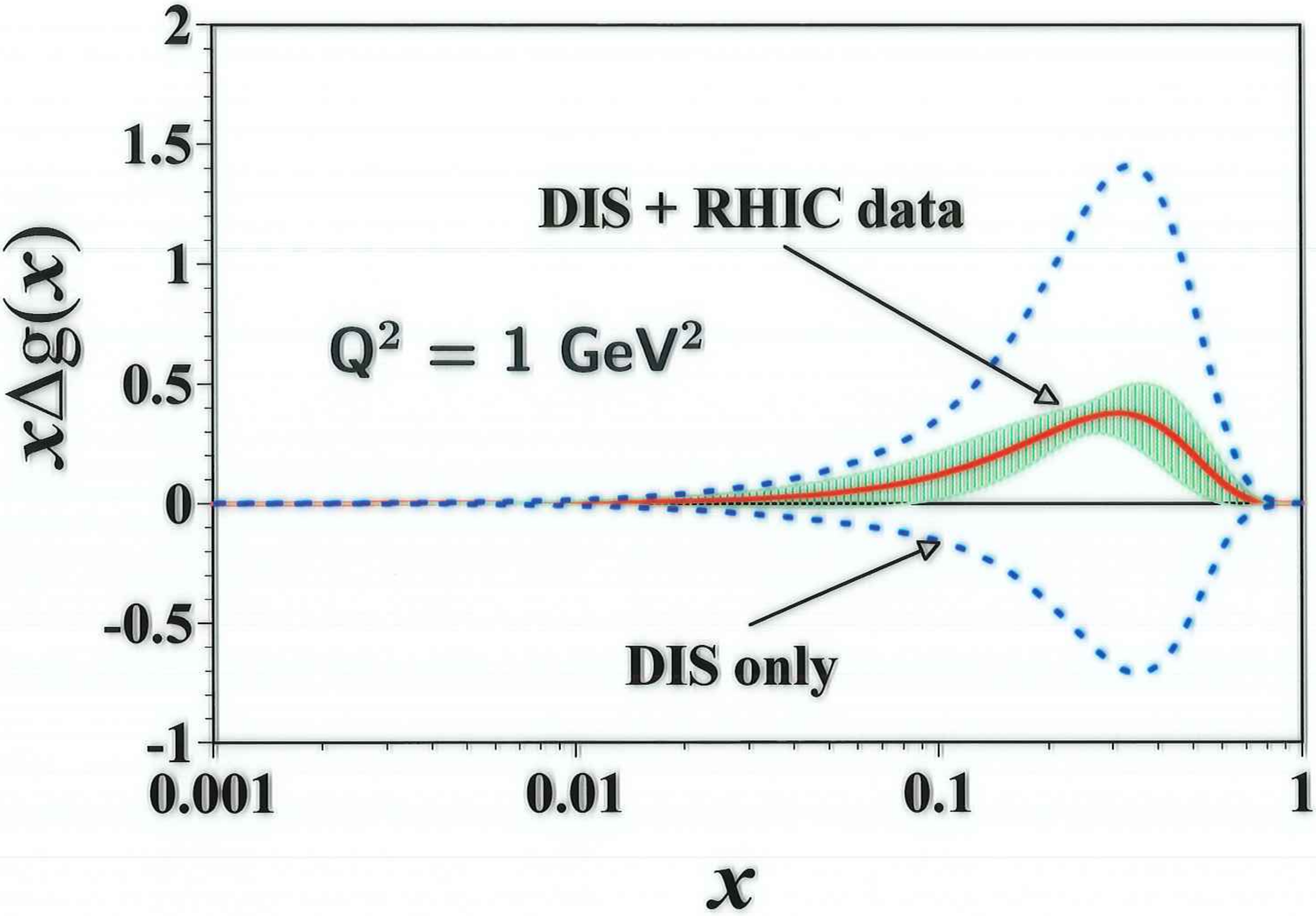
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

AAC: Polarized Valence Quark Densities



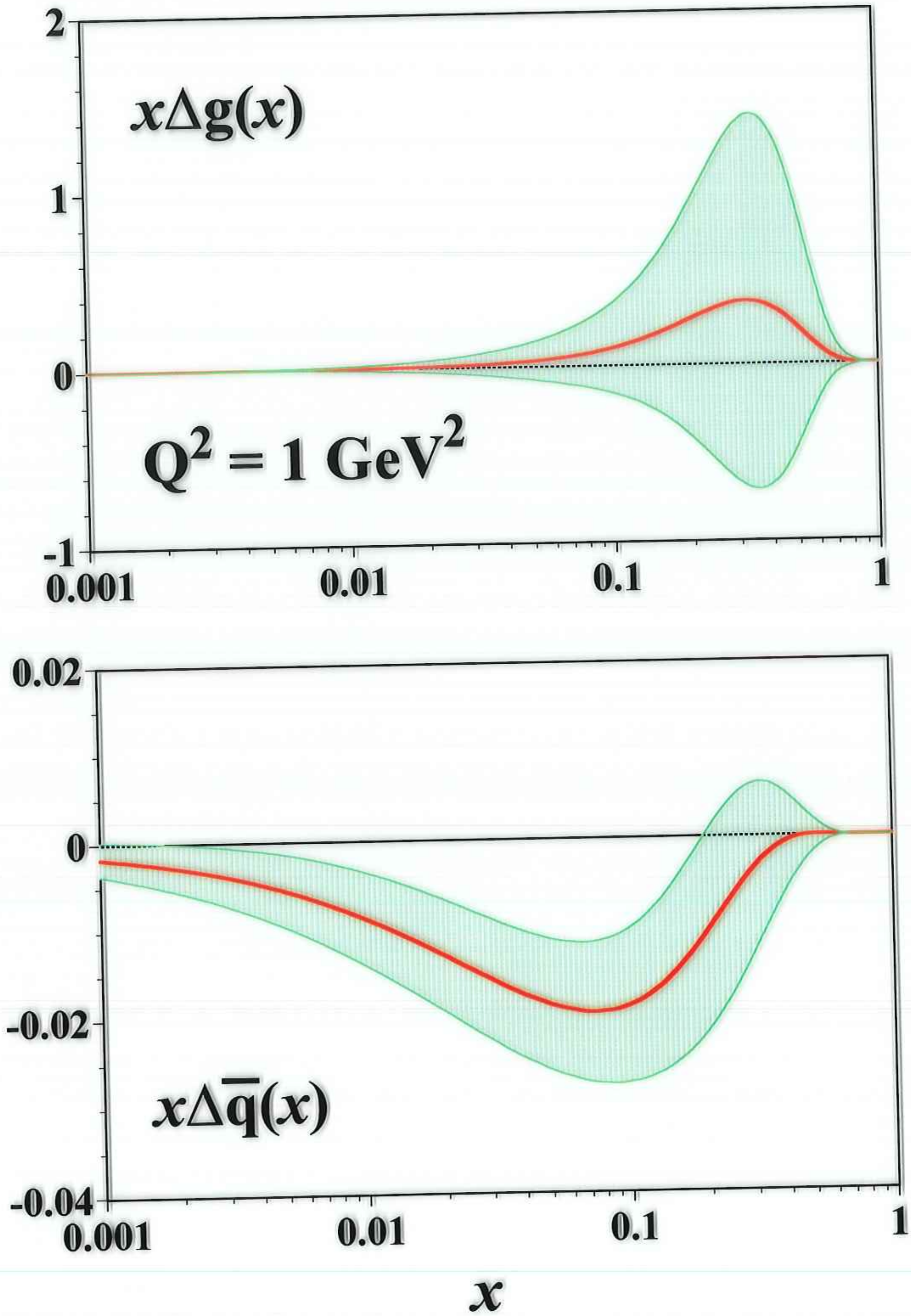
⇒ **Green band:** Fully correlated 1σ statistical error band.

AAC: Polarized Gluon Density with RHIC



⇒ Green band: Fully correlated 1σ statistical error band.

AAC: Polarized Gluon and Sea Densities

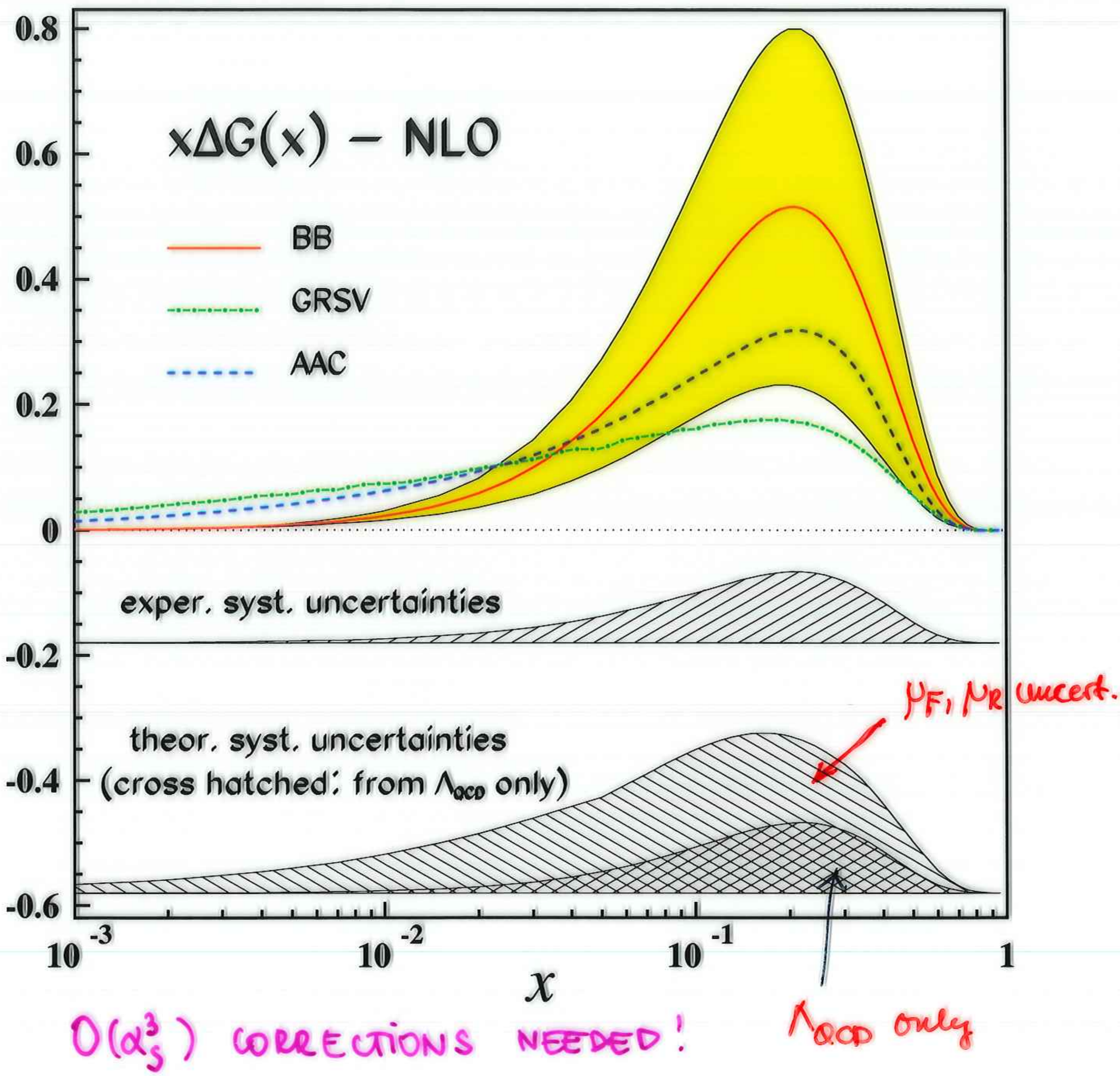


⇒ Green band: Fully correlated 1σ statistical error band.

Theoretical Systematics for the Gluon

- 7+1 Parameter Fit based on the Asymmetry Data:

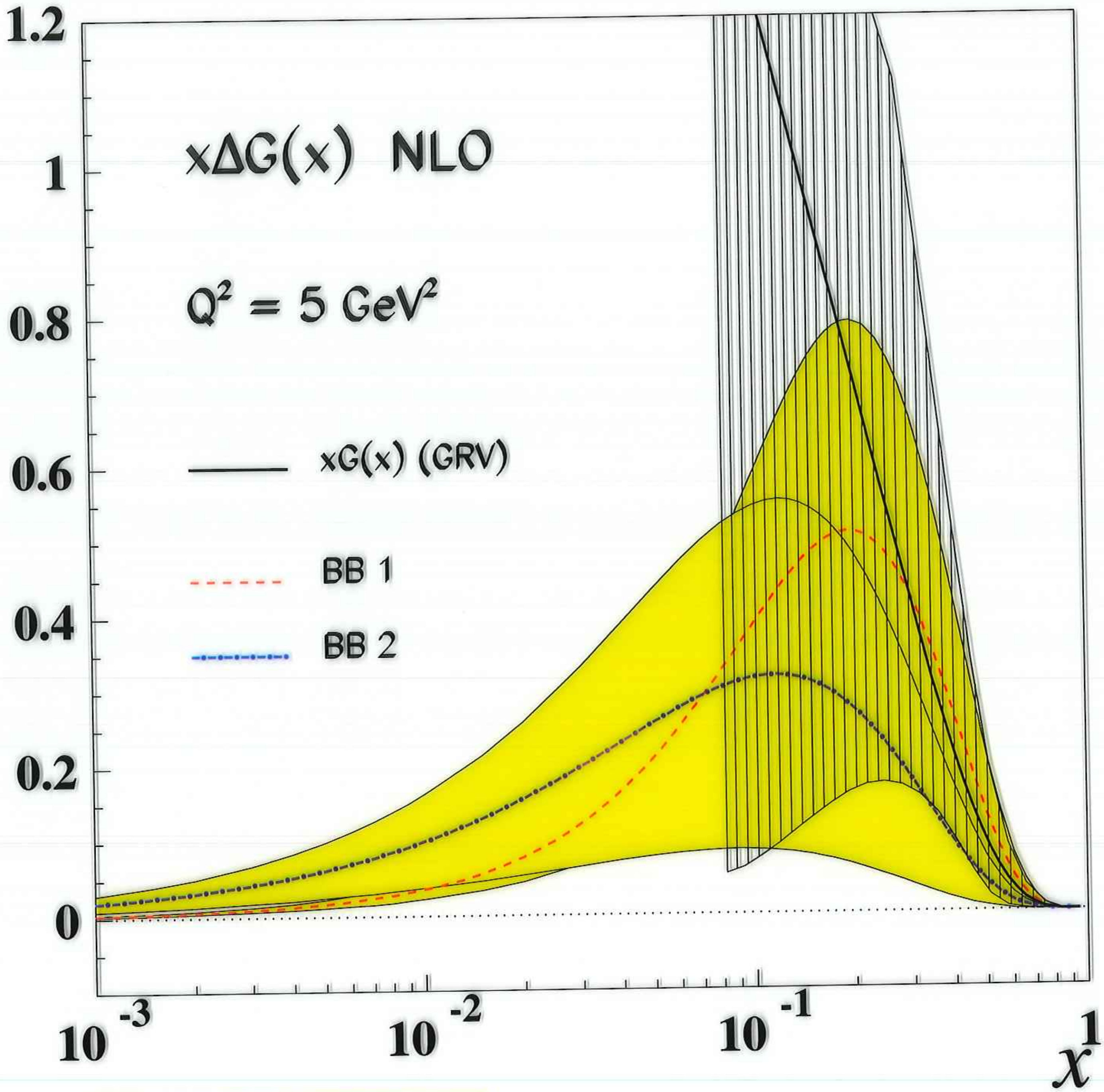
  ⟹ Scenario 1



$O(\alpha_s^3)$ CORRECTIONS NEEDED!

The Polarized Gluon at $Q_0^2 = 5.0\,GeV^2$

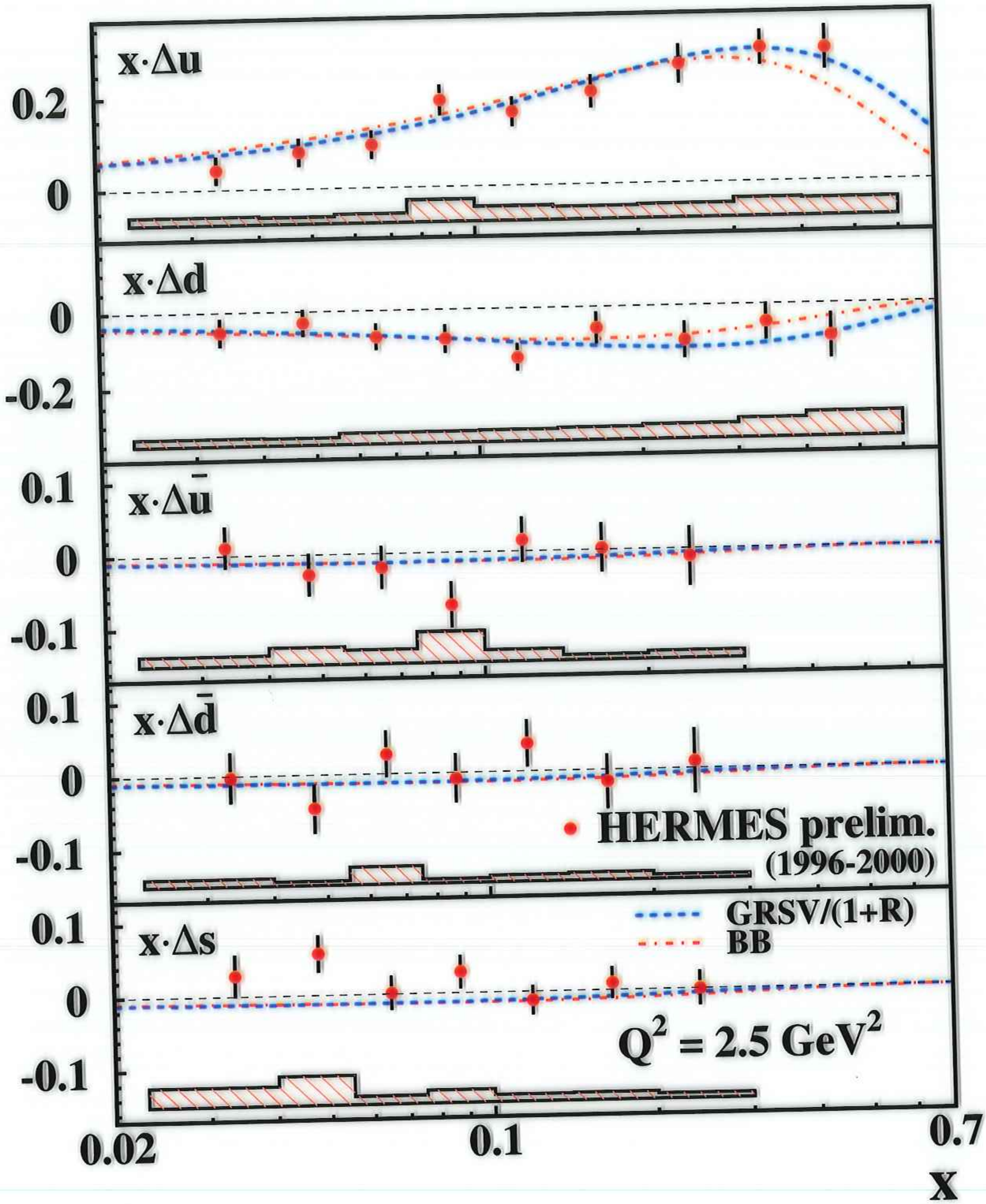
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error bands: Fully correlated 1σ Gaussian error propagation at $Q^2 = 5.0\,GeV^2$.

⇒ Hatched Area: Error Band taken from H1 and laid over the GRV curve.

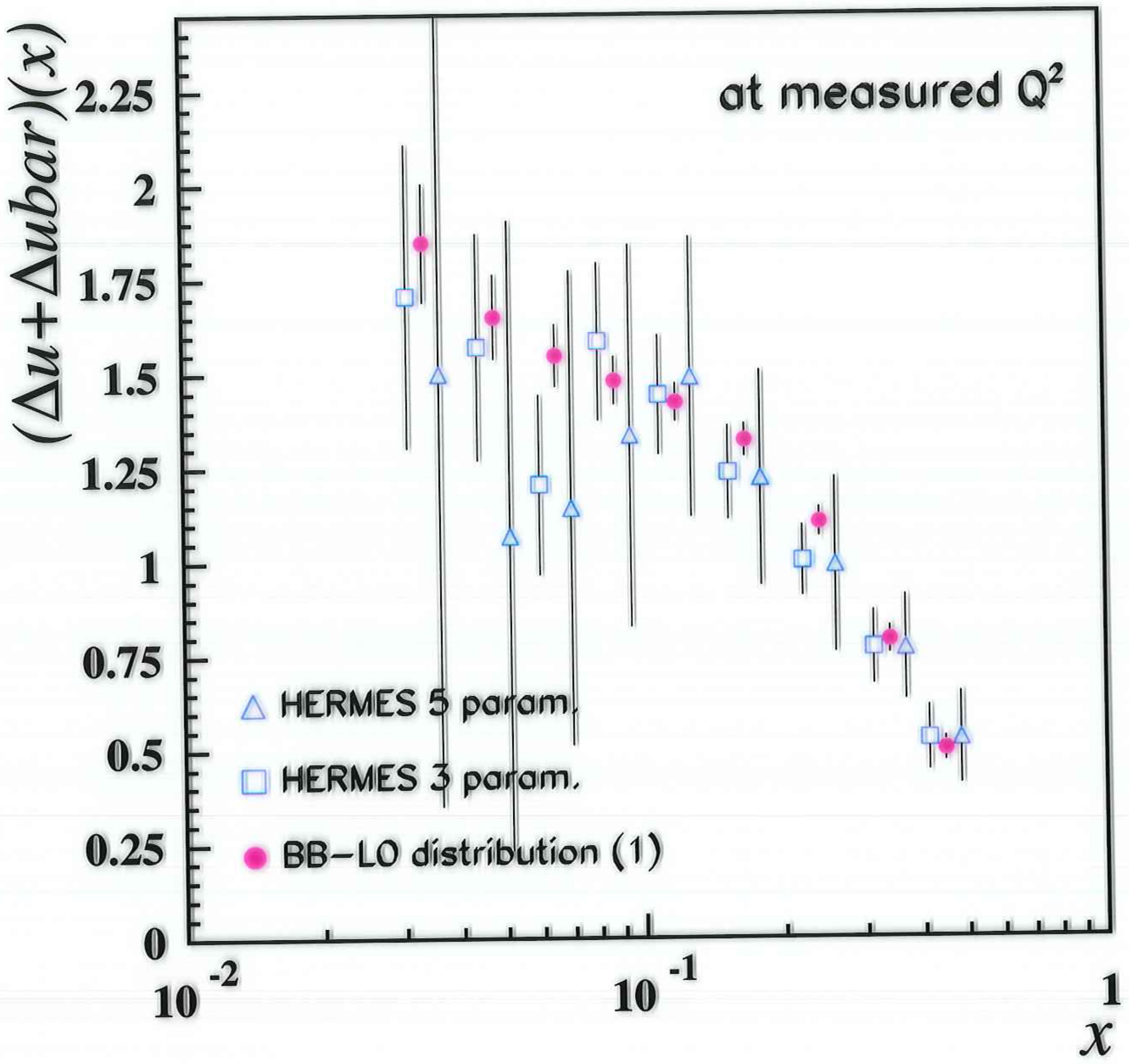
Extraction of Δq from Semi-Incl. Data



⟹ z–range in Semi–Incl. Analysis: $0.2 < z < 0.7$

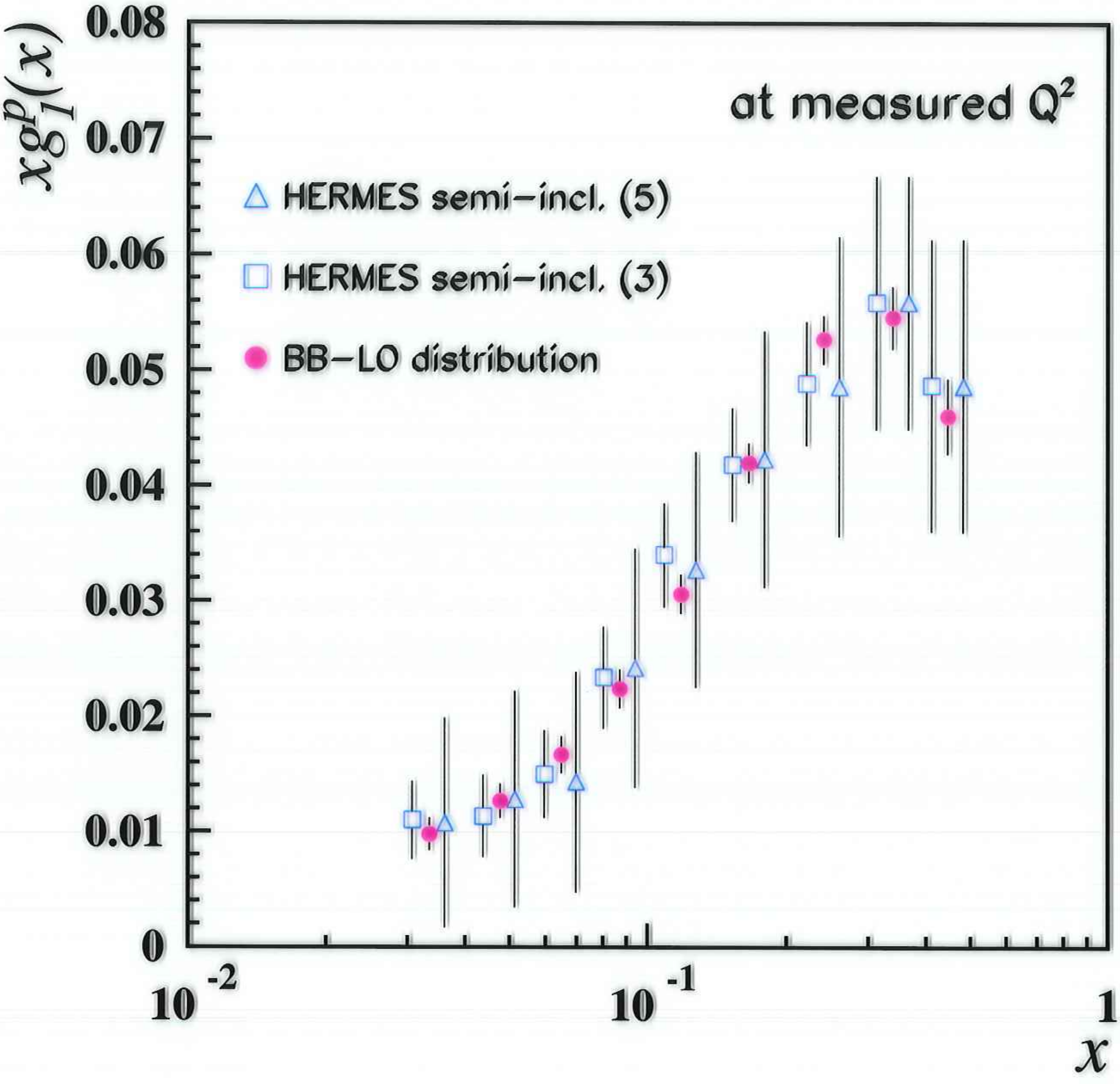
⟹ Symmetric Sea assumed in both the Semi–Inclusive Analysis and the LO QCD Analyses.

Comparison with Δq from Semi-Incl. Data



$\Longrightarrow$ z–range in Semi–Incl. Analysis: $0.2 < z < 0.7$

Comp. with $xg_1^p(x)$ from Semi-Incl. Data



⟹ z–range in Semi–Incl. Analysis: $0.2 < z < 0.7$

Sum Rules and Integral Relations:

Twist 2:

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

Wandzura, Wilczek, 1977;

Piccione, Ridolfi 1998; J.B., A. Tkabladze, 1998 : with TM

$$g_3(x,Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4(y,Q^2)$$

J.B., N. Kochelev, 1996; J.B., A. Tkabladze, 1998 : with TM

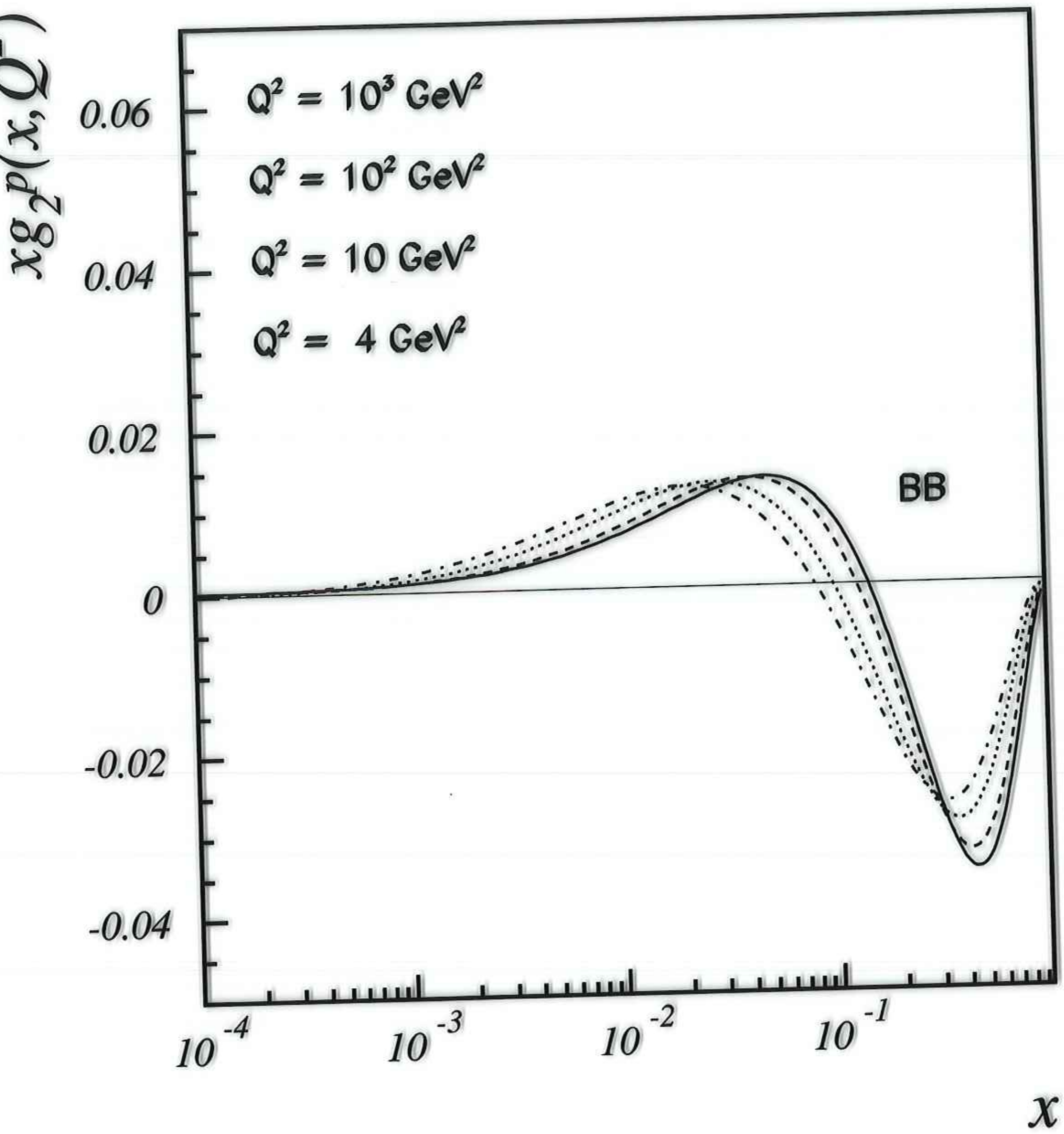
Twist 3:

Include nucleon mass effects.

J.B., A. Tkabladze, 1998

$$\begin{aligned}
g_1(x,Q^2) &= \frac{4M^2x^2}{Q^2}\left[g_2(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2(y,Q^2)\right] \\
\frac{4M^2x^2}{Q^2} g_3(x,Q^2) &= g_4(x,Q^2)\left(1 + \frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g_4(y,Q^2) \\
2x g_5(x,Q^2) &= -\int_x^1 \frac{dy}{y} g_4(y,Q^2)
\end{aligned}$$

$g_2^{\text{light}}(x, Q^2)$



$$g_2^{\text{II}}(x, Q^2) = -g_1^{\text{II}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, g_1^{\text{II}}(x, Q^2)$$

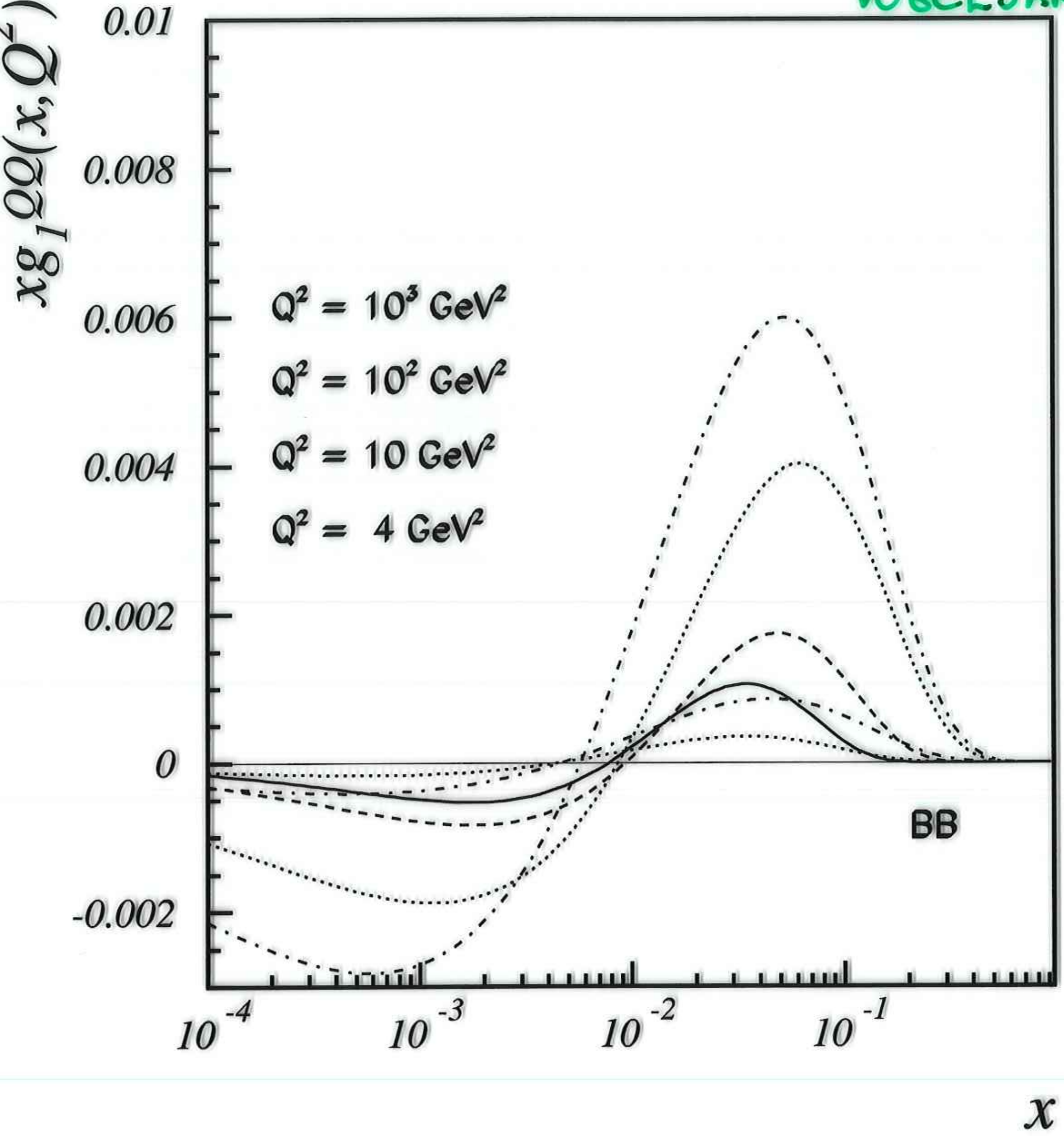
WW - RELATION

$$\int_0^1 dz\, g_2^{\text{II}}(z, Q^2) = 0$$ BC - RELATION

$g_1^{Q\overline{Q}}(x, Q^2)$

J.B., V. Ravindran, and W.L. van Neerven, hep-ph/0304292, Phys. Rev. D to appear.

WATSON VOGELSANG



LO

Upper lines: $c\bar{c}$, Lower lines $b\bar{b}$

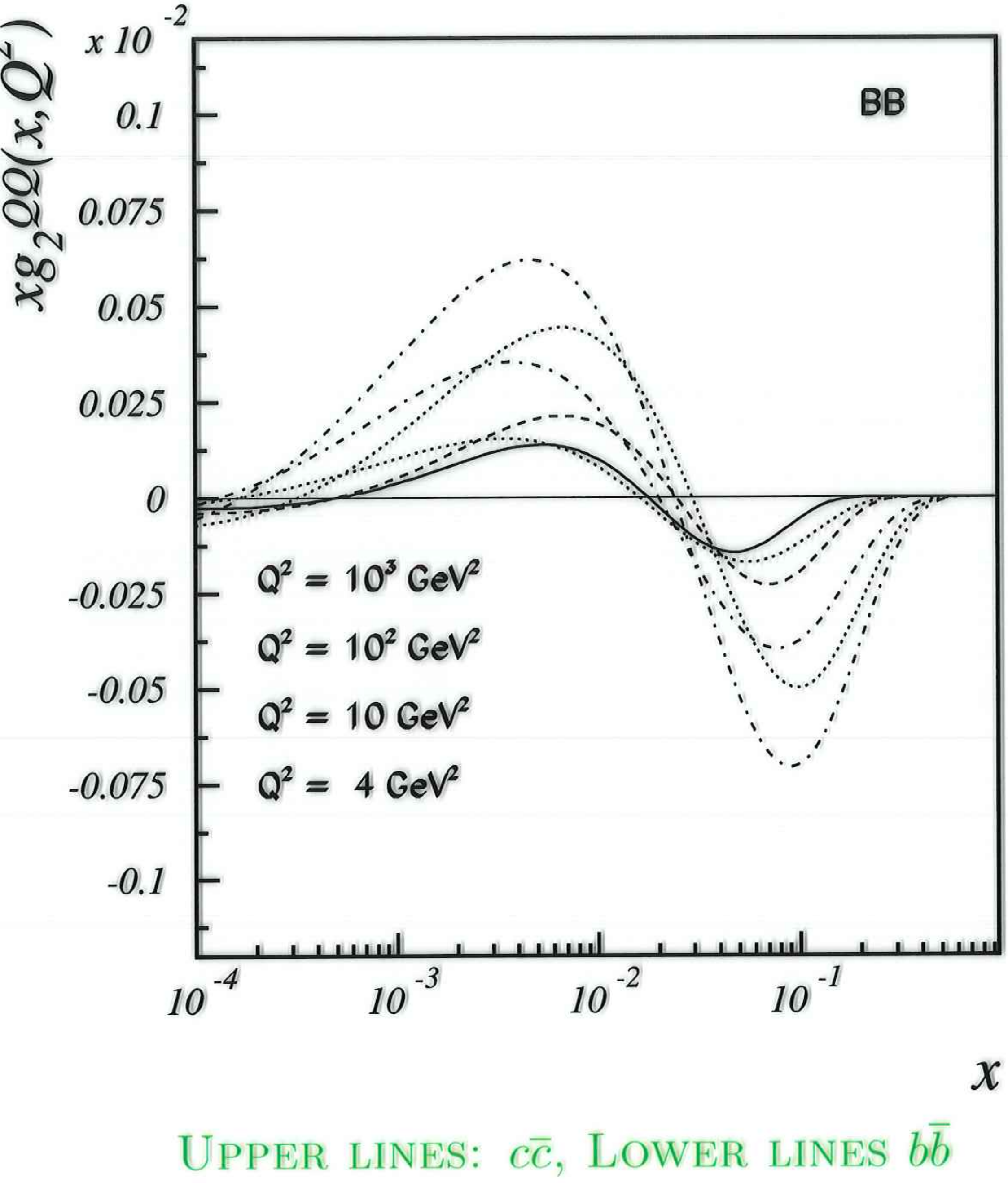
$m_c = 1.5$ GeV $\quad m_b = 4.5$ GeV

$$\int_0^1 dz\, g_1^{Q\bar{Q}}(z, Q^2) = 0$$

LO

NLO ?

$g_2^{Q\overline{Q}}(x, Q^2)$



UPPER LINES: $c\bar{c}$, LOWER LINES $b\bar{b}$

$m_c = 1.5$ GEV $\quad m_b = 4.5$ GEV

$$g_2^{\mathrm{II}\,Q\overline{Q}}(x, Q^2) = -g_1^{\mathrm{II}\,Q\overline{Q}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, g_1^{\mathrm{II}\,Q\overline{Q}}(z, Q^2)$$

System : $g_1(x,Q^2), \partial g_1/\partial t(x,Q^2)$

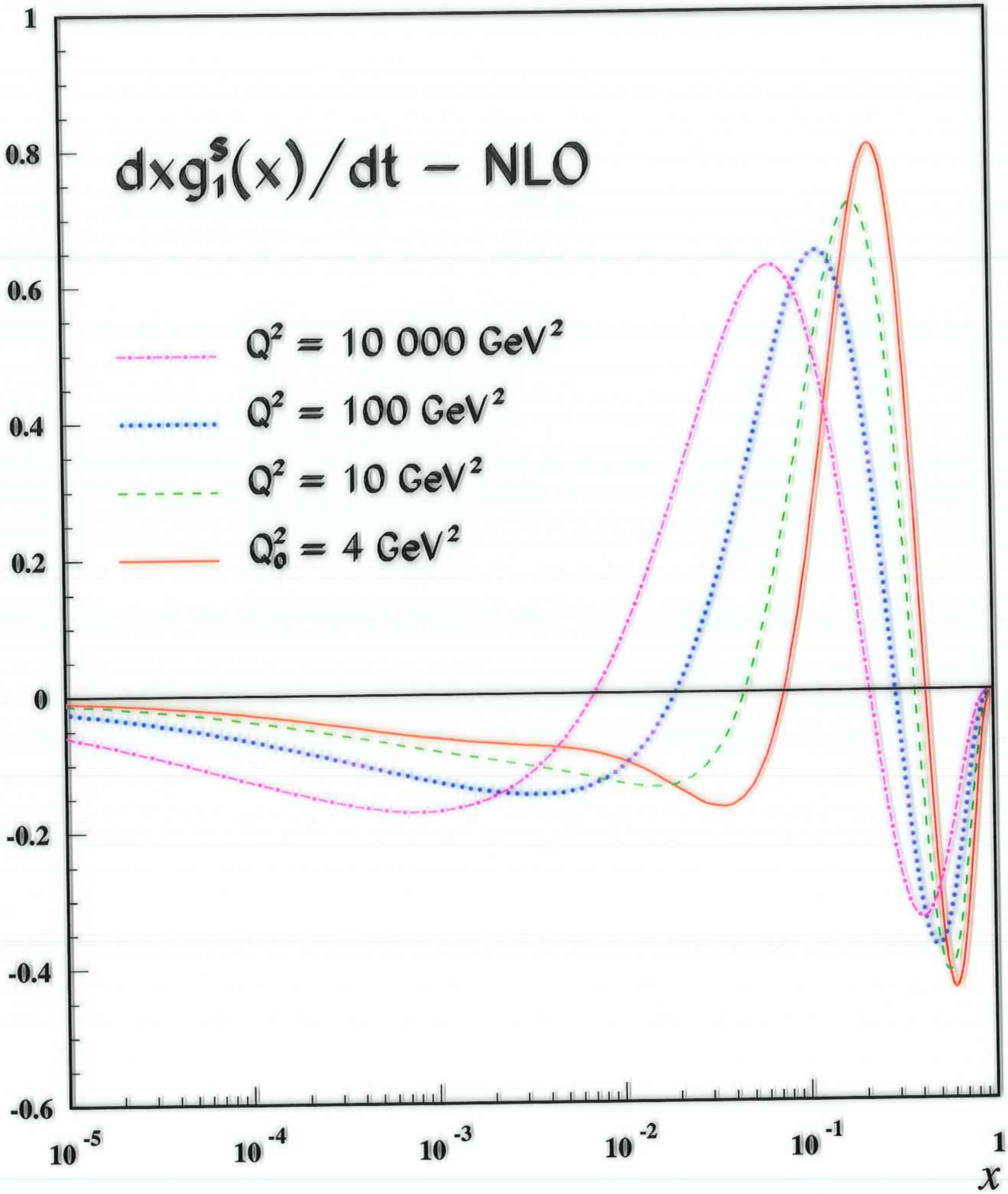
Leading Order :

$$
\begin{aligned}
K_{22}^{N(0)} &= 0 \\
K_{2d}^{N(0)} &= -4 \\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned}
$$

Next-to-Leading Order : [W. Furmanski and R. Petronzio, Z. Phys. **C 11** (1982) 293.]

$$
\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right] \\
&\quad - \frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right) \\
&\quad + \frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad - \frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\gamma_{qq}^{N(0)2} - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
&\quad - \frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right) \\
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 \\
&\quad - \frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}
$$

$\partial x g_1^S/\partial t(x, Q^2)$ and shift of $\Lambda_{QCD}^{(4)}$



$Sc1: \ \Lambda_{QCD}^{(4)} : 0.235 \rightarrow 0.223\ , \ \alpha_s(M_Z^2) : 0.113 \rightarrow 0.112$

$Sc2: \ \Lambda_{QCD}^{(4)} : 0.240 \rightarrow 0.228\ , \ \alpha_s(M_Z^2) : 0.114 \rightarrow 0.113$

Fac. Scheme Invariant Combinations

- Instead of PROCESS-INDEPENDENT SCHEME-DEPENDENT Evolution Equations for PARTONS one may think of PROCESS-DEPENDENT SCHEME-INDEPENDENT Evolution Equations for OBSERVABLES, F_A, F_B.

⇒ The input densities are measured! Control over the input directly.
⇒ No ΔG–Ansatz necessary.
⇒ A one parameter fit only – Λ_{QCD}.

Evolution Equations : [J. Blümlein, V. Ravindran, and W. L. van Neerven, Nucl. Phys **B586** (2000) 349.]

$$\frac{\partial}{\partial t}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{pmatrix}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}$$

evolution variable :

$$t = -\frac{2}{\beta_0}\log\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right)$$

⟹ The evolution kernels K_{IJ}^N are also Physical Quantities! The Factorization Scheme Independence holds order by order.

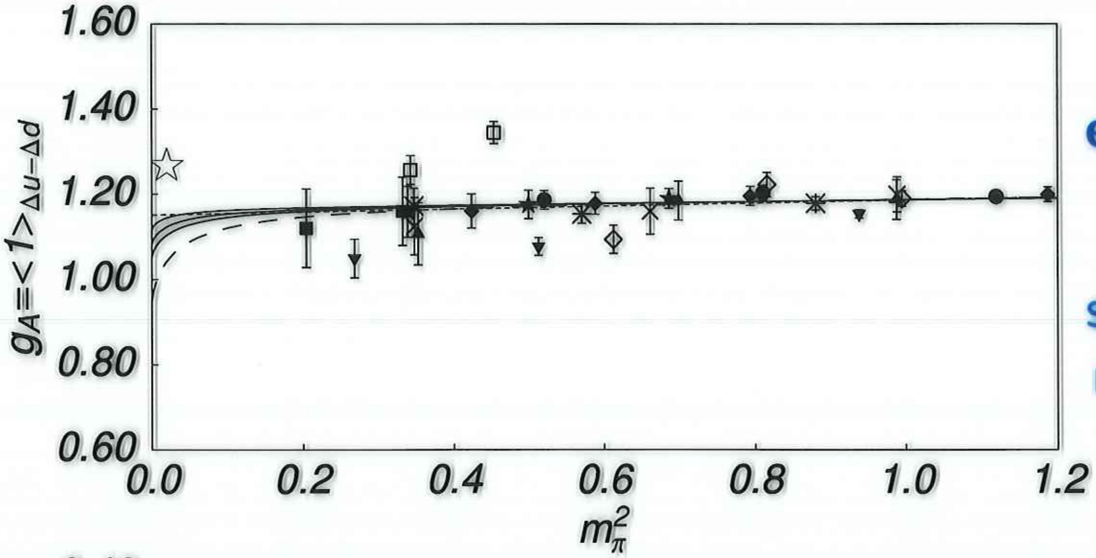
The Renormalization Scale Dependence disappears only with more higher orders.

⟹ A possible choice: $F_A = g_1$ and $F_B = \partial g_1/\partial t$.

'Prediction' of Moments

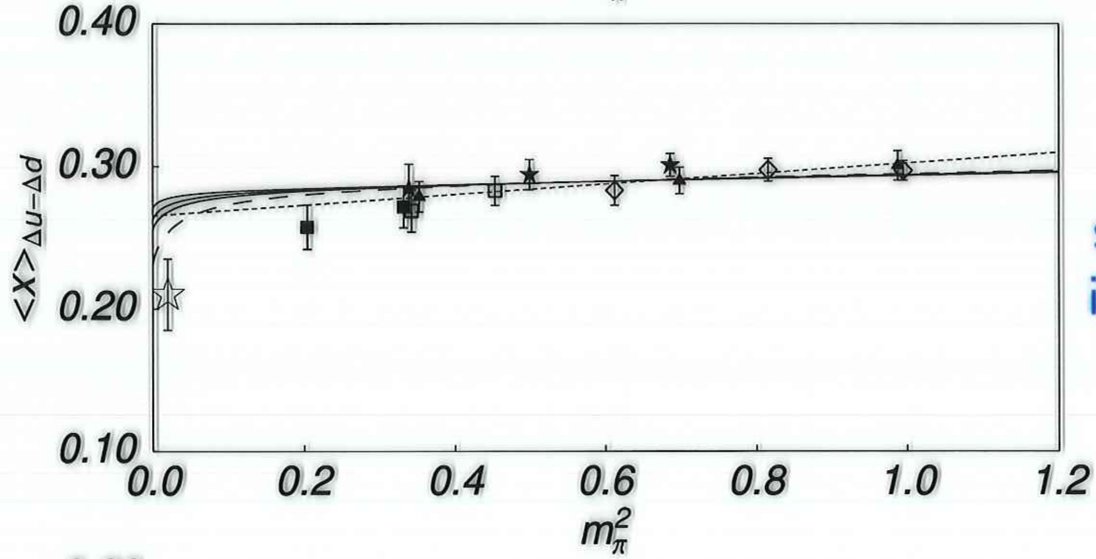
	n	QCD Scenario 1	
		value at $Q^2 = 4\,GeV^2$	value out of measured range
Δu	-1	0.851 ± 0.075	0.152|4E−4
	0	0.160 ± 0.014	8E−4|3E−4
	1	0.055 ± 0.006	1E−5|3E−4
	2	0.024 ± 0.003	0|3E−4
Δd	-1	-0.415 ± 0.124	-0.144|-7E−5
	0	-0.050 ± 0.022	-7E−4|-6E−5
	1	-0.015 ± 0.009	-1E−5|-5E−5
	2	-0.006 ± 0.005	0|-5E−5
$\Delta \overline{q}$	-1	-0.074 ± 0.017	-0.04|0
	0	-0.003 ± 0.001	-2E−4|0
	1	$-4E{-}4 \pm 1E{-}4$	0|0
	2	$-8E{-}5 \pm 2E{-}5$	0|0
ΔG	-1	1.026 ± 0.549	0.04|1E−5
	0	0.184 ± 0.103	5E−4|1E−5
	1	0.050 ± 0.028	1E−5|1E−5
	2	0.017 ± 0.010	0|1E−5

Lattice: The lowest moments of $\Delta u - \Delta d$

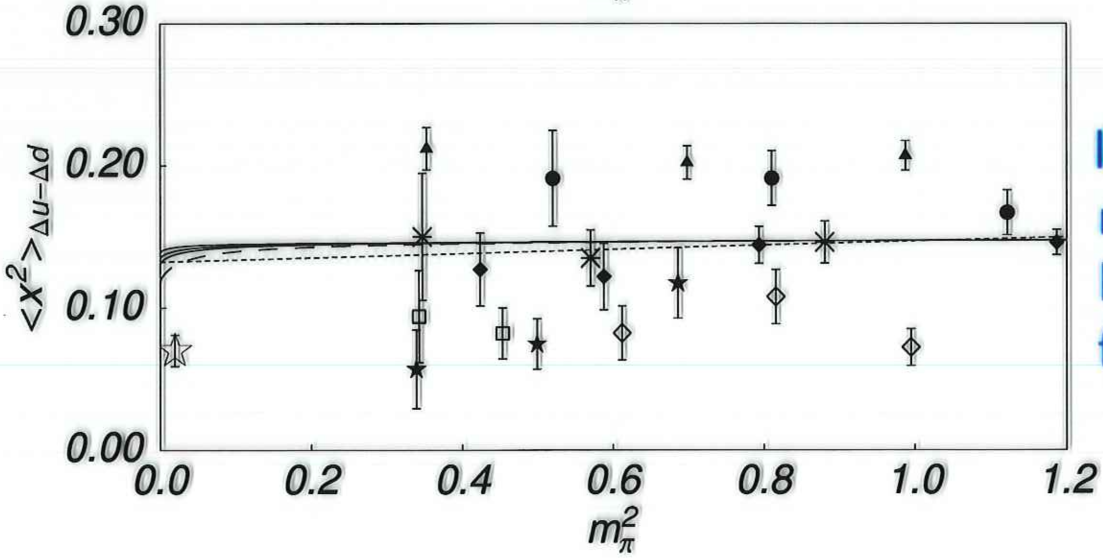


extrapolations:

short–dashed: naive linear



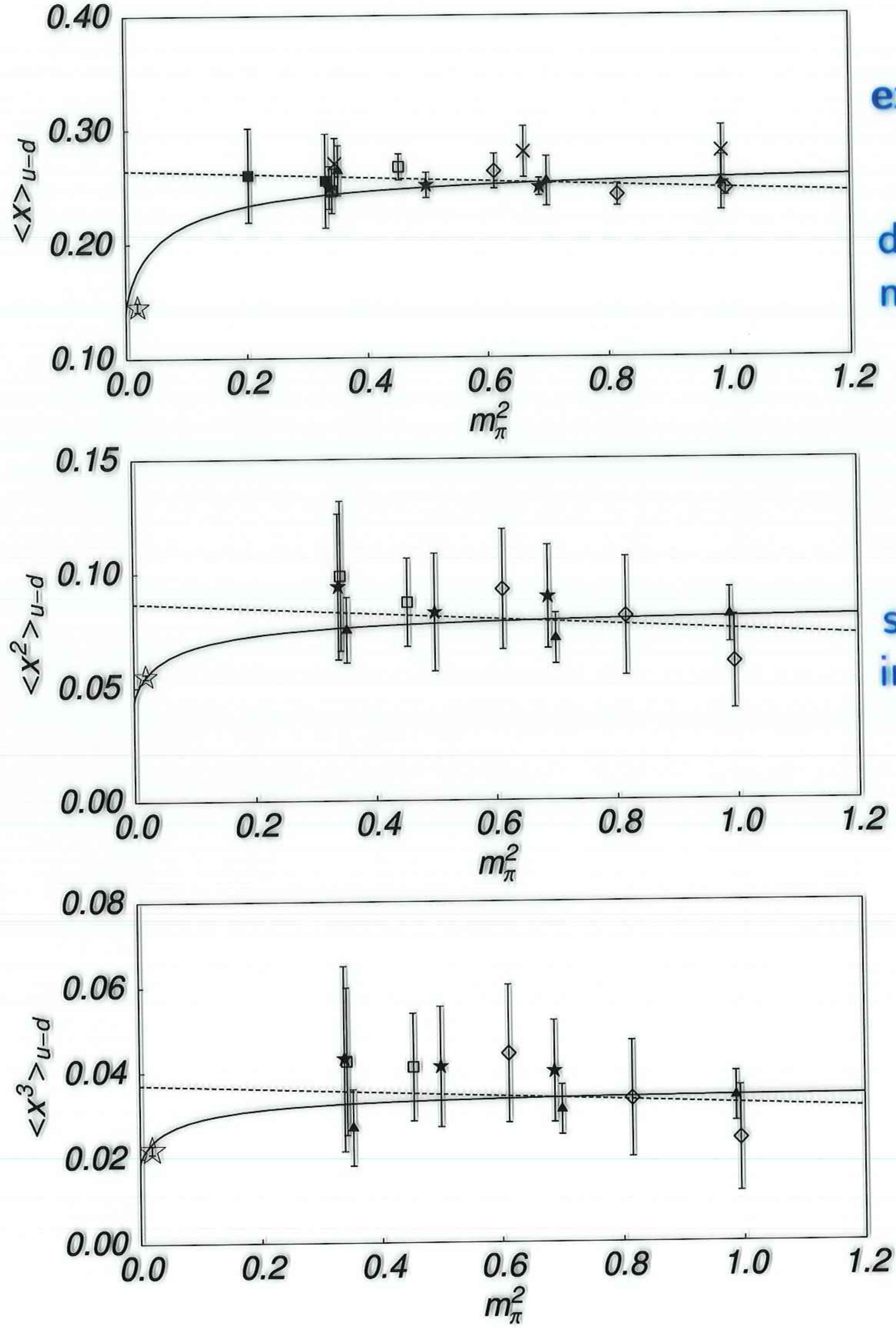
solid: improved chiral



long-dashed: no Δ and LNA coeff. from χPT

Ref.: M. Detmold, W. Melnitchouk, A.W. Thomas; hep-lat/0206001.

Lattice: The lowest moments of $u - d$



extrapolations:

dashed:
naive linear

solid:
improved chiral

Ref.: M. Detmold, W. Melnitchouk, A.W. Thomas; hep-lat/0206001.

Comparison of Moments

Δf	n	`QCD Scenario 1` moment at $Q^2 = 4\,GeV^2$	lattice results: QCDSF	lattice results: LHPC/SESAM
Δu_v	–1	0.926 ± 0.071	0.889(29)	0.860(69)
	0	0.163 ± 0.014	0.198(8)	0.242(22)
	1	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	–1	-0.341 ± 0.123	-0.236(27)	-0.171(43)
	0	-0.047 ± 0.021	-0.048(3)	-0.029(13)
	1	-0.015 ± 0.009	-0.028(2)	0.001(25)
Δu–Δd	–1	1.267 ± 0.142	1.14(3)	1.031(81) *
	0	0.210 ± 0.025	0.246(9)	0.271(25)
	1	0.070 ± 0.011	0.069(9)	0.115(49)

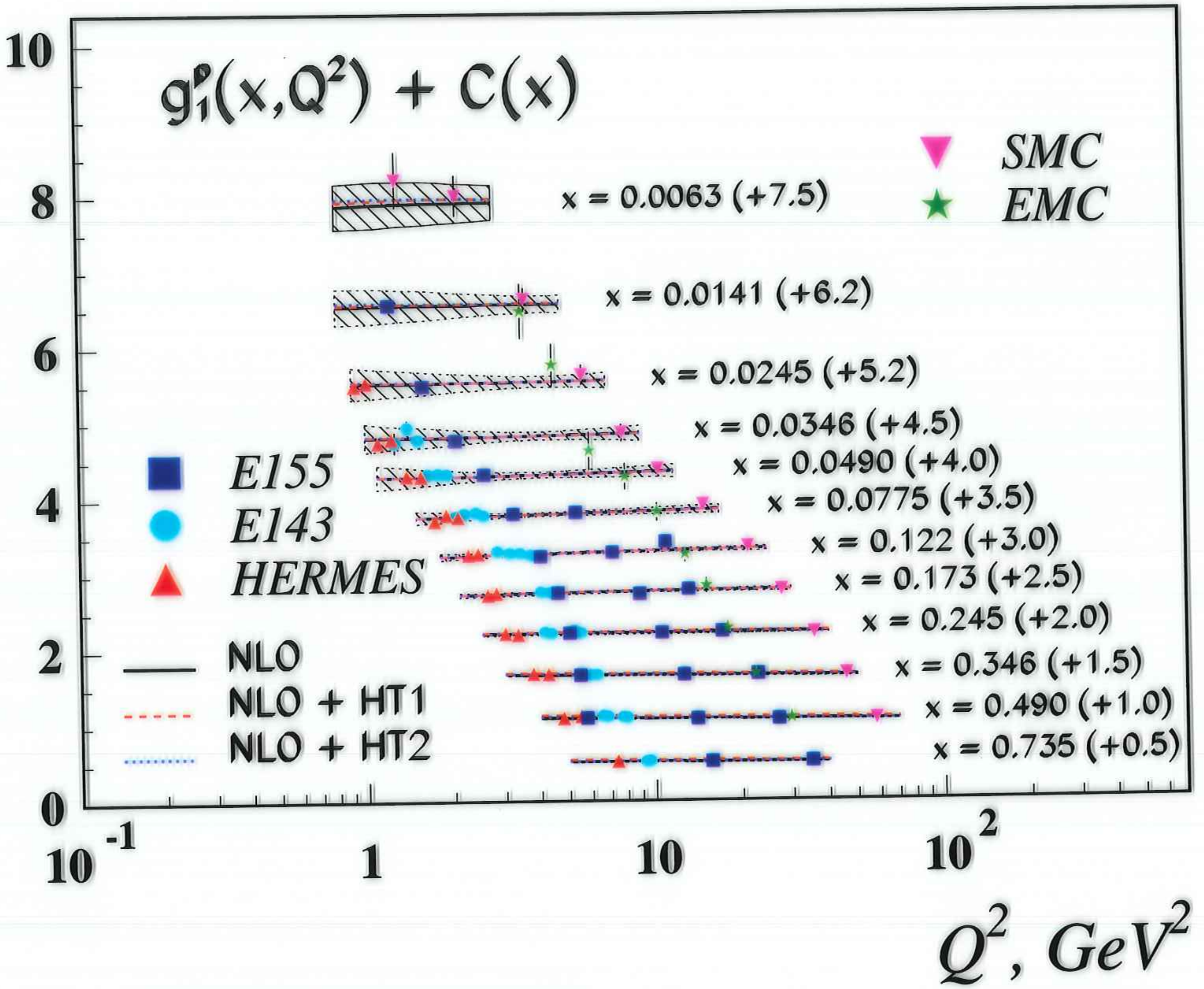
$$\Longrightarrow \Gamma_{\Delta f}(Q^2) = \int_0^1 x^{n+1} \Delta f(x, Q^2) dx$$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4\,GeV^2$. For the $n = 0, 1$ values of the QCDSF Coll. no continuum extrapolation was performed.

[**Refs:** M.Göckeler et al., QCDSF Coll., Phys.Rev. **D53** (1996) 2317; Phys.Lett. **B414** (1997) 340; hep-ph/9711245; Phys.Rev. **D63** (2001) 074506; S.Capitani et al., Nucl.Phys.(Proc. Suppl.) **B79** (1999) 548; S.Güsken et al., SESAM Coll., hep-lat/9901009; D.Dolgov et al., LHPC and SESAM Coll., hep-lat/0201021.]

* IMPROVED BY : SASAKI et al. hep-lat/0306007

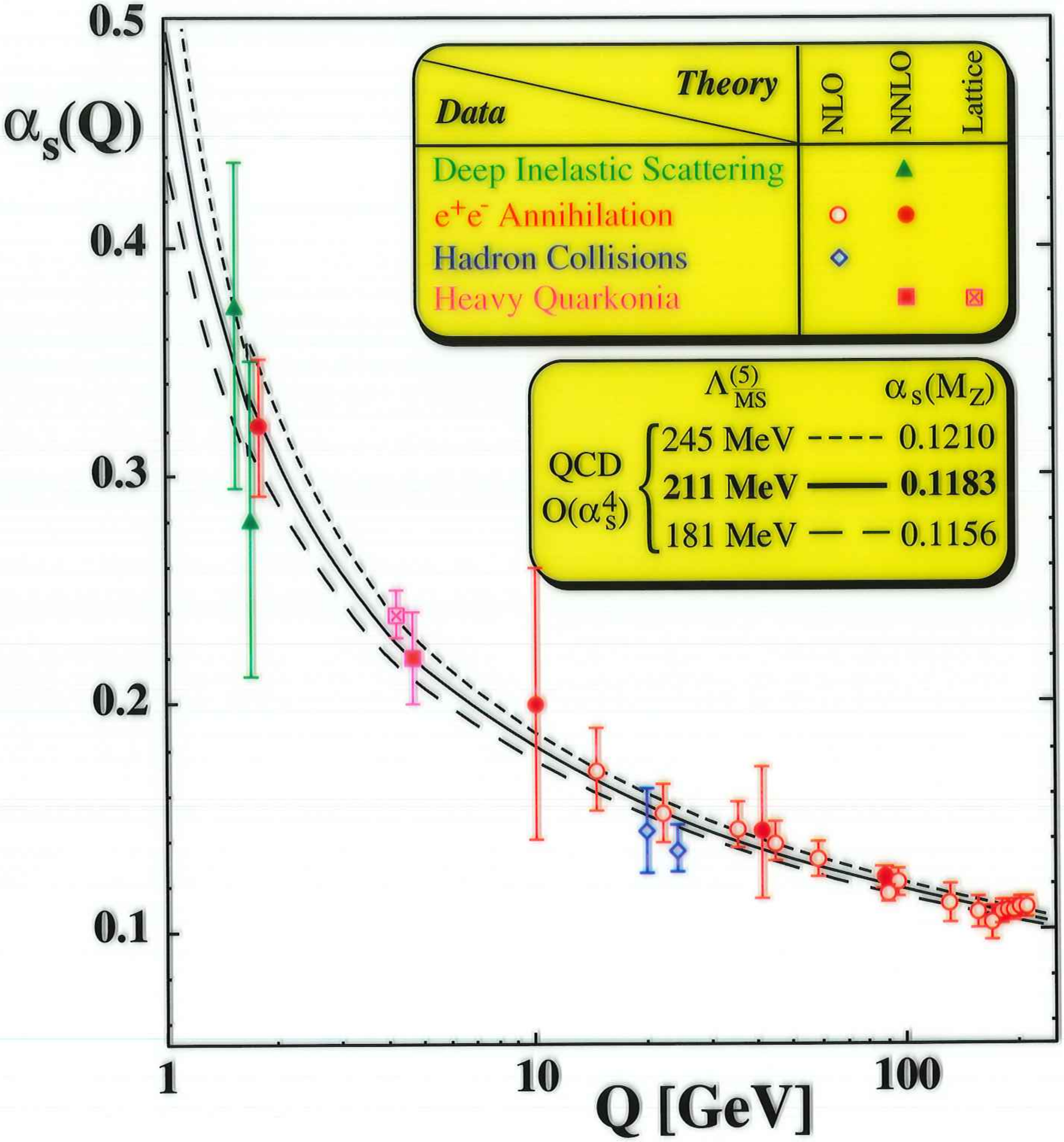
$g_1^p(x)$ + Higher Twist - Scenario 1



⇒ Hatched error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

- Higher Twist Contribution: $g_1(x,Q^2)[1 + HT(x,Q^2)]$
  - HT1: $(1/Q^2)(x^a(1-x)^b)$
  - HT2: $(1/Q^2)(a + bx + cx^2)$

The QCD Running Coupling Constant



S. Bethke, 2002.

7+1 parameter NLO fit: $\Lambda^{(4)}_{QCD} \Rightarrow \alpha_s(M_Z^2)$

$\Lambda^{(4)}_{QCD}$ [Gev]	Scenario 1		Scenario 2	
	value	error	value	error
FS/RS=1.0/1.0	0.235	± 0.053	0.240	± 0.060
FS/RS=0.5/1.0	0.188	– 0.047	0.195	– 0.045
FS/RS=2.0/1.0	0.296	+ 0.061	0.298	+ 0.058
FS/RS=1.0/0.5	0.349	+ 0.114	0.363	+ 0.123
FS/RS=1.0/2.0	0.174	– 0.061	0.174	– 0.066

- Sc. 1:

$$\alpha_s(M_Z^2) = 0.113 \;\begin{matrix}+0.004\\-0.004\end{matrix}\;\begin{matrix}+0.004\\-0.004\end{matrix}\;\begin{matrix}+0.008\\-0.005\end{matrix}$$

(fit) (fac) (ren)

- Sc. 2:

$$\alpha_s(M_Z^2) = 0.114 \;\begin{matrix}+0.004\\-0.005\end{matrix}\;\begin{matrix}+0.004\\-0.004\end{matrix}\;\begin{matrix}+0.008\\-0.006\end{matrix}$$

- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

E154: $0.108 - 0.116 \quad (bad\, for \geq 0.120)$

ABFR: $0.120 \;\begin{matrix}+0.004\\-0.005\end{matrix}\;(exp)\;\begin{matrix}+0.009\\-0.006\end{matrix}\;(theor)$

⇒ world average (PDG): 0.118 ± 0.002

⇒ H1 + BCDMS data: [Eur.Phys.J. **C21**(2001)33]

$0.1150 \pm 0.0017(exp) \;\begin{matrix}+0.0009\\-0.0005\end{matrix}\;(model) \pm 0.0050(thy)$

ANGULAR MOMENTUM

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

$$J_g = \Delta g + \text{"}L_g\text{"}$$

DECOMPOS. NOT GAUGE INV.

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} J_q \\ J_g \end{pmatrix} = \frac{\alpha_s}{2\pi}\,\frac{1}{9}\begin{pmatrix} -16 & 3N_f \\ 16 & -3N_f \end{pmatrix}\begin{pmatrix} J_q \\ J_g \end{pmatrix}$$

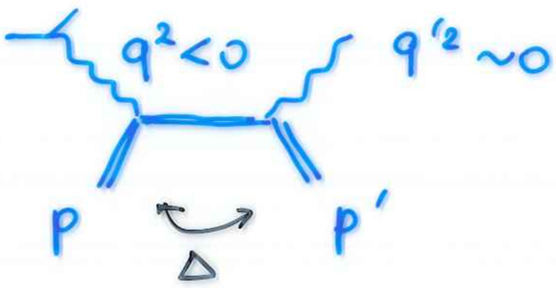
$$\mu^2 \to \infty \qquad J_q \to \frac{1}{2}\,\frac{3n_f}{16+3n_f} \simeq 0.214 \; (N_f = 4)$$

$$J_g \to \frac{1}{2}\,\frac{16}{16+3n_f} \simeq 0.286 \; (N_f = 4)$$

MEASUREMENT: NEED 2^{nd} VECTOR

→ NON-FORWARD SCATTERING

DVCS



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda n/2) \gamma^\mu \psi(\lambda n/2) | p \rangle$$

$$= H(x, \Delta^2, \xi)\, \bar{u}(p') \gamma^\mu u(p)$$

$$+ E(x, \Delta^2, \xi)\, \bar{u}(p') \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} u(p) + \dots$$

$$J_q = \frac{1}{2} \int_{-1}^{+1} dx\, x \left[H_q(x, \Delta^2 = 0, \xi) + E_q(x, \Delta^2 = 0, \xi) \right]$$

↑ extrapolation.

ξ – nonforwardness.

Conclusions

- An LO and NLO QCD Analysis of the current World-Data of Polarized Structure Functions was performed.

- New parametrizations of the parton densities including their fully correlated 1σ Error Bands were derived. They are available via a fast FORTRAN program for the range:

$$1\ GeV^2 < Q^2 < 10^6\ GeV^2 \text{ and } 10^{-9} < x < 1.$$

- The following values for $\alpha_s(M_Z^2)$ were obtained:

  – Scenario 1:

$$\alpha_s(M_Z^2) = 0.113 \begin{array}{c} +0.004 \\ -0.004 \end{array} (\mathrm{fit}) \begin{array}{c} +0.004 \\ -0.004 \end{array} (\mathrm{fac}) \begin{array}{c} +0.008 \\ -0.005 \end{array} (\mathrm{ren})\ ,$$

  – Scenario 2:

$$\alpha_s(M_Z^2) = 0.114 \begin{array}{c} +0.004 \\ -0.005 \end{array} (\mathrm{fit}) \begin{array}{c} +0.004 \\ -0.004 \end{array} (\mathrm{fac}) \begin{array}{c} +0.008 \\ -0.006 \end{array} (\mathrm{ren})\ ,$$

  compatible with results from other QCD analyses and with the world average.

Conclusions (cont'd)

- First steps in a factor. scheme invariant QCD evolution based on the structure function $g_1(x, Q^2)$ and $\partial g_1(x, Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$.

  Such an analysis is a very promising way to proceed in the future, since it allows to extract $\Lambda_{\rm QCD}$ fixing all the input distributions by direct measurement.

- Comparing the QCD low moments with values from Lattice simulations the errors improved during recent years and the values became closer. However, more work has yet to be done in the future on systematic effects and even more precise experimental data are welcome to improve precision.

9. Future Avenues

HERA:

- Collect high luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.
- Measure : $F_L(x,Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized.

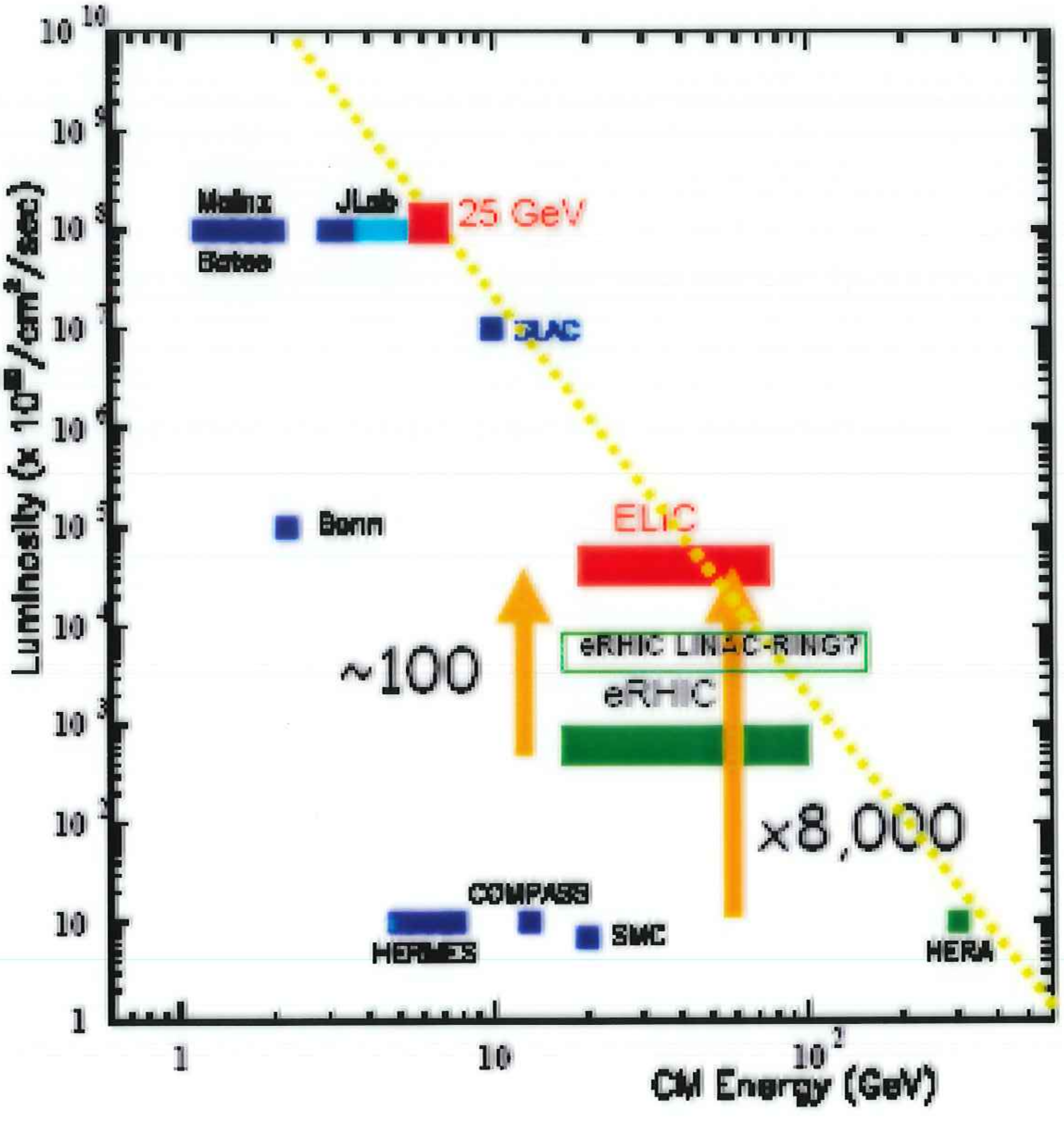
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

ELIC:

- High precision measurements in the medium x domain; both unpolarized and polarized

The quest for large luminosity !



- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\text{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor.[Theory & Experiment]
- Flavor Structure of Sea-Quarks: More studies needed.[All Experiments]
- Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed.[Theory]
- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions [High Precision polarized experiments, JLAB, EIC]
- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Transversity, Angular Momentum.
- Calculation of more hard scattering reactions at the 3–loop level: ILC, LHC
- Further perfection of the mathematical tools:
  $\Longrightarrow$ Algorithmic simplification of Perturbation theory in higher orders.
- Even higher order corrections needed ?

- DIS AS A TERRITORY FOR PERTURBATIVE AND NON-PERTURBATIVE PRECISION CALCULATIONS
  $\Longrightarrow$ B3