QCD Analysis of Polarized Structure Functions

Johannes Blümlein* and Helmut Böttcher

1. Formalism
2. World Data
3. Parton Distributions
4. $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z)$
5. Scheme Invariant Evolution
6. New Parton Parametrizations with Errors
7. Conclusions

Polarized Structure Function $g_1(x)$

- Parton Model (LO):

$$g_1(x) = \frac{1}{2}\sum_{i=1}^{n_f} e_i^2 \left[\Delta q_i(x) + \Delta \overline{q}_i(x)\right]$$

$$\Delta q_i(x) \quad = \quad (q_i^+(x) - q_i^-(x)) \ ,$$

$q_i^{+(-)}$ are the quark density with helicity aligned(anti-aligned) to the helicity of the parent nucleon

- QCD Improved Quark Parton Model (NLO):

$$\begin{aligned} g_1(x, Q^2) \quad &= \quad \frac{1}{2}\left(\frac{1}{n_f}\sum\nolimits_{i=1}^{n_f} e_i^2\right)\left[\delta C_S \otimes \Delta\Sigma \right.\\ &\left. +\delta C_G \otimes \Delta G + \delta C_{NS} \otimes \Delta q^{NS}\right] \end{aligned}$$

The symbol $\otimes$ denotes convolution w.r.t. x with the Wilson coeffient functions $\delta C_i(x, \alpha_s(Q^2))$:

$$f(x) \otimes g(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) f(x_1) g(x_2)$$

Polarized Structure Function $g_1(x)$

– Singlet Polarized Quark Distribution :

$$\Delta\Sigma(x, Q^2) = \sum_{i=1}^{n_f} \Delta^+ q_i(x, Q^2)$$

– Polarized Gluon Distribution : $\Delta G(x, Q^2)$

– Non-Singlet Polarized Quark Distribution :

$$\Delta q^{NS}(x, Q^2) = \frac{\sum_{i=1}^{n_f} \left(e_i^2 - \frac{1}{n_f} \sum_{k=1}^{n_f} e_k^2 \right) \Delta^+ q_i(x, Q^2)}{\frac{1}{n_f} \sum_{k=1}^{n_f} e_k^2}$$

For $n_f = 3\,(u, d, s, \bar{u}, \bar{d}, \bar{s})$:

$$\Delta\Sigma = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s})$$

$$\Delta q^{NS}_{p(n)} = +(-)\frac{3}{4}\Delta q_3 + \frac{1}{4}\Delta q_8$$

- with

$$\Delta q_3 = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})$$

$$\Delta q_8 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s})$$

Evolution Equations

- Evolution Equations to go from Q_0^2 to Q^2 ($t \equiv lnQ^2$)
  - for $\Delta\Sigma$ and ΔG:

$$\frac{d}{dt}\begin{pmatrix} \Delta\Sigma(x,t) \\ \Delta G(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} \Delta P_{qq}(x) & \Delta P_{qG}(x) \\ \Delta P_{Gq}(x) & \Delta P_{GG}(x) \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma(x,t) \\ \Delta G(x,t) \end{pmatrix}$$

  - for Δq^{NS}:

$$\frac{d}{dt}\Delta q^{NS}(x,t) = \frac{\alpha_s(t)}{2\pi}\Delta P_{qq}^{NS}(x) \otimes \Delta q^{NS}(x,t)$$

with $\Delta P_{ij}(x)$ the polarized splitting functions.

- The polarized Wilson coefficient functions $\delta C_i(x, \alpha_s(Q^2))$ and the polarized splitting functions $\Delta P_{ij}(x, \alpha_s(Q^2))$ are known in the $\overline{MS}$ scheme up to $\mathcal{O}(\alpha_s^2)$. [E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B417 (1994) 61, R. Mertig and W.L. van Neerven, Z. Phys. C70 (1996) 637, W. Vogelsang, Phys. Rev. D54 (1996) 2023]

$\Longrightarrow$ NLO QCD Analysis; Determination of Λ_{QCD} and the Parton densities with Errors.

Parametrization

- General choice for the parametrization of the polarized parton distributions at Q_0^2:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})$$

- Normalization:

$$A_i^{-1} = \left(1+\gamma_i \frac{a_i}{a_i+b_i+1}\right)\frac{\Gamma(a_i)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1)} + \rho_i \frac{\Gamma(a_i+0.5)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1.5)}$$

such that

$$\int_0^1 dx \Delta q_i(x, Q_0^2) = \eta_i$$

are the first moment of $\Delta q_i(x, Q_0^2)$

- The polarized parton distributions to be fitted are:

$$\Delta u_v \, , \, \Delta d_v \, , \, \Delta \bar{q} \, , \, \Delta G$$

where the index v denotes the *valence* quark

Note that: $\Delta q + \Delta \bar{q} = \Delta q_v + 2\Delta \bar{q}$.

Choice of Parameters

- $Q_0^2 = 4.0\, GeV^2$

  SU(3) flavour symmetry assumed

  η_{u_v} and η_{d_v} determined from F and D the SU(3) parameters involved in the matrix elements describing the neutron and hyperon β-decays:

  $\eta_{u_v} = 2F = 0.926$; $\eta_{d_v} = F - D = -0.341$

- Flavor symmetric sea assumed

  $\Delta\bar{u}(x, Q_0^2) = \Delta\bar{d}(x, Q_0^2) = \Delta\bar{s}(x, Q_0^2) = \Delta\bar{q}(x, Q_0^2)$

- No assumption made concerning positivity and helicity retention

- For u_v and d_v : $\rho_{u_v} = \rho_{d_v} = 0$

- For the gluon : $\gamma_G = \rho_G = 0$ (Gluon A)

- For the sea : $\gamma_{\bar{q}} = \rho_{\bar{q}} = 0$ (Sea A)

- The normalizations of the different data sets against each other were fitted and fixed afterwards

- The remaining 12 parameters to be fitted are then:

  Δu_v: a_u , b_u , γ_u $\quad$ Δd_v: a_d , b_d , γ_d

  $\Delta\bar{q}$: $\eta_{\bar{q}}$, $a_{\bar{q}}$, $b_{\bar{q}}$ $\quad$ ΔG: η_G , a_G , b_G

The World Data: $g_1(x, Q^2)$

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	x-range	Q^2-range $[GeV^2]$	ΔN [%]	# data points
E143	0.031 – 0.749	1.27 – 9.52	3.7	28
HERMES	0.028 – 0.660	1.13 – 7.46	3.0	39
E155 (*)	0.015 – 0.750	1.22 – 34.73	7.6	24
SMC	0.005 – 0.480	1.30 – 58.0	4.0	12
proton				103
E143	0.031 – 0.749	1.27 – 9.52	4.9	28
E155	0.015 – 0.750	1.22 – 34.79	7.6	24
SMC	0.005 – 0.479	1.30 – 54.8	4.0	12
deuteron				64
E142	0.035 – 0.466	1.10 – 5.50	3.0	8
HERMES	0.033 – 0.464	1.22 – 5.25	5.0	9
E154	0.017 – 0.564	1.20 – 15.0	3.0	17
neutron				34
total				201

(*): PhD Thesis G. S. Mitchell, 10% normalization correction in addition

The World Data: $g_1/F_1(x, Q^2)$ or $A_1(x, Q^2)$

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	type	x–range	Q^2–range $[GeV^2]$	ΔN [%]	# data points
E143	g1/F1	0.027 – 0.749	1.17 – 9.52	3.7	82
HERMES	g1/F1	0.028 – 0.660	1.13 – 7.46	3.0	39
E155	g1/F1	0.015 – 0.750	1.22 – 34.72	7.6	24
SMC	A1	0.005 – 0.480	1.30 – 58.0	4.0	59
EMC	A1	0.015 – 0.466	3.50 – 29.5	14.0	10
proton					214
E143	g1/F1	0.027 – 0.749	1.17 – 9.52	4.9	82
E155	g1/F1	0.015 – 0.750	1.22 – 34.79	7.6	24
SMC	A1	0.005 – 0.479	1.30 – 54.8	4.0	65
deuteron					171
E142	A1	0.035 – 0.466	1.10 – 5.50	3.0	28
HERMES	A1	0.033 – 0.464	1.22 – 5.25	5.0	9
E154	A1	0.017 – 0.564	1.20 – 15.0	3.0	11
neutron					48
total					433

Parameter Values at $Q_0^2 = 4.0\,GeV^2$

8 Parameter Fit based on $A1(g1/F1)$ Data:

Parameter	LO value	LO error	NLO value	NLO error
η_{u_v}	0.926	fixed	0.926	fixed
a_{u_v}	0.176	0.024	0.292	0.064
b_{u_v}	2.479	0.193	3.326	0.333
γ_{u_v}	21.34	fixed (*)	27.22	fixed (*)
η_{d_v}	-0.341	fixed	-0.341	fixed
a_{d_v}	0.116	0.059	0.122	0.083
b_{d_v}	2.575	1.089	2.818	1.270
γ_{d_v}	38.50	fixed (*)	22.95	fixed (*)
$\eta_{\bar{q}}$	-0.533	0.203	-0.580	0.126
$a_{\bar{q}}$	0.274	0.200	0.472	0.271
$b_{\bar{q}}$	4.98	fixed (*)	5.82	fixed (*)
η_G	1.663	0.778	1.167	0.593
a_G	2.094	0.813	1.662	0.694
b_G	4.65	fixed (*)	4.91	fixed (*)
χ^2 / NDF	0.91		0.85	

$\Rightarrow$ The present data do not constrain the parameters marked by (*) well enough

Parameter Values at $Q_0^2 = 4.0\, GeV^2$

7 Parameter Fit based on $A1(g1/F1)$ Data:

Parameter	LO		NLO	
	value	error	value	error
η_{u_v}	0.926	fixed	0.926	fixed
a_{u_v}	0.132	0.011	0.252	0.044
b_{u_v}	2.471	0.132	3.260	0.272
γ_{u_v}	30.60	fixed (*)	29.01	fixed (*)
η_{d_v}	-0.341	fixed	-0.341	fixed
a_{d_v}	0.141	0.032	0.124	0.065
b_{d_v}	$b_{u_v} + 1$		$b_{u_v} + 1$	
γ_{d_v}	45.33	fixed (*)	43.50	fixed (*)
$\eta_{\bar{q}}$	-0.480	0.053	-0.549	0.143
$a_{\bar{q}}$	0.225	0.027	0.415	0.269
$b_{\bar{q}}$	5.14	fixed (*)	6.87	fixed (*)
η_G	1.647	0.659	1.122	0.560
a_G	2.106	0.722	1.614	0.705
b_G	4.98	fixed (*)	4.98	fixed (*)
χ^2 / NDF	0.91		0.86	

$\Longrightarrow$ The present data do not constrain the parameters marked by (*) well enough

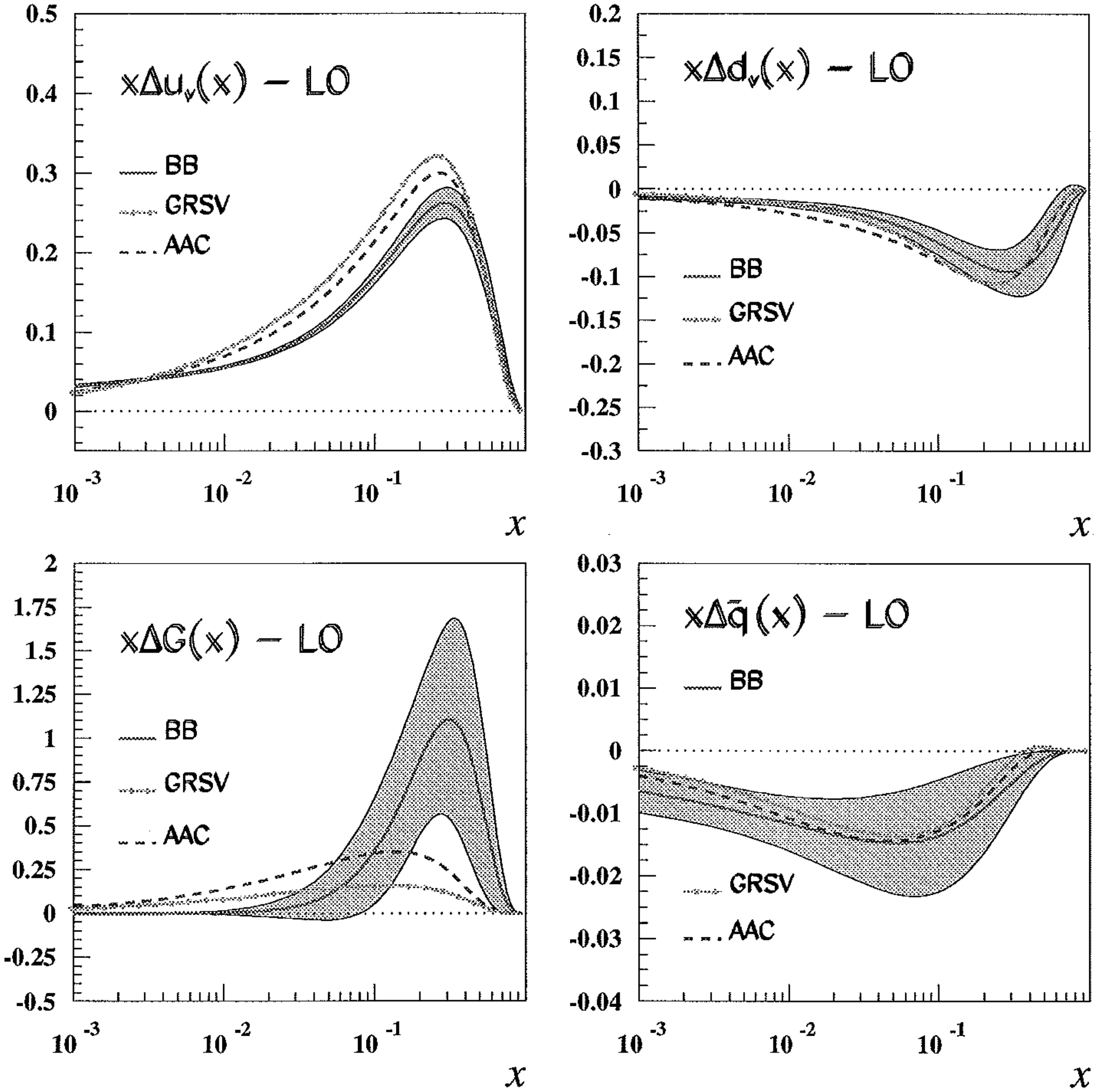
Covariance Matrices – 8 Parameter Fit

LO	$\Lambda_{QCD}^{(4)}$	a_{uv}	b_{uv}	a_{dv}	b_{dv}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda_{QCD}^{(4)}$	2.74e-2								
a_{uv}	-6.12e-6	5.53e-4							
b_{uv}	-9.49e-5	3.15e-3	3.72e-2						
a_{dv}	1.29e-5	-5.92e-4	-3.46e-3	3.43e-3					
b_{dv}	-5.06e-4	-8.97e-3	2.42e-2	3.78e-2	1.19e-0				
$\eta_{\bar{q}}$	1.48e-4	2.99e-3	1.50e-2	-6.17e-3	-5.90e-2	4.12e-2			
$a_{\bar{q}}$	-2.72e-5	3.77e-3	2.11e-2	-7.96e-3	-9.36e-2	3.75e-2	4.01e-2		
η_G	-6.51e-4	-5.06e-3	-1.94e-2	1.15e-2	4.47e-2	-7.75e-2	-5.67e-2	6.06e-1	
a_G	3.42e-4	2.38e-3	2.30e-2	-1.14e-2	-2.14e-1	5.00e-2	5.05e-2	-2.43e-1	6.61e-1

NLO	$\Lambda_{QCD}^{(4)}$	a_{uv}	b_{uv}	a_{dv}	b_{dv}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda_{QCD}^{(4)}$	3.64e-3								
a_{uv}	-2.66e-6	4.11e-3							
b_{uv}	8.46e-5	1.86e-2	1.11e-1						
a_{dv}	4.23e-5	-3.54e-3	-1.64e-2	6.88e-3					
b_{dv}	-1.82e-4	-1.57e-2	3.63e-2	4.67e-2	1.61e-0				
$\eta_{\bar{q}}$	3.87e-5	-8.45e-4	-8.14e-3	2.34e-3	2.27e-2	1.58e-2			
$a_{\bar{q}}$	6.21e-5	1.42e-2	6.11e-2	-1.50e-2	-4.71e-2	1.27e-2	7.35e-2		
η_G	-1.55e-4	8.14e-3	4.76e-2	-8.57e-3	-4.95e-2	-6.32e-2	-4.24e-2	3.51e-1	
a_G	1.00e-5	2.78e-2	1.08e-1	-3.07e-2	-6.82e-2	2.67e-2	1.60e-1	-1.33e-1	4.82e-1

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

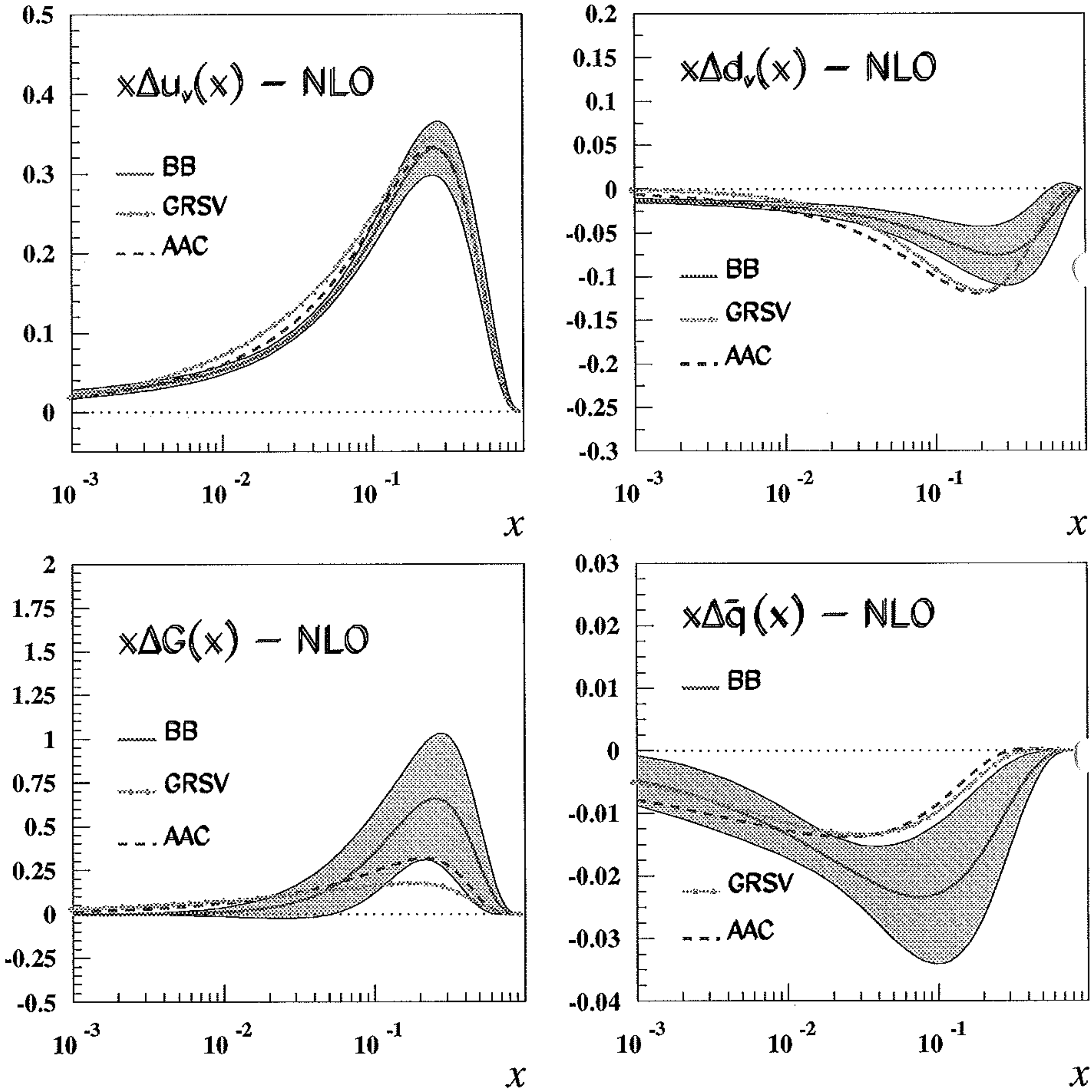
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow error band: Gaussian error propagation with fully taking into account the parameter correlations.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow error band: Gaussian error propagation with fully taking into account the parameter correlations

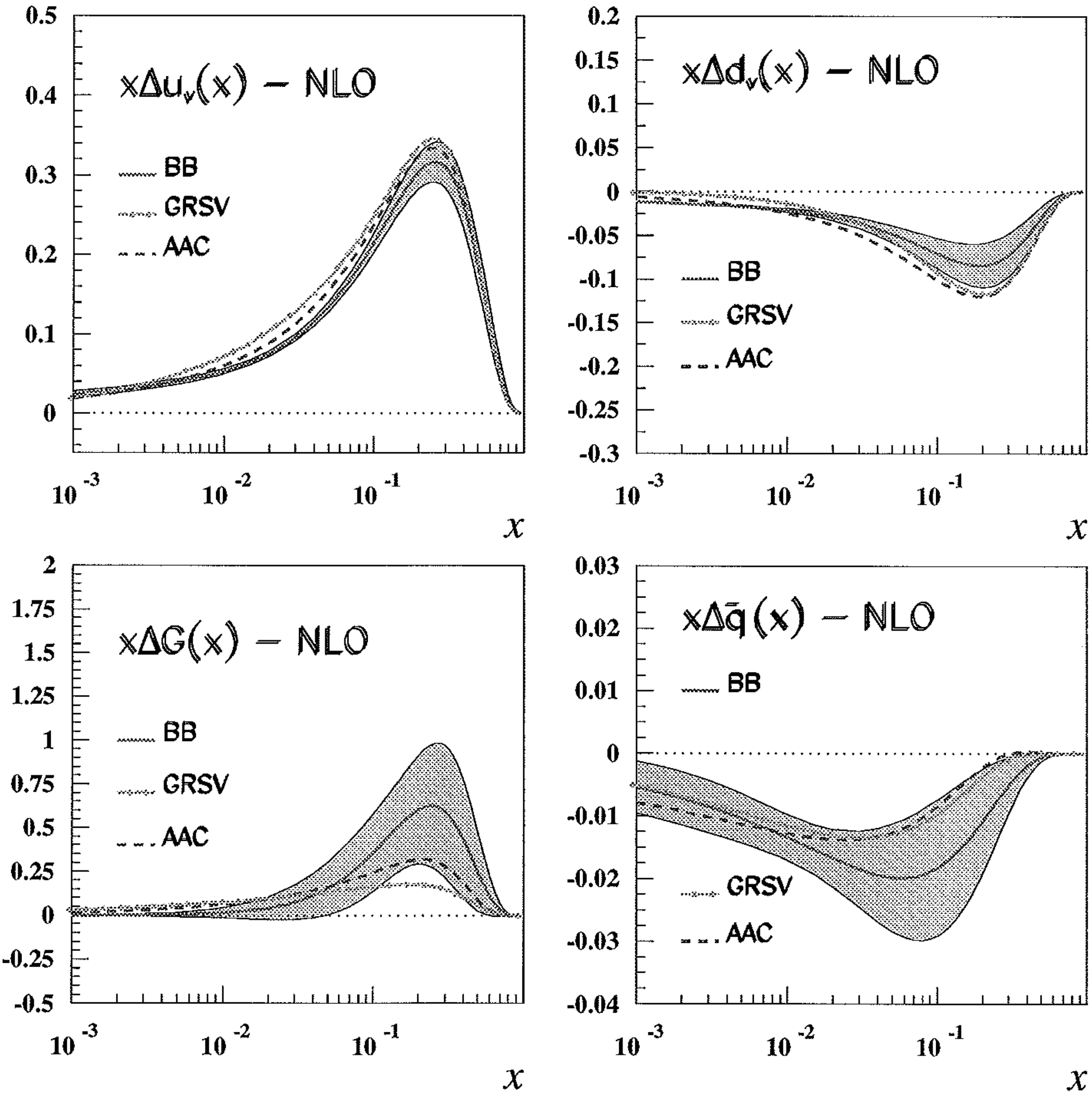
Covariance Matrices – 7 Parameter Fit

LO	$\Lambda^{(4)}_{QCD}$	a_{uv}	b_{uv}	a_{dv}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda^{(4)}_{QCD}$	5.43e-3							
a_u	-1.15e-4	1.15e-4						
b_u	-1.86e-3	9.33e-4	1.74e-2					
a_d	2.00e-4	1.32e-4	1.18e-3	1.03e-3				
$\eta_{\bar{q}}$	-1.71e-3	-3.41e-4	-3.75e-3	-2.30e-4	2.78e-3			
$a_{\bar{q}}$	-1.58e-3	9.20e-5	1.07e-3	-3.50e-5	-1.72e-4	7.02e-4		
η_G	-6.07e-3	2.86e-4	1.53e-2	8.83e-4	-1.02e-2	6.89e-3	4.34e-1	
a_G	7.83e-3	-1.81e-3	-5.26e-3	4.32e-3	5.86e-3	-2.72e-3	-1.58e-1	5.22e-1

NLO	$\Lambda^{(4)}_{QCD}$	a_{uv}	b_{uv}	a_{dv}	$\eta_{\bar{q}}$	$a_{\bar{q}}$	η_G	a_G
$\Lambda^{(4)}_{QCD}$	2.99e-3							
a_u	2.45e-5	1.96e-3						
b_u	-4.25e-4	1.06e-2	7.42e-2					
a_d	-1.96e-4	-1.64e-3	-1.03e-2	4.28e-3				
$\eta_{\bar{q}}$	-1.94e-4	9.78e-4	1.42e-3	-6.44e-4	2.05e-2			
$a_{\bar{q}}$	-3.24e-4	9.25e-3	4.80e-2	-1.16e-2	2.46e-2	7.23e-2		
η_G	3.55e-4	9.00e-3	5.57e-2	-1.08e-2	-5.82e-2	-1.68e-2	3.14e-1	
a_G	-4.68e-4	1.88e-2	8.48e-2	-2.46e-2	5.50e-2	1.64e-1	-7.95e-2	4.98e-1

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

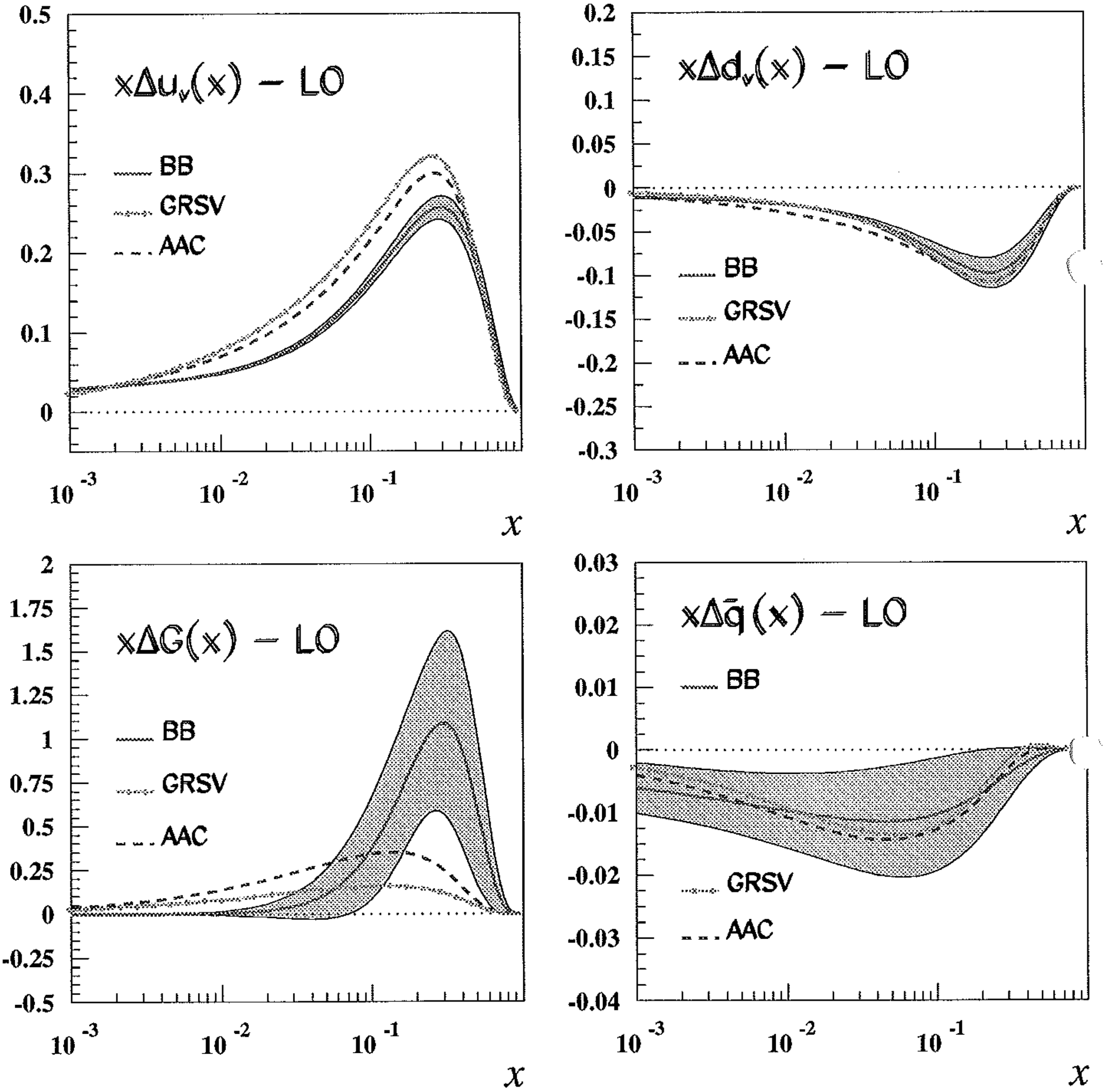
- 7 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow error band: Gaussian error propagation with fully taking into account the parameter correlations

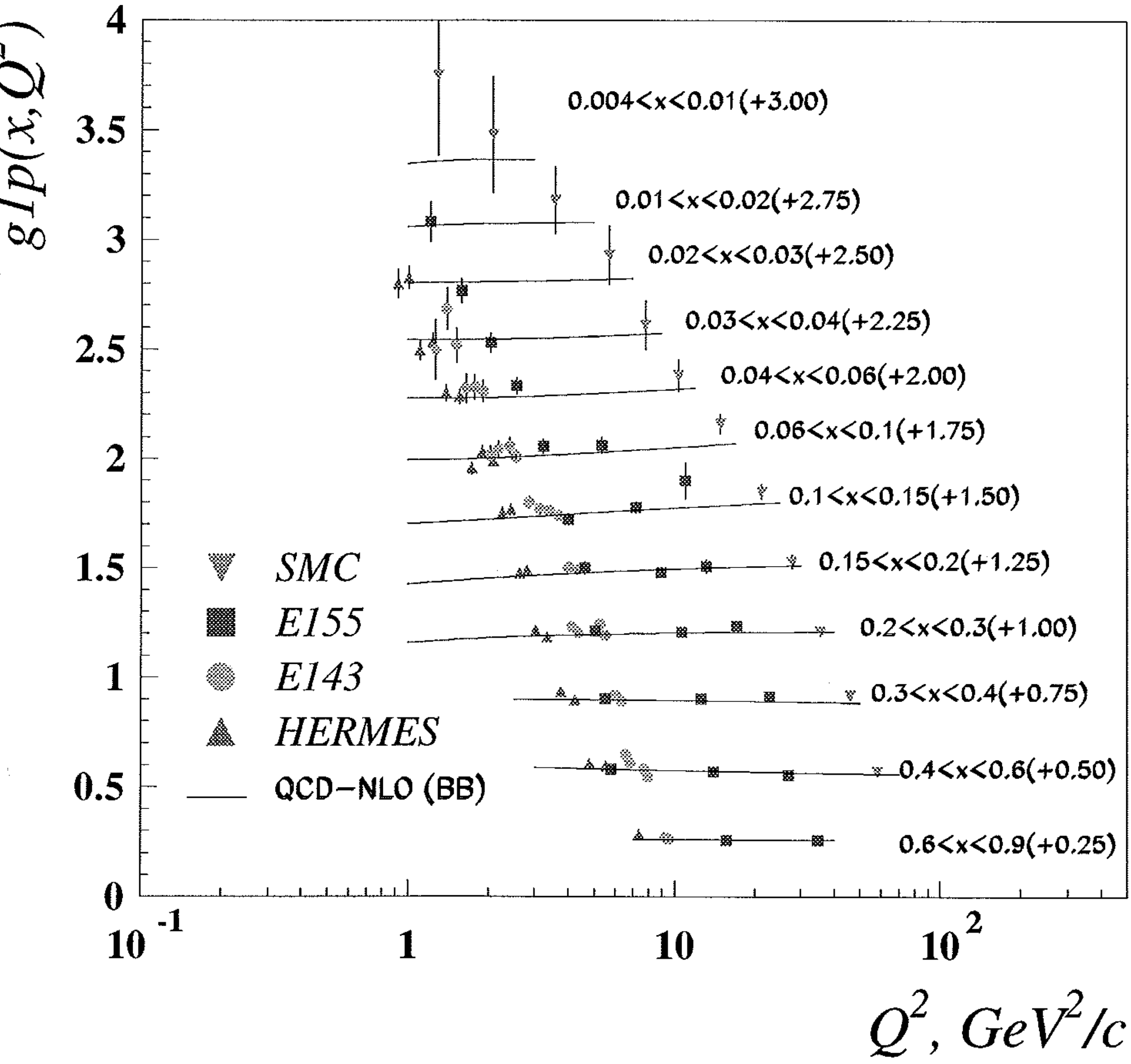
Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7 Parameter Fit based on $A1(g1/F1)$ Data:

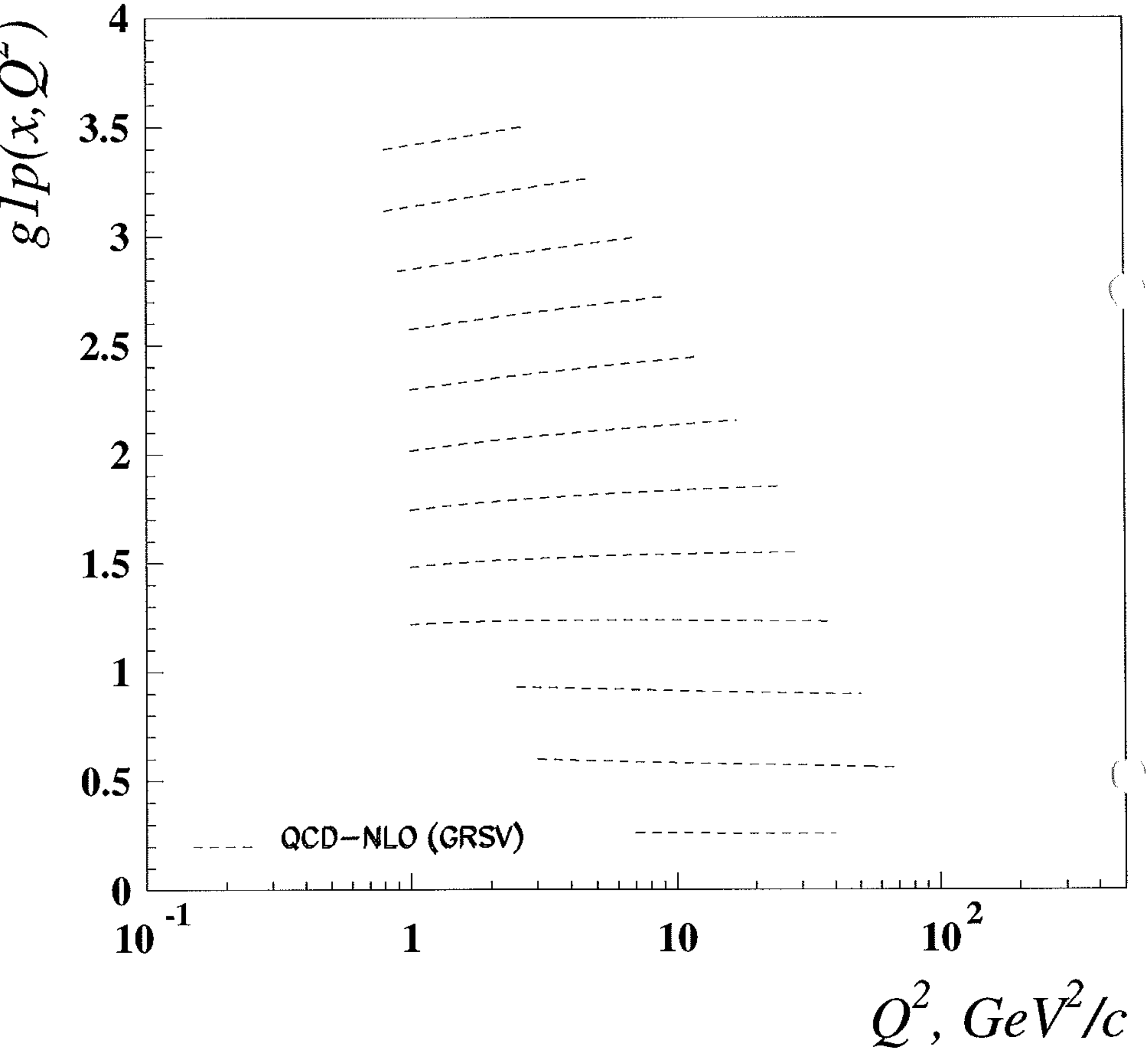


$\Rightarrow$ Yellow error band: Gaussian error propagation with fully taking into account the parameter correlations

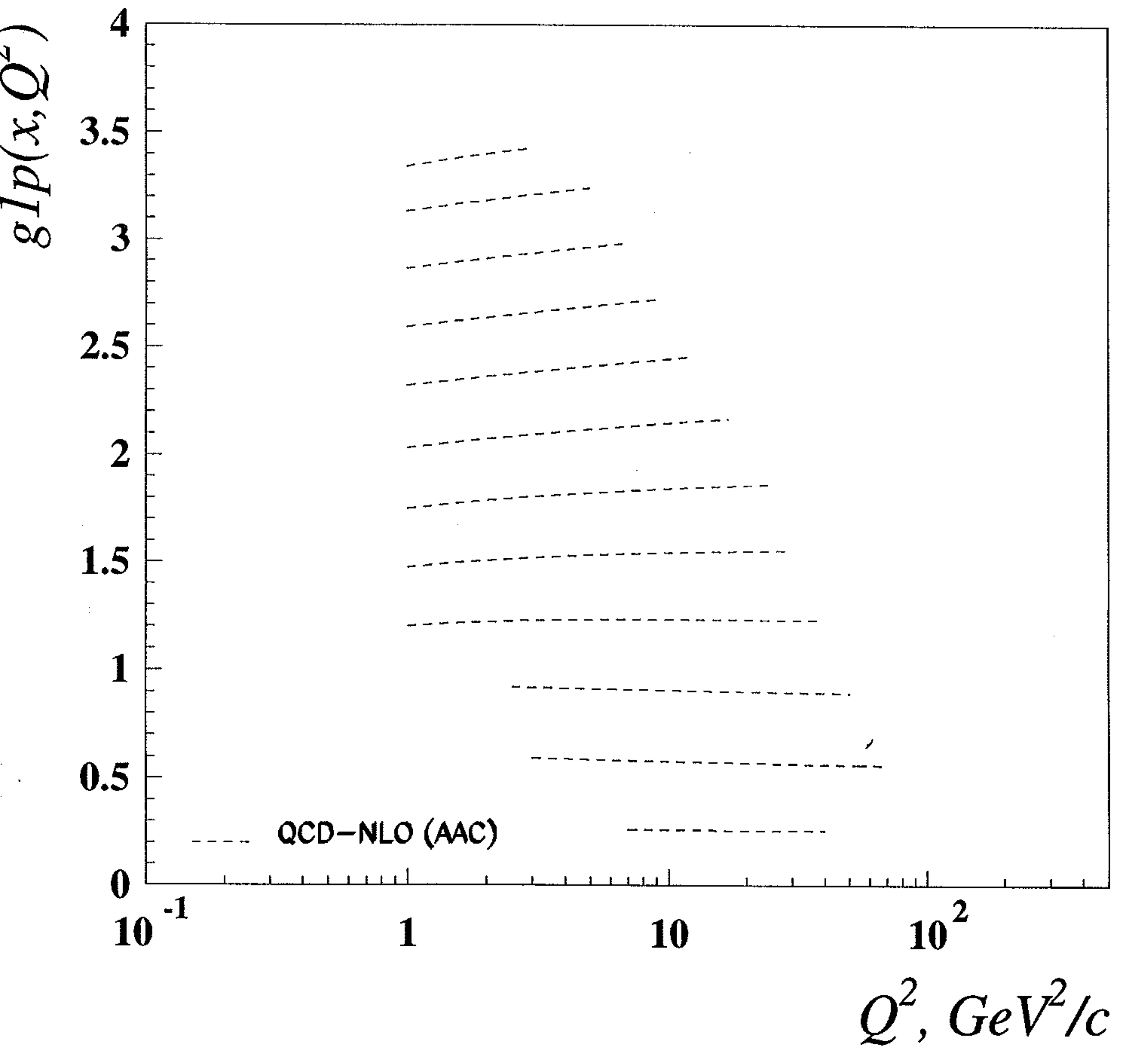
$g_1^p(x)$ versus Q^2



$g_1^p(x)$ versus Q^2



$g_1^p(x)$ versus Q^2



$\Lambda_{QCD}^{(4)} \Longleftrightarrow \alpha_s(M_Z^2)$
8 parameter fit

$\Lambda_{QCD}^{(4)}$	$A1$		$g1$	
	VALUE	ERROR	VALUE	ERROR
FS/RS=1.0/1.0	0.235	$\pm$ 0.060	0.242	$\pm$ 0.067
FS/RS=0.5/1.0	0.185	– 0.050	0.193	– 0.049
FS/RS=2.0/1.0	0.293	+ 0.058	0.318	+ 0.076
FS/RS=1.0/0.5	0.330	+ 0.095	0.349	+ 0.107
FS/RS=1.0/2.0	0.175	– 0.060	0.187	– 0.055
SYST. ERROR $\Longrightarrow$		+ 0.121 – 0.077		+ 0.130 – 0.084

- $A1$: $\alpha_s(M_Z^2) = 0.113 \begin{matrix}+0.004\\-0.005\end{matrix} (exp) \begin{matrix}+0.004\\-0.004\end{matrix} (fac) \begin{matrix}+0.007\\-0.005\end{matrix} (ren)$

- $g1$: $\alpha_s(M_Z^2) = 0.114 \begin{matrix}+0.005\\-0.006\end{matrix} (exp) \begin{matrix}+0.005\\-0.004\end{matrix} (fac) \begin{matrix}+0.008\\-0.005\end{matrix} (ren)$

- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

  E154: $0.108 - 0.116 (> 0.120\ bad)$

  ABFR: $0.120 \begin{matrix}+0.004\\-0.005\end{matrix} (exp) \begin{matrix}+0.009\\-0.006\end{matrix} (theor)$

  WORLD AVERAGE: 0.118 ± 0.002

System : $g_1(x,Q^2), \partial g_1/\partial t(x,Q^2)$

Leading Order :

$$\begin{aligned}
K_{22}^{N(0)} &= 0 \\
K_{2d}^{N(0)} &= -4 \\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned}$$

Next-to-Leading Order :

[Furmanski, Petronzio 1982]

$$\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right] \\
&\quad - \frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right) \\
&\quad + \frac{\beta_0}{2} C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad - \frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[(\gamma_{qq}^{N(0)})^2 - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
&\quad - \frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right)
\end{aligned} \qquad (1)$$

5. Scheme Invariant Combinations

Evolution Equations of Structure or Fragmentation Functions do normally exhibit FACTORIZATION AND RENORMALIZATION SCHEME DEPENDENCES. INSTEAD OF PROCESS-INDEPENDENT SCHEME-DEPENDENT EVOLUTION EQUATIONS FOR PARTONS ONE MAY THINK OF PROCESS-DEPENDENT SCHEME-INDEPENDENT EVOLUTION EQUATIONS FOR **Observables.**

Evolution Equations :

BLÜMLEIN, RAVINDRAN, VAN NEERVEN, NPB 586 (2000) 349.

$$\frac{\partial}{\partial t}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{pmatrix}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix},$$

evolution variable

$$t = -\frac{2}{\beta_0}\ln\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right),$$

physical evolution kernels

$$K_{IJ}^N = \left[-4\frac{\partial C_{I,m}^N(t)}{\partial t}\left(C^N\right)^{-1}_{m,J}(t) - \frac{\beta_0 a_s(Q^2)}{\beta(a_s(Q^2))}C_{I,m}^N(t)\gamma_{mn}^N(t)\left(C^N\right)^{-1}_{n,J}(t)\right]$$

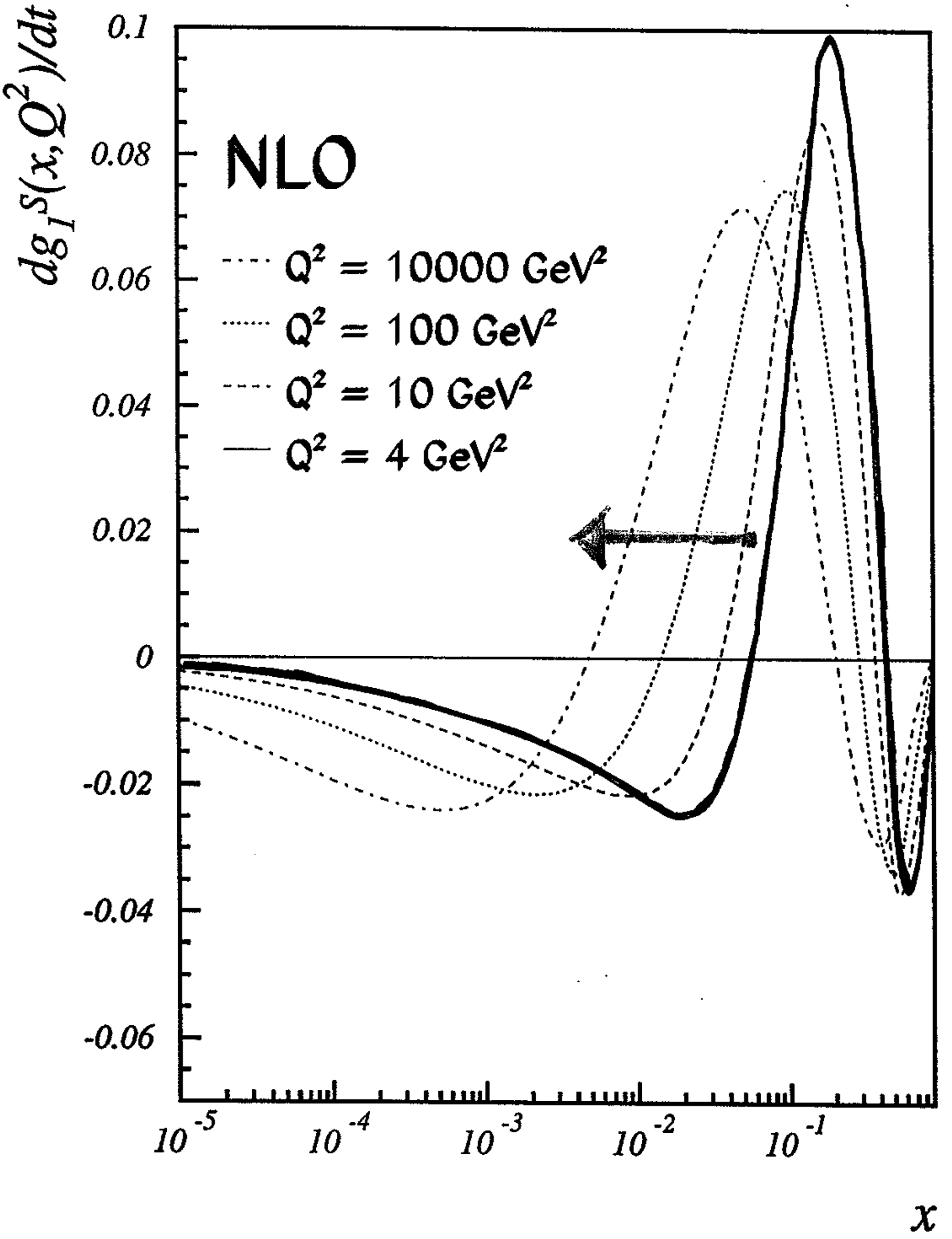
with

$$K_{IJ}^N = \sum_{n=0}^{\infty} a_s^n(Q^2)\left(K^N\right)^{(n)}_{IJ}$$

Possible choices for F_A and F_B are g_1 and $\partial g_1/\partial t$.

The dependence on the renormalization scheme is only removed if the perturbation series is summed to all orders.

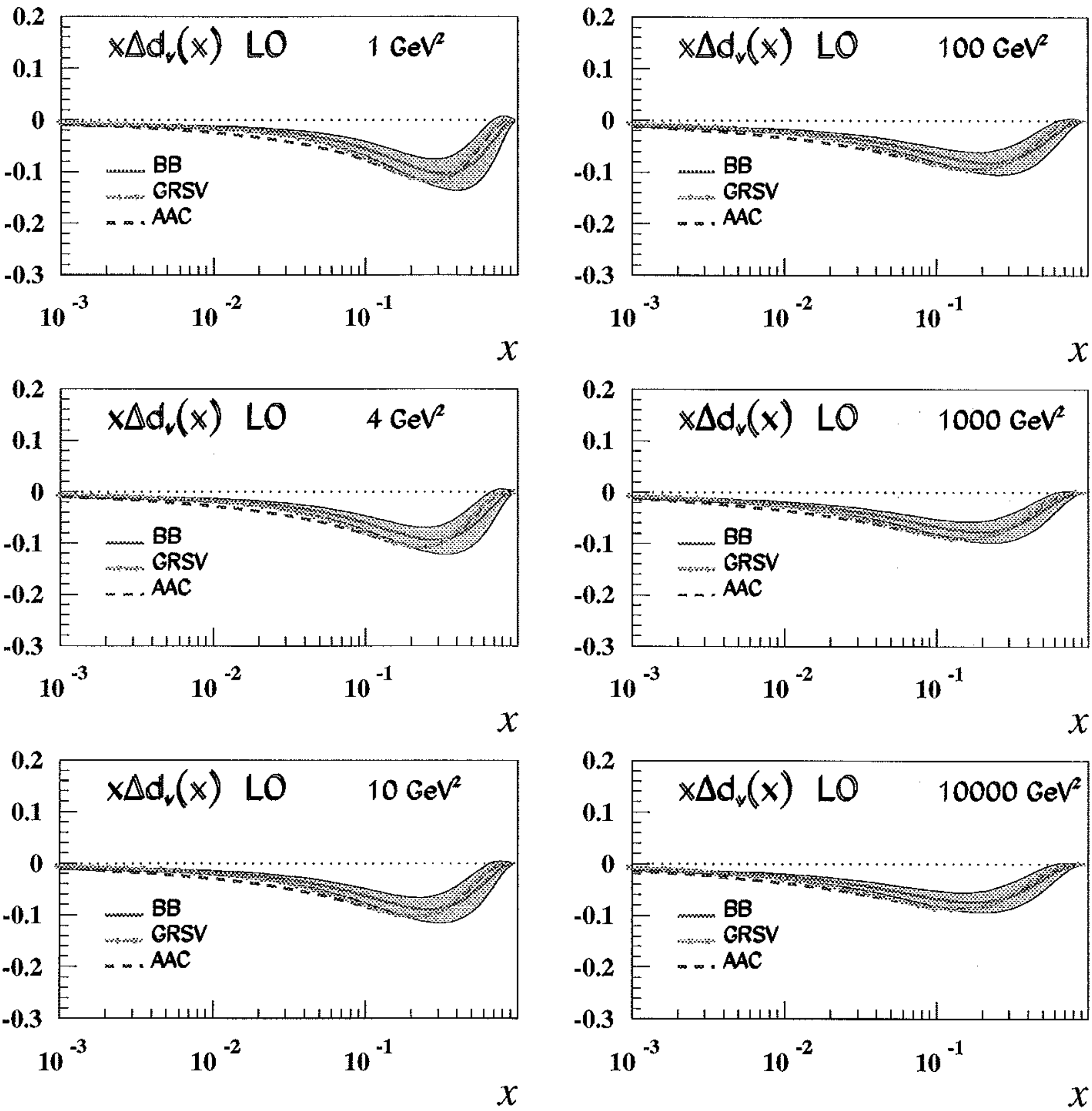
$$
\begin{aligned}
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 \\
&\quad - \frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}
$$



NLO
Q² = 10000 GeV²
Q² = 100 GeV²
Q² = 10 GeV²
Q² = 4 GeV²
dg₁^S(x,Q²)/dt
x
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
10^-5
10^-4
10^-3
10^-2
10^-1

Evolution of Polarized Parton Densities

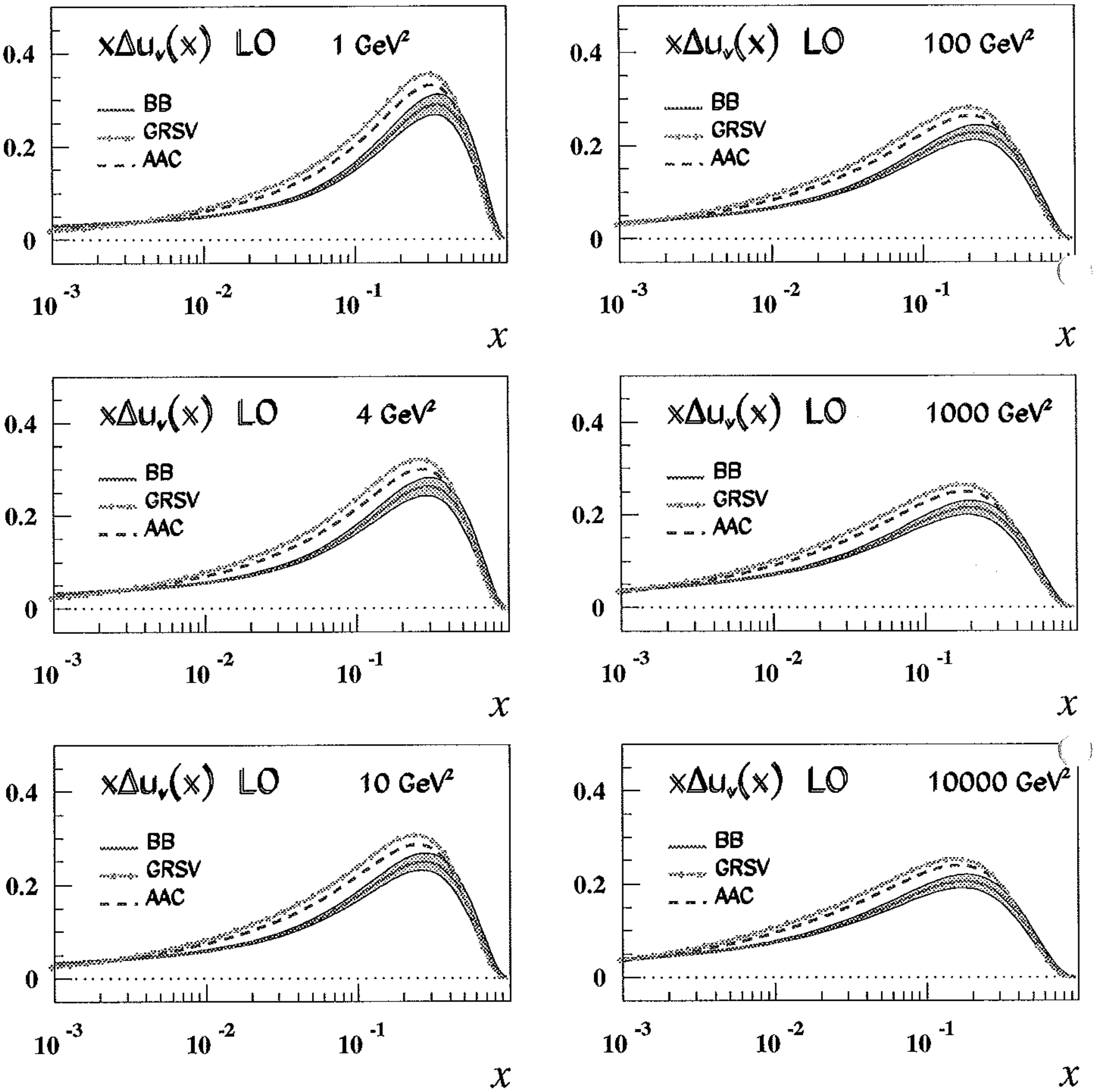
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

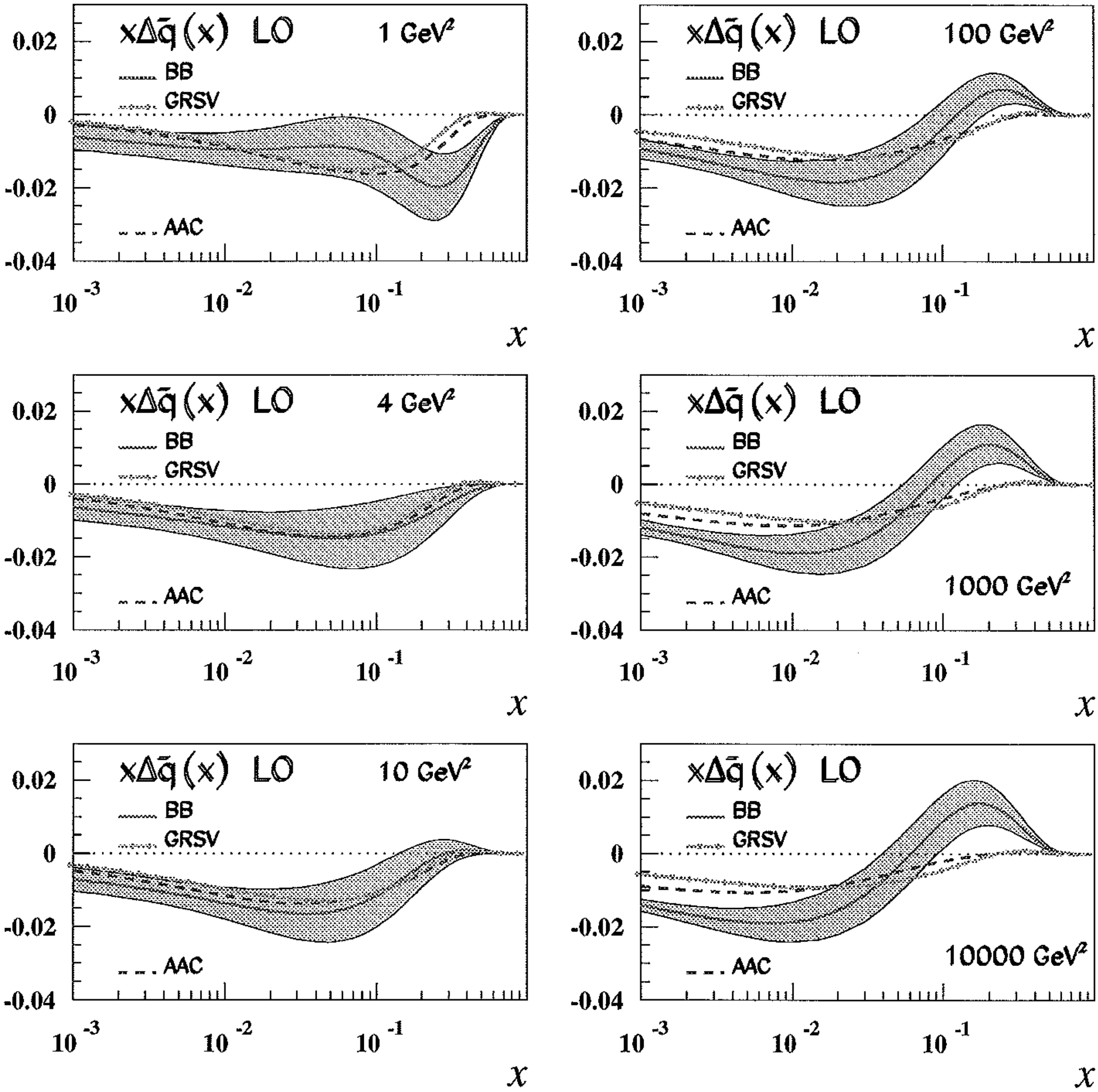
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

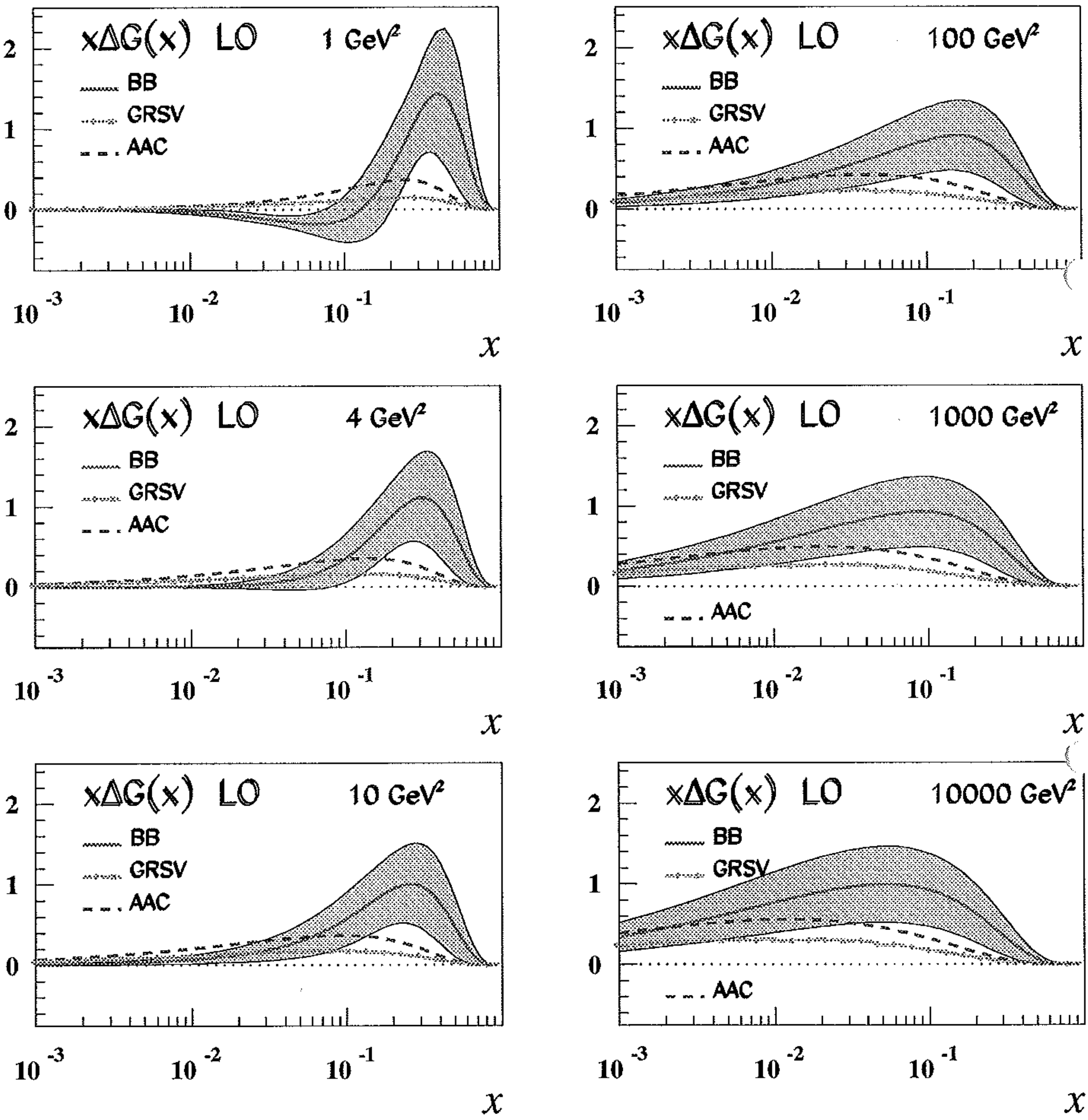
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

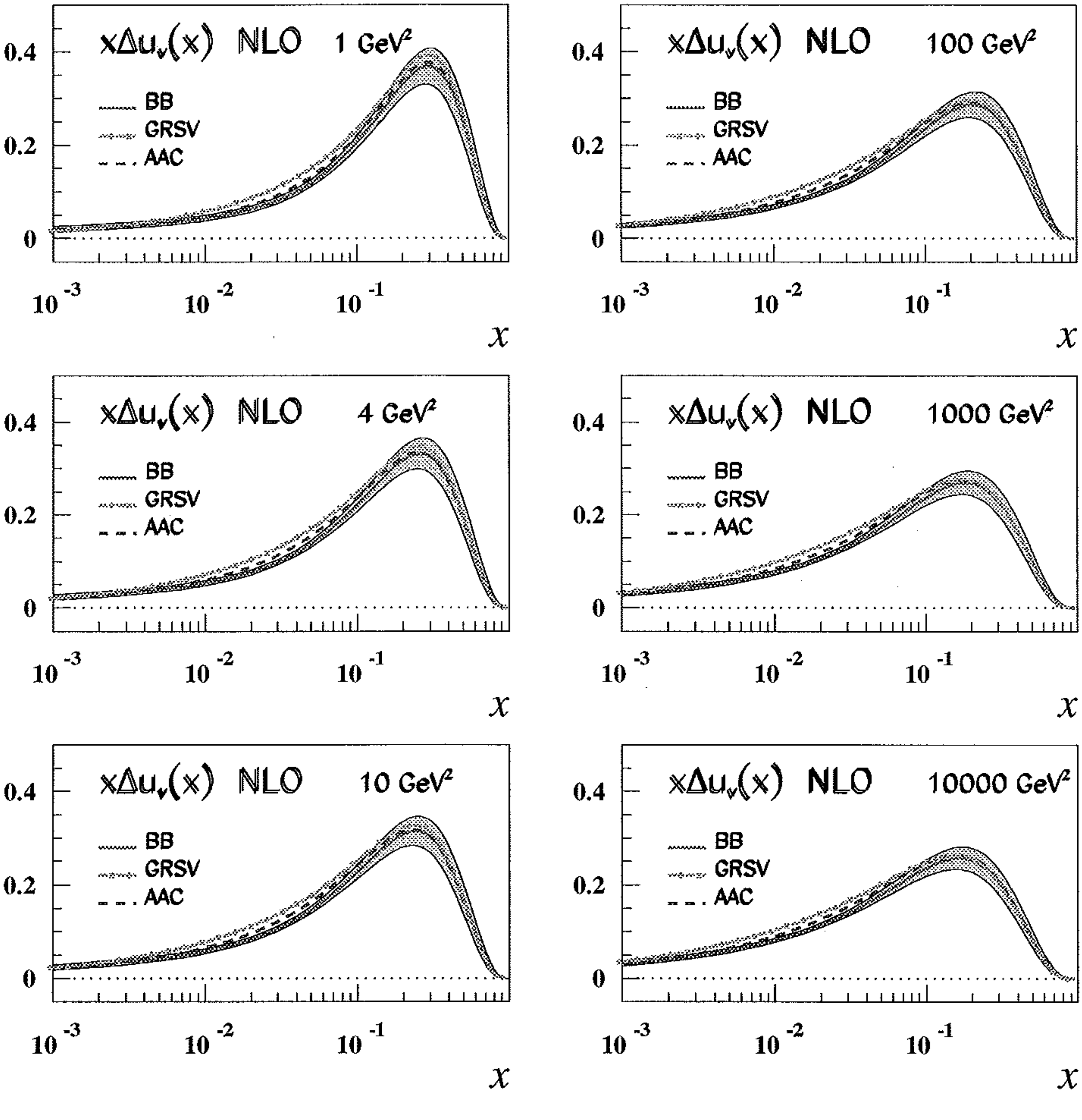
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



⇒ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

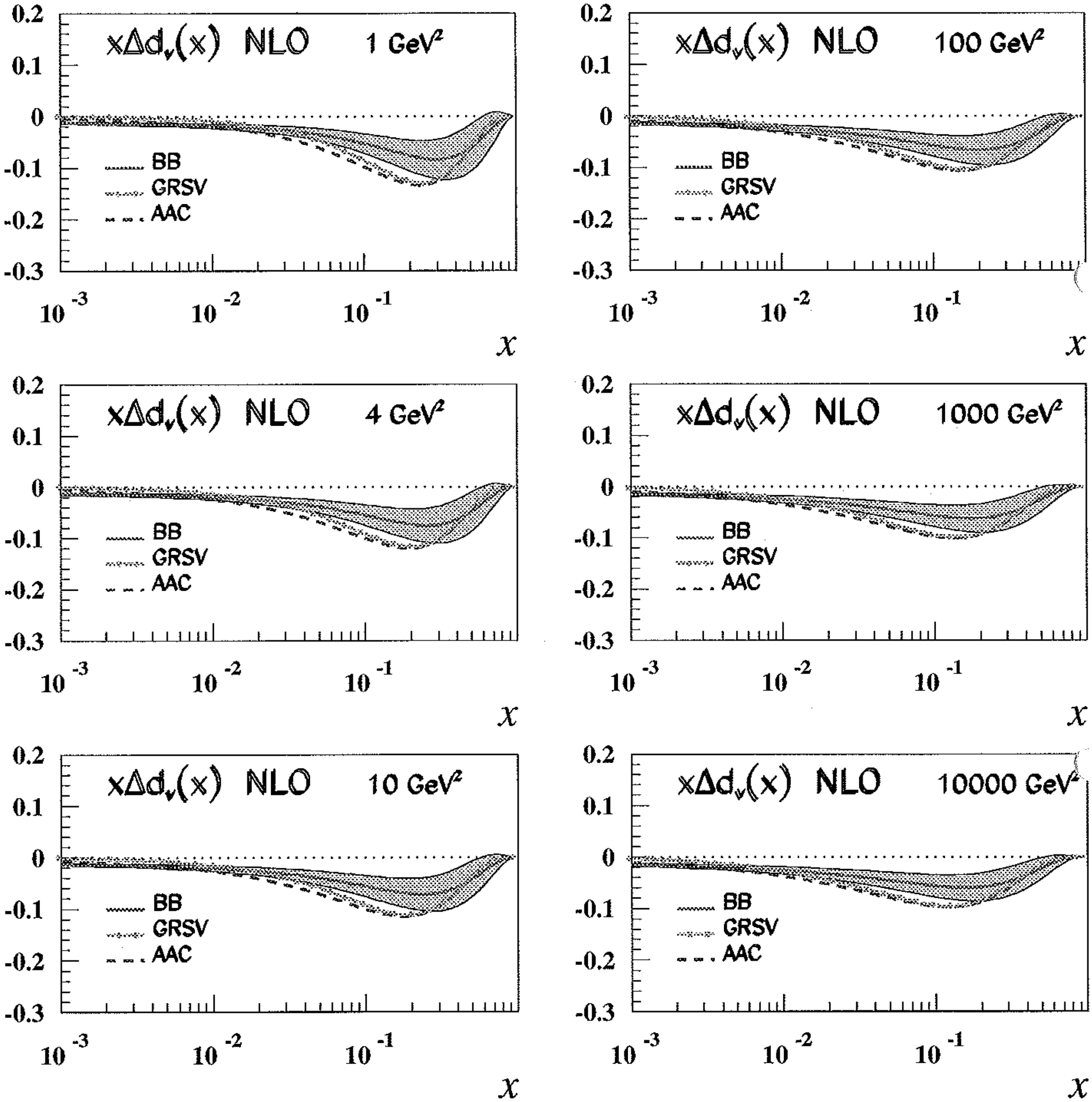
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

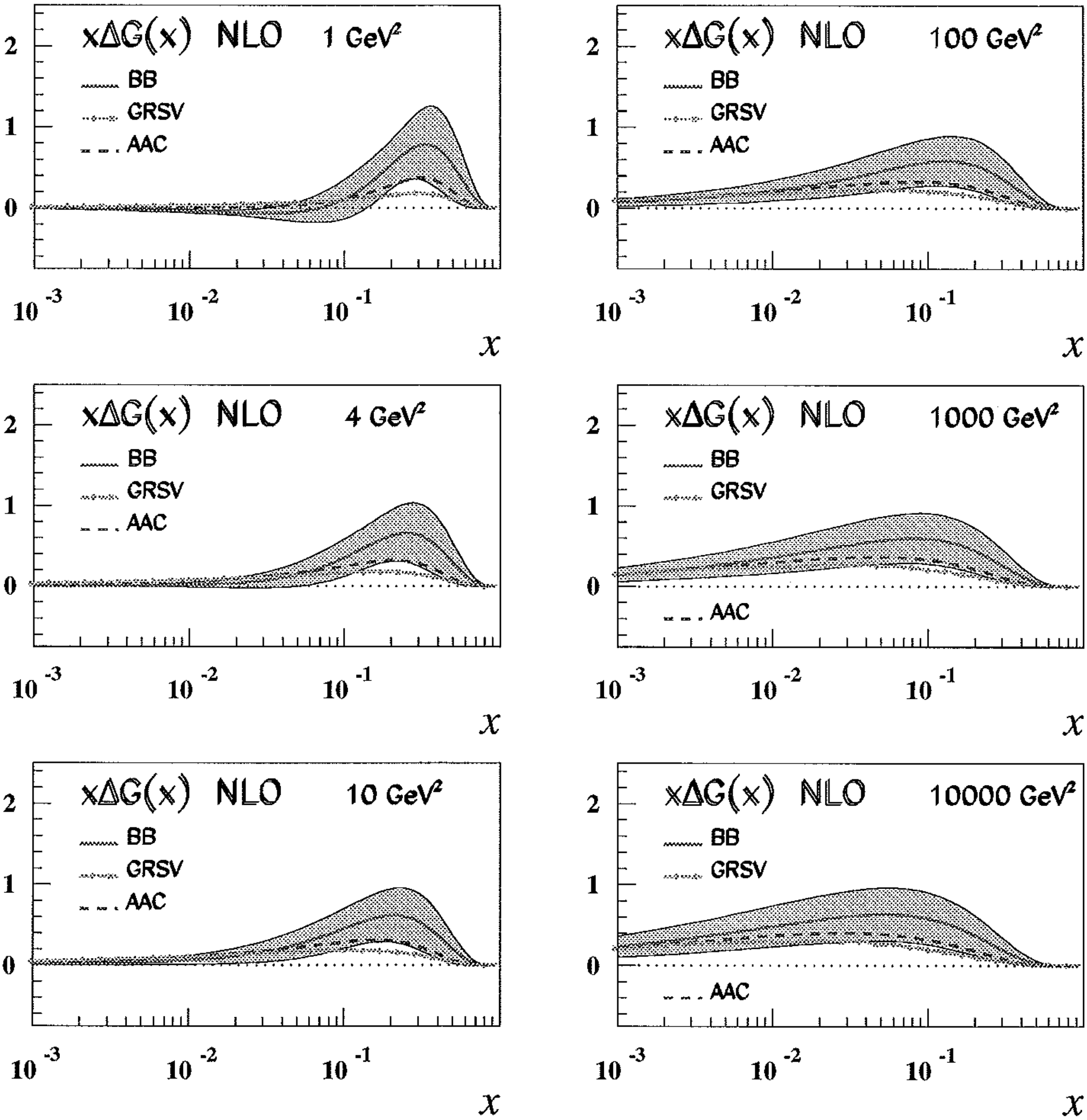
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



⇒ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

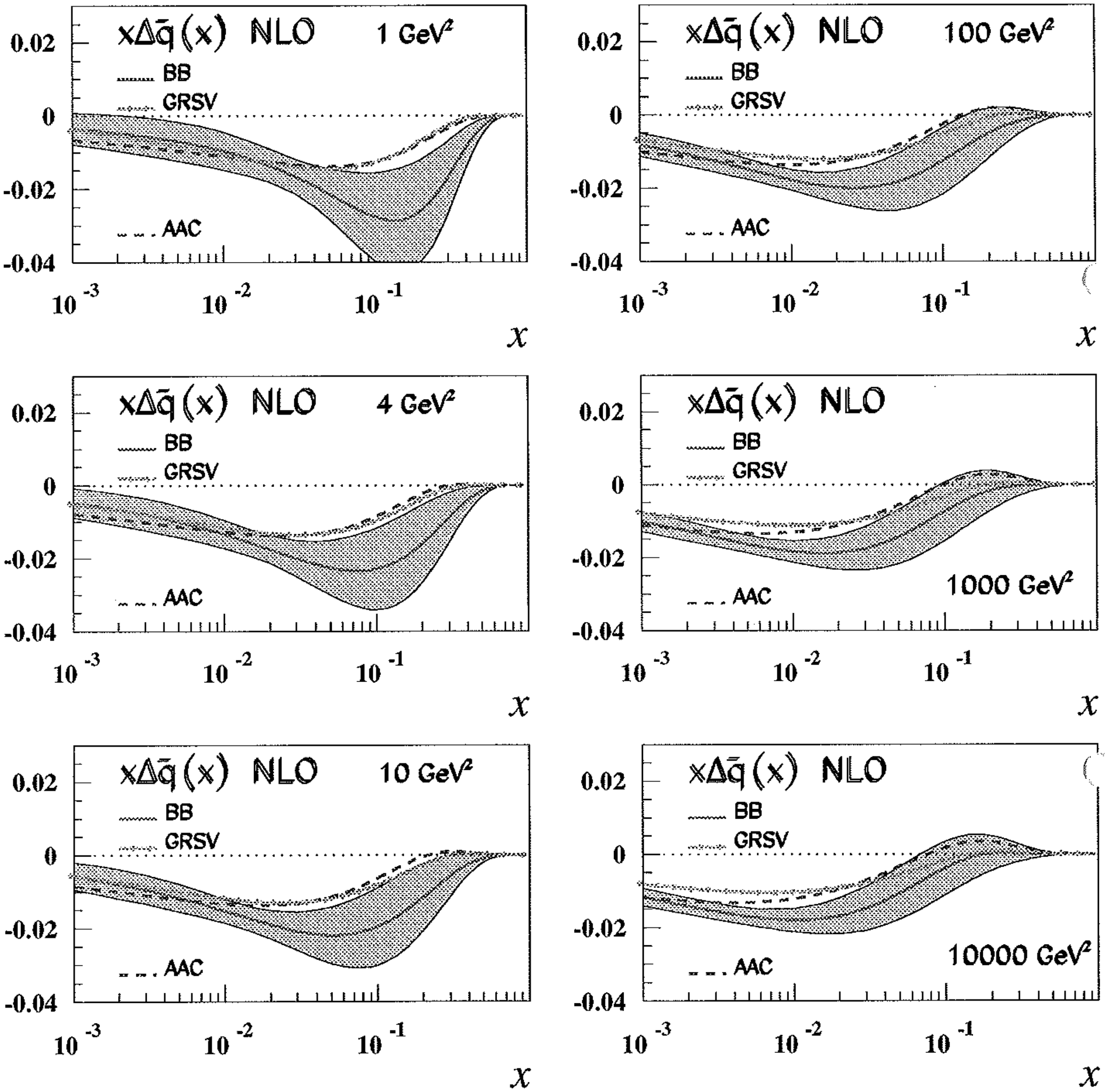
- 8 Parameter Fit based on $A1(g1/F1)$ Data:



$\Rightarrow$ Yellow Error Band evolved to the Q^2 indicated.

Evolution of Polarized Parton Densities

- 8 Parameter Fit based on $A1(g1/F1)$ Data:



⇒ Yellow Error Band evolved to the Q^2 indicated.

7. Conclusions

- A NLO QCD Analysis of the current World-Statistics of Polarized Structure Functions was performed.
- New parton parametrizations, including those of the Errors of the parton densities, were derived. They are available in form of fast **FORTRAN** programs for the range $1 < Q^2 < 10^6$ GeV^2 and $10^{-4} < x < 1$.
- The following results for $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$ were obtained (9-parameter fit):

$$\Lambda^{(4)}_{\rm QCD} = 235 \pm 60 \;({\rm exp}) \;\; {}^{+58}_{-50} \;({\rm fac}) \;\; {}^{+121}_{-77} \;({\rm ren})$$

$$\alpha_s(M_Z^2) = 0.113 \;\; {}^{+0.004}_{-0.005} \;({\rm exp}) \;\; {}^{+0.004}_{-0.004} \;({\rm fac}) \;\; {}^{+0.007}_{-0.005} \;({\rm ren})$$

- First steps in a scheme–invariant QCD evolution based on the structure function $g_1(x,Q^2)$ and $\partial g_1(x,Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$.
- The latter analysis is a very promising way to proceed in the future, since it allows to extract $\Lambda_{\rm QCD}$ fixing all the input distributions by direct measurement.