The QCD Structure of the Non–Forward Compton Amplitude

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1. Introduction

Deeply Virtual Compton Scattering: A New Test Ground for QCD

- Scaling Violations: Non-Forward
- New Evolution Equations
- Generalization of the Light Cone Expansion
- New Integral Relations Specific to the Non-Forward Case

Generalization of the CALLAN–GROSS and WANDZURA–WILCZEK Relations to the Amplitude Level

Conceptional Problem: Non-Forward Light Cone Expansion & Current Conservation

How to extract the Twist–2 Contributions ?

How to resum the Spin Towers ? → Also a Problem for the Higher Twist Operators for Forward-Scattering!

3. Non–Forward Anomalous Dimensions

Non–local Operators ≡ Taylor Summed-up Local Operators

$$
\begin{aligned}
O^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O_5^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \tilde{x}^\nu \, \frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})F_{\nu\rho}^a(\kappa_2\tilde{x}) + F_\mu^{a\,\rho}(\kappa_2\tilde{x})F_{\nu\rho}^a(\kappa_1\tilde{x})\right]\\
O_5^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O_5^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu \, \tilde{x}^\nu \, \frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_2\tilde{x}) - F_\mu^{a\,\rho}(\kappa_2\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_1\tilde{x})\right]
\end{aligned}
$$

Renormalization Group Equation:

$$
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1\tilde{x},\kappa_2\tilde{x};\mu^2) = \int_{\kappa_2}^{\kappa_1} d\kappa_1' d\kappa_2' \gamma^{AB}(\kappa_1,\kappa_2,\kappa_1',\kappa_2';\mu^2) O^B(\kappa_1'\tilde{x},\kappa_2'\tilde{x};\mu^2)\ .
$$

Argument-relations of the anomalous dimension

$$
\begin{aligned}
\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(\kappa_1-\kappa,\kappa_2-\kappa;\kappa_1'-\kappa,\kappa_2'-\kappa)\\
&= \lambda^{d_{AB}}\gamma^{AB}(\lambda\kappa_1,\lambda\kappa_2;\lambda\kappa_1',\lambda\kappa_2')\ ,
\end{aligned}
$$

$$
d_{AB} = 2 + d_A - d_B,
$$

$$
d_q = 1 \quad \text{and} \quad d_G = 2\ .
$$

$$
\begin{aligned}
(\kappa_2-\kappa_1)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(0,1;\alpha_1,1-\alpha_2)\\
&\equiv \widehat{K}^{AB}(\alpha_1,\alpha_2),\\
4(\kappa_-)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= 4\gamma^{AB}(-1,+1;w_1,w_2)\\
&\equiv \widetilde{K}^{AB}(w_1-w_2,w_1+w_2)
\end{aligned}
$$

$$
\alpha_1=\frac{\kappa_1'-\kappa_1}{\kappa_2-\kappa_1}, \qquad -\alpha_2=\frac{\kappa_2'-\kappa_2}{\kappa_2-\kappa_1},
$$

$$
w_1=\alpha_1-\alpha_2=\frac{\kappa'_+-\kappa_+}{\kappa_-}, \qquad w_2=1-\alpha_1-\alpha_2=\frac{\kappa'_-}{\kappa_-},
$$

Evolution Equations for Operators:

$$
\begin{aligned}
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int D\alpha\,(\kappa_2-\kappa_1)^{d_B-d_A}\widehat{K}^{AB}(\alpha_1,\alpha_2)O^B(\kappa_1',\kappa_2'),\\
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int Dw\,(\kappa_-)^{d_B-d_A}\widetilde{K}^{AB}(w_1,w_2)O^B(\kappa_1',\kappa_2')\\
&= \int_0^1 dw_2\int_{-1+w_2}^{1-w_2} dw_1\,(\kappa_-)^{d_B-d_A}\widetilde{K}^{AB}_{\rm sym}(w_1,w_2)\\
&\quad\times O^B(\kappa_1',\kappa_2'),
\end{aligned}
$$

$$
\widetilde{K}^{AB}_{\rm sym}(w_1,w_2) = \frac{1}{2}\left[\widetilde{K}^{AB}_0(w_1,w_2)+(-1)^{d_B}\widetilde{K}^{AB}_0(w_1,-w_2)\right].
$$

Spin-Towers : Two-fold Moment Expansion

$$\mu^2 \frac{d}{d\mu^2} O^A_{n_1 n_2} = \sum_{n_1', n_2'} \gamma^{AB}_{n_1, n_2; n_1', n_2'} O^B_{n_1' n_2'} ~,$$

$$\begin{aligned}\gamma^{AB}_{n_1,n_2;n_1',n_2'} &= \frac{\partial^{n_1}}{\partial \kappa_1^{n_1}} \frac{\partial^{n_2}}{\partial \kappa_2^{n_2}} \int_{\kappa_2}^{\kappa_1} d\kappa_1' \int_{\kappa_2}^{\kappa_1} d\kappa_2' \frac{(\kappa_1')^{n_1'}}{n_1'!} \frac{(\kappa_2')^{n_2'}}{n_2'!} \\ &\times \gamma^{AB}(\kappa_1, \kappa_2; \kappa_1', \kappa_2')_{\kappa_1=\kappa_2=0} \qquad .\end{aligned}$$

(2)

$$\begin{aligned}
\gamma^{qq}_{nn'} &= \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qq}(w_1, w_2) \right\}, \\
\gamma^{qG}_{nn'} &= n \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qG}(w_1, w_2) \right\} \\
\gamma^{Gq}_{nn'} &= \frac{1}{n} \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{Gq}(w_1, w_2) \right\}, \\
\gamma^{GG}_{nn'} &= \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{GG}(w_1, w_2) \right\},
\end{aligned}$$

with

$$\begin{aligned}
\sigma^{(\pm)}_{n\,n'} &= \frac{1}{4}\left(1 + (-1)^{n-n'}\right)\left(1 \pm (-1)^{n'-2d_B}\right) \\
&= \frac{1}{4}(1 \pm (-1)^n)(1 \pm (-1)^{n'})
\end{aligned}$$

1) Unpolarized anomalous dimensions

$$\widehat{K}_0^{qq}(\alpha_1,\alpha_2) = C_F\left\{1-\delta(\alpha_1)-\delta(\alpha_2)+\delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+ +\delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+ + \frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\right\},$$

$$\widehat{K}_0^{qG}(\alpha_1,\alpha_2) = -2N_fT_R\left\{1-\alpha_1-\alpha_2+4\alpha_1\alpha_2\right\},$$

$$\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) = -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)+2\right\},$$

$$\widehat{K}_0^{GG}(\alpha_1,\alpha_2) = C_A\left\{4(1-\alpha_1-\alpha_2)+12\alpha_1\alpha_2 +\delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ -2+\alpha_2\right) +\delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ -2+\alpha_1\right)\right\} + \frac{1}{2}\beta_0\,\delta(\alpha_1)\delta(\alpha_2),$$

where $C_F = (N_c^2-1)/2N_c \equiv 4/3$, $T_R = 1/2$, $C_A = N_c \equiv 3$, and the β–function in leading order, $\beta_0 = (11C_A - 4T_RN_f)/3$.

$$\int_0^1 dx\,[f(x,y)]_+\,\varphi(x) = \int_0^1 dx\,f(x,y)\,[\varphi(x)-\varphi(y)],$$

if the singularity of f is of the type $\sim 1/(x-y)$.

2) Polarized anomalous dimensions

$$\Delta\widehat{K}_0^{qq}(\alpha_1,\alpha_2) = \widehat{K}_0^{qq}(\alpha_1,\alpha_2),$$

$$\Delta\widehat{K}_0^{qG}(\alpha_1,\alpha_2) = -2N_fT_R\left\{1-\alpha_1-\alpha_2\right\},$$

$$\Delta\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) = -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)-2\right\},$$

$$\Delta\widehat{K}_0^{GG}(\alpha_1,\alpha_2) = \widehat{K}_0^{GG}(\alpha_1,\alpha_2) - 12C_A\alpha_1\alpha_2$$

1) Unpolarized anomalous dimensions:

$$\gamma_{nn'}^{qq} = C_F \left\{ \left[\frac{1}{2} - \frac{1}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1} \frac{1}{j} \right] \delta_{nn'} - \left[\frac{1}{(n+1)(n+2)} + \frac{2}{n-n'} \frac{n'+1}{n+1} \right] \theta_{nn'} \right\}$$

$$\gamma_{nn'}^{qG} = -N_f T \frac{1}{(n+1)(n+2)(n+3)} \left[(n^2+3n+4) - (n-n')(n+1) \right]$$

$$\gamma_{nn'}^{Gq} = -C_F \frac{1}{n(n+1)(n+2)} \left[(n^2+3n+4)\delta_{nn'} + 2\theta_{nn'} \right],$$

$$\gamma_{nn'}^{GG} = C_A \left\{ \left[\frac{1}{6} - \frac{2}{n(n+1)} - \frac{2}{(n+2)(n+3)} + 2\sum_{j=2}^{n+1} \frac{1}{j} + \frac{2N_f T}{3C_A} \right] \delta_{nn'} + \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'} \right) - (n-n'+2)\left(\frac{1}{n(n+1)} + \frac{1}{(n+2)(n+3)} \right) \right] \theta_{nn'} \right\},$$

with the following notation

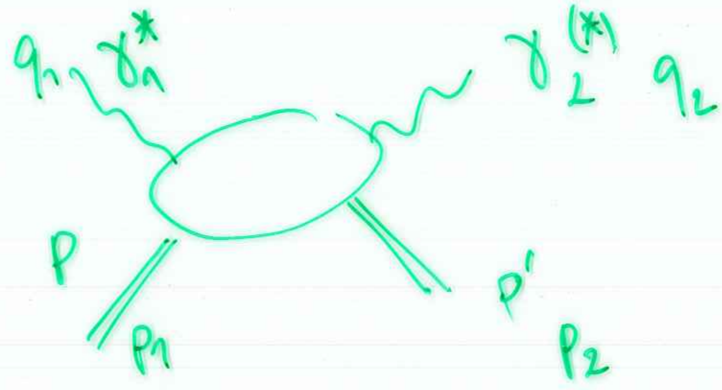
$$\sigma_{n\,n'}^{(\pm)} = \tfrac{1}{4}(1 \pm (-1)^n)(1 \pm (-1)^{n'})$$

$$\theta_{n\,n'} = \begin{cases} 1 & \text{for } n' < n, \\ 0 & \text{otherwise}. \end{cases}$$

2) Polarized local anomalous dimensions:

$$
\begin{aligned}
\Delta\gamma^{qq}_{nn'} &= \gamma^{qq}_{nn'}, \\
\Delta\gamma^{qG}_{nn'} &= -N_f T \frac{n'}{(n+1)(n+2)}, \\
\Delta\gamma^{Gq}_{nn'} &= \frac{1}{(n+1)(n+2)}\left[(n+3)\delta_{nn'} - \frac{2}{n}\theta_{nn'}\right], \\
\Delta\gamma^{GG}_{nn'} &= C_A\left\{\left[\frac{1}{6} - \frac{4}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1}\frac{1}{j} + \frac{2N_fT}{3C_A}\right]\delta_{nn'}\right. \\
&+ \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'}\right)\right. \\
&\left.\left. -(n-n'+2)\frac{2}{(n+1)(n+2)}\right]\theta_{nn'}\right\}.
\end{aligned}
$$

2 The Compton Amplitude



$$T_{\mu\nu}(p_+, p_-, q) = i \int d^4x\, e^{iqx} \langle p_2, S_2 \| T(J_\mu(x/2) J_\nu(-x/2)) \| p_1, S_1 \rangle \ .$$

$$\begin{aligned} p_+ &= p_2 + p_1, \qquad & p_- = p_2 - p_1 = q_1 - q_2, \\ q &= \frac{1}{2}(q_1 + q_2), \qquad & p_1 + q_1 = p_2 + q_2 \ , \end{aligned}$$

$$\nu = q p_+ \longrightarrow \infty, \qquad -q^2 \longrightarrow \infty \ ,$$

$$\xi = -\frac{q^2}{q p_+}, \qquad \eta = \frac{q p_-}{q p_+} = \frac{q_1^2 - q_2^2}{2\nu}$$

$$\begin{aligned} q_1 &= q + \frac{1}{2} p_- \\ q_2 &= q - \frac{1}{2} p_- \ . \end{aligned}$$

3 Kinematic Relations

BREIT-FRAME:

$$\begin{aligned} p_+ &= p_1 + p_2 = (2E_p;\ \vec{0}) \\ -p_- &= p_1 - p_2 = (0;\ 2\vec{p}) = (0;\ 0, 0, 2p_3) \\ q &= \frac{1}{2}(q_1 + q_2) = (q_0;\ q_1, 0, q_3)\ . \end{aligned}$$

$$\begin{aligned} q_1 . q_1 &= -\nu(\xi - \eta) \\ q_2 . q_2 &= -\nu(\xi + \eta) \\ q . p_+ &= \nu \\ q . p_- &= \eta\nu \\ q . q &= -\xi\nu \\ q . p_z &= q^2 - Q^2 = (z_+ + z_-\eta)\nu \equiv t\,\nu \\ \nu \gg \quad p_+^2 &\approx p_-^2 \approx p_+ p_- \approx 0\ . \end{aligned}$$

$\nu \equiv q p_+$ (LARGE.)

STUDY: HELICITY PROJECTIONS: γ_1^*, $\gamma_2^{(*)}$

$$\mathsf{T}_{kl} = \varepsilon^{\mu}_{2,k} T_{\mu\nu} \varepsilon^{\nu}_{1,l}, \quad k, l \in \{0, 1, 2, 3\}$$

$$\begin{aligned} n_0 &= (1;\ 0, 0, 0) && \propto p_+ \\ n_2 &= (0;\ 0, 1, 0)\ . && \text{NEW.} \end{aligned}$$

$$
\begin{aligned}
&e^2\left\langle p_2, S_2 \middle| O^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1\right\rangle \\
&= i \int Dz e^{-i\tilde{x}p_z/2} \underline{F(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma^\mu u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma\tilde{x} u(p_1, S_1)\right] \\
&+ i \int Dz e^{-i\tilde{x}p_z/2} \underline{G(z_1, z_2)} \left[\overline{u}(p_2, S_2)\sigma^{\mu\nu} p_{-\nu} u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\sigma^{\alpha\beta}\tilde{x}_\alpha p_{-\beta} u(p_1, S_1)\right]
\end{aligned}
$$

UNPOL.

($\propto p_- \to 0$ FORW.)

$p_z = p_1 z_1 + p_2 z_2$.

$$
\begin{aligned}
&e^2\left\langle p_2, S_2 \middle| O_5^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1\right\rangle \\
&= i \int Dz e^{-i\tilde{x}p_z/2} \underline{F_5(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma_5\gamma^\mu u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma_5\gamma\tilde{x} u(p_1, S_1)\right] \\
&+ i \int Dz e^{-i\tilde{x}p_z/2} \underline{G_5(z_1, z_2)} \left[\overline{u}(p_2, S_2)\gamma_5\sigma^{\mu\nu} p_{-\nu} u(p_1, S_1) - \frac{i}{2} p_z^\mu \overline{u}(p_2, S_2)\gamma_5\sigma^{\alpha\beta}\tilde{x}_\alpha p_{-\beta} u(p_1, S_1)\right],
\end{aligned}
$$

POL.

4 Current Conservation

$$\partial_\mu^x J^\mu(x) = 0$$

$$\begin{aligned} T_{\mu\nu}(p_+, p_-, q) &= i \int d^4x\, e^{-iq_2 x} \langle p_2, S_2 \| RT(J_\mu(0) J_\nu(x) \| p_1, S_1 \rangle \\ &= i \int d^4x\, e^{-iq_1 x} \langle p_2, S_2 \| RT(J_\mu(-x) J_\nu(0) \| p_1, S_1 \rangle \end{aligned}$$

$$q_2^\mu T_{\mu\nu} = T_{\mu\nu} q_1^\nu = 0\ .$$

$$\begin{aligned} \overline{u}(p_2, S_2)\gamma_\mu q^\mu u(p_1, S_1) &\propto \nu \\ \varepsilon_{\alpha,\beta,\gamma,\delta}\, p_\pm^\gamma q^\delta &\propto \nu \end{aligned}$$

$$\Longrightarrow \int Dz\, H(z_+, z_-) = \int_{-1}^{+1} dz_+ \int_{-1+|z_+|}^{+1-|z_+|} H(z_+, z_-) = 0$$ CRUCIAL.

THE LIGHT-CONE EXPANSION BREAKS EM-CURRENT CONSERVATION.

→ FIND SUBSETS WHICH PRESERVE IT.

WHAT HAPPENS AT LOWEST TWIST ?

5 The Helicity Projections of the Compton Amplitude

UNPOLARIZED: DIRAC

$$\mathsf{T}^F_{11} = 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta) + \varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right]$$

$$\mathsf{T}^F_{22} = 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta) + \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right]$$

$$\mathsf{T}^F_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \quad \text{for the other projections } k,l \in \{1,2,3\} \text{ and } n \geq 0\,,$$

POLARIZED: DIRAC

$$\mathsf{T}^{F5}_{12} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon^{(2)}_{1\mu}\varepsilon^{(1)}_{2\nu}\int Dz\frac{q_\lambda}{Q^2+i\varepsilon}\left[S_{21,\sigma} + \frac{q.S_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right]F_5(z_+,z_-)\,.$$

$$\mathsf{T}^{F5}_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \quad \text{for the other projections } k,l \in \{1,2,3\} \text{ and } n \geq 0\,.$$

$$\varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2\mu\nu}(\xi,\eta) = \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2\mu\nu}(\xi,\eta) = \int Dz\frac{q.p_z}{(Q^2+i\varepsilon)^2}F(z_+,z_-)$$

$$\mathsf{T}^F_{11} = \mathsf{T}^F_{22}\,.$$

$$\mathsf{T}^{F5}_{12} = -\mathsf{T}^{F5}_{21}$$

NON-FORWARD 'SPIN' VECTOR.

$$S^\sigma_{21} := -\frac{1}{2}\overline{u}(p_2,S_2)\gamma_5\gamma^\sigma u(p_1,S_1)\,.$$

6 The Integral Relations

6.1 Unpolarized Contributions

$$\mathsf{T}_{11}^{F,G} = \mathsf{T}_{22}^{F,G}$$

$$F_2(x_B) = 2xF_1(x_B) \equiv \sum_q e_q^2 x \left[q(x_B) + \overline{q}(x_B)\right]$$

FORWARD

↑ PARTON DENSITIES

BUT :

$$\begin{aligned}
\mathsf{H}_1(\xi,\eta) &= \int Dz \frac{\nu}{Q^2 + i\varepsilon} H(z_+, z_-) = -\int Dz \frac{H(z_+, z_-)}{\xi + t - i\varepsilon} \\
\mathsf{H}_2(\xi,\eta) &= \int Dz \frac{\nu\ q.p_z}{(Q^2 + i\varepsilon)^2} H(z_+, z_-) = \int Dz \frac{tH(z_+, z_-)}{(\xi + t - i\varepsilon)^2} .
\end{aligned}$$

$t = t(z_+, z_-)$

6.2 Polarized Contributions

$$\mathsf{T}_{12}^{H5} = i\, \varepsilon^{\mu\lambda\nu\sigma} \varepsilon_{1\mu}^{(2)} \varepsilon_{2\nu}^{(1)} B_{\lambda\sigma}$$

$$B_{\lambda\sigma} = \int Dz \frac{q_\lambda}{Q^2 + i\varepsilon} \left[S_{21,\sigma}^H + \frac{q.S_{21}^H}{Q^2 + i\varepsilon} p_{z\sigma} \right] H_5(z_+, z_-),$$

$S_{21}^H = S_{21}(\Sigma_{21})$ for $H = F(G)$. It may be rewritten as

$$B_{\lambda\sigma} = -\frac{1}{\nu} \int Dz \frac{q_\lambda}{\xi + t - i\varepsilon} \left[S_{21,\sigma}^H - \frac{1}{\nu} \frac{t\, q.S_{21}^H}{\xi + t - i\varepsilon} p_{+\sigma} + \frac{1}{\nu} \frac{q.S_{21}^H}{\xi + t - i\varepsilon} z_- \pi_\sigma \right] H_5(z_+, z_-)$$

$$\begin{aligned} B_{\lambda\sigma} \;=\; & -\frac{1}{\nu} q_\lambda S_{21,\sigma}^H \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}_5(z, \eta) \\ & -\frac{1}{\nu^2} q_\lambda p_{+\sigma}\; q.S_{21}^H \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \left[\hat{h}_5(t, \eta) - \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}_5(z, \eta) \right] \\ & -\frac{1}{\nu^2} q_\lambda \pi_\sigma\; q.S_{21}^H \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \widetilde{h}_5(z, t, \eta)\,. \end{aligned}$$

$$\widetilde{h}_5(z, t, \eta) = \left(\frac{t}{z} \right) \int_{\rho_{\min}}^{\rho_{\max}} d\rho\; \rho h\left(z - \eta\rho, \rho\right)$$

$q_\mu \pi^\mu$, $p_{\pm\mu} \pi^\mu$, $n_{2\mu} \pi^\mu \propto O(\mu^2)$.

8. Conclusions

1. The virtual Compton Amplitude for deep-inelastic Nonforward Scattering was studied in the Generalized Bjorken Region for the Twist–2 contributions.

2. There exist several equivalent methods to derive the Nonforward Evolution Kernels and anomalous dimensions, which yield the same results. The problem of Spin Towers can be solved in terms of integral representations. This is likely the solution of the Spin Tower problem arising for Higher Twist Operators, in generalized form, for Forward Scattering too.

3. The Nonforward Compton Amplitude consists of unpolarized and polarized DIRAC and PAULI –type contributions at leading twist. For the Operator-Expectaion Values an expansion in $1/\nu$ has to be performed to find the Leading Twist terms, in the spirit of the Bjorken Limit.

4. For the unpolarized terms only the Amplitude matrix elements T_{11} and T_{22} and the polarized terms the projections T_{12} and T_{21} contribute in this order.

5. In this order the Light-Cone expansion conserves the electromagnetic current. This property has to be studied twist by twist for the remaining contributions.

6. Generalizations of the CALLAN-GROSS and WANDZURA-WILCZEK relations known in the forward case for the matrix element square level for the Nonforward Case were derived at the Matrix Element Level.