Aachen
QCD 20yr Lkn
6/92

STRUCTURE FUNCTIONS AT SMALL X

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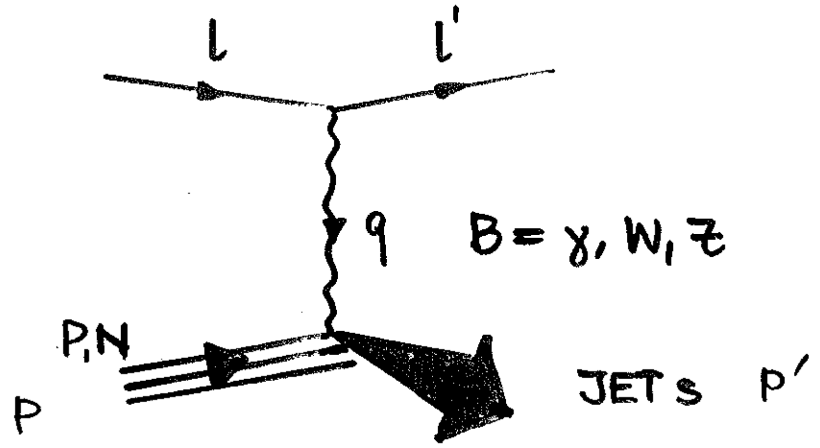
DESY – ZEUTHEN

1. INTRODUCTION
2. EXTRAPOLATION OF AP, $O(\alpha_s)$, $O(\alpha_s^2)$ RESULTS
3. THE FKL EQUATION
4. THE GLR EQUATION
5. CONCLUSIONS

1. INTRODUCTION

DEEP INELASTIC SCATTERING :



$$Q^2 = -(l-l')^2 \quad , \quad x = \frac{Q^2}{2Pq} \quad , \quad y = Q^2/sx$$

$$x \ll 1$$

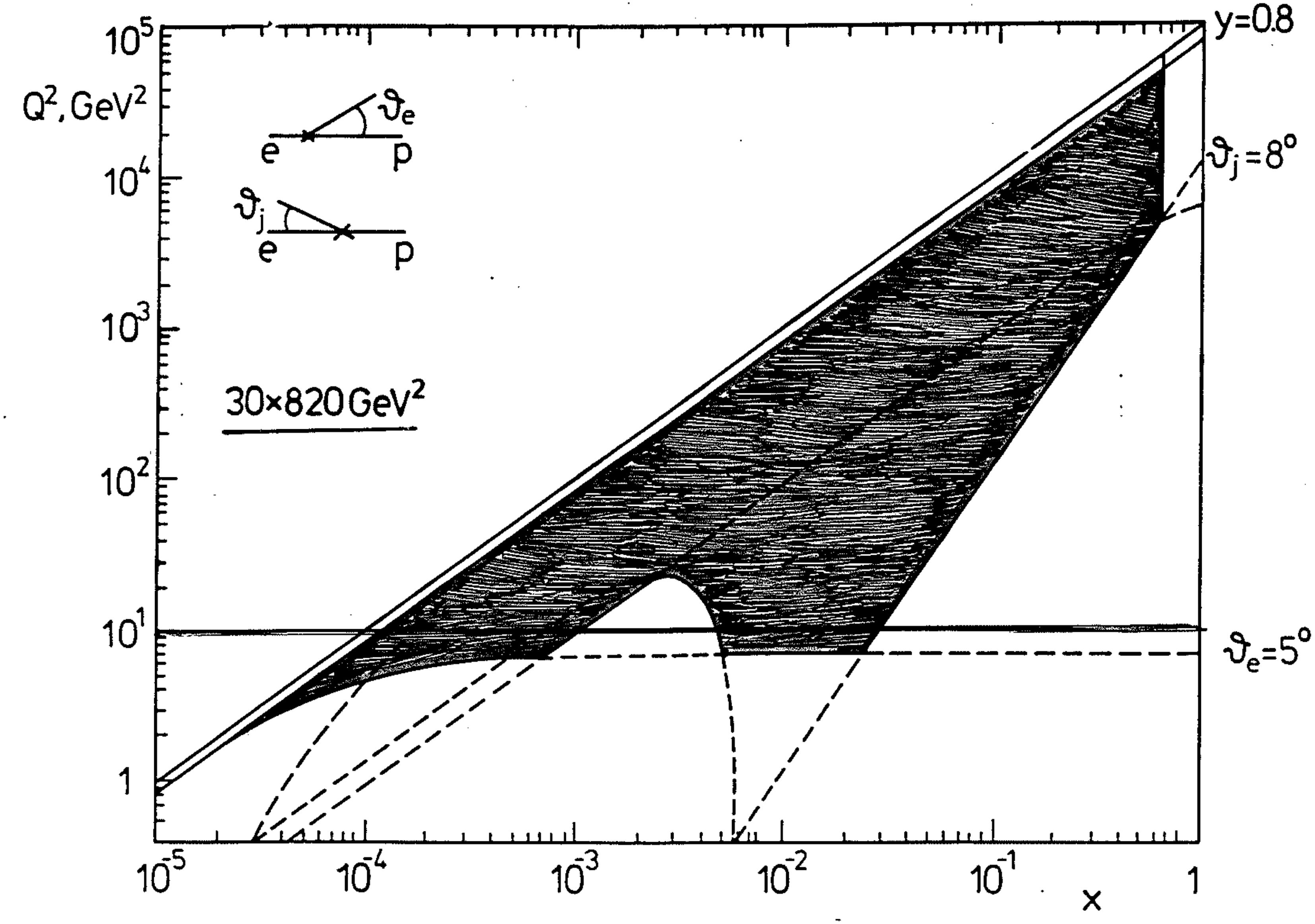
$$\frac{d^2\sigma^{e^\pm p}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[2xF_1(x,Q^2)\,y^2 + F_2(x,Q^2)\,2(1-y)\right]$$

EVOLUTION:

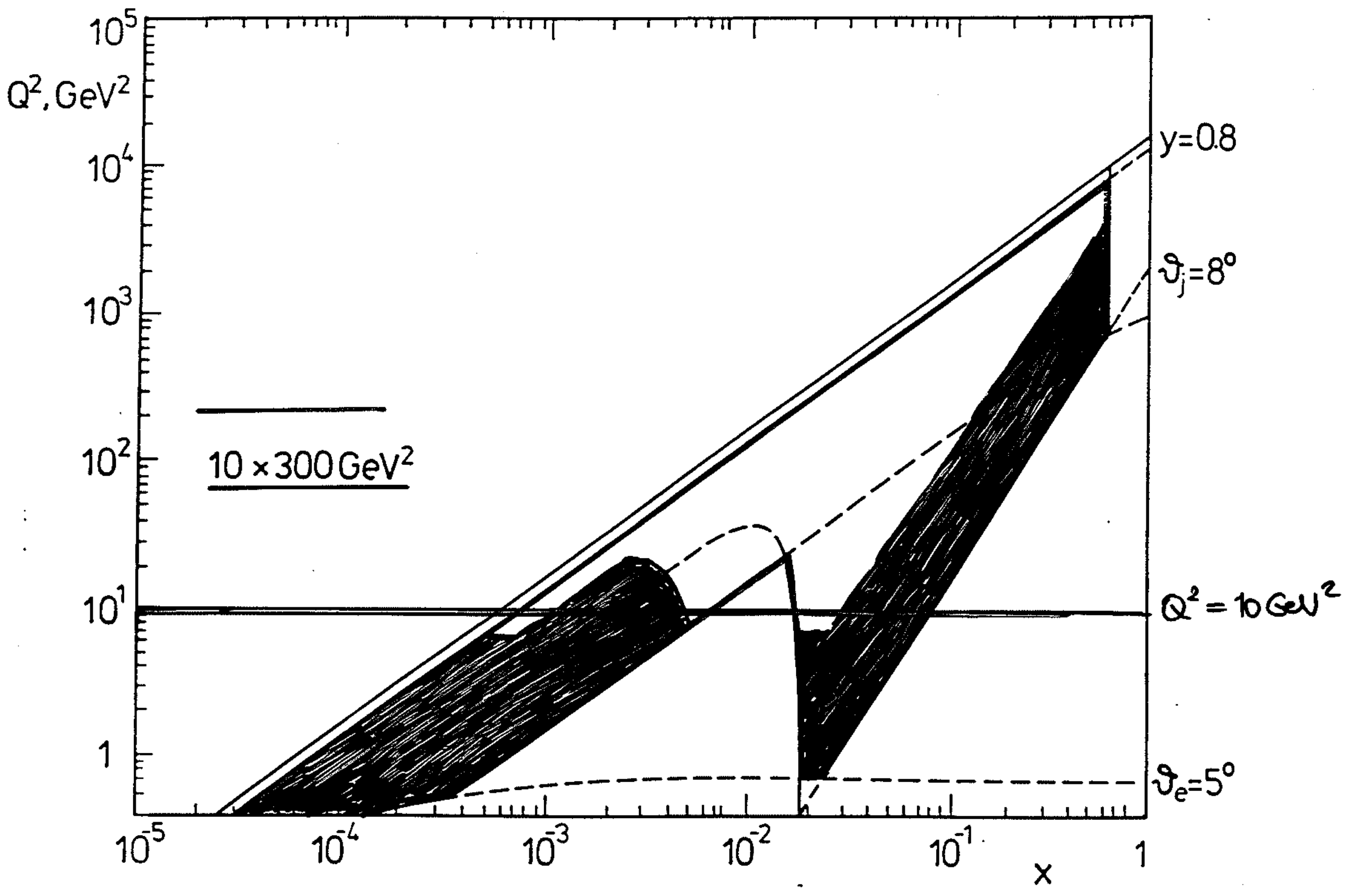
$x = 0.01 \ldots 1$ AP-EQU.

$$\begin{pmatrix} q_i(x,Q^2) \\ G(x,Q^2) \end{pmatrix} = \left(A_{kj}(Q^2, Q_0^2, x; \Lambda^2)\right) \otimes \begin{pmatrix} q_i(x,Q_0^2) \\ G(x,Q_0^2) \end{pmatrix}$$

$$(A \otimes B)(x) = \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x - x_1x_2)\, A(x_1)\, B(x_2)$$



Q², GeV²
10⁵
10⁴
10³
10²
10¹
1
e
p
ϑe
ϑj
30×820 GeV²
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
x
y=0.8
ϑj=8°
ϑe=5°


Q², GeV²
10⁵
10⁴
10³
10²
10¹
1
10×300 GeV²
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
x
y=0.8
ϑj=8°
Q²=10 GeV²
ϑe=5°

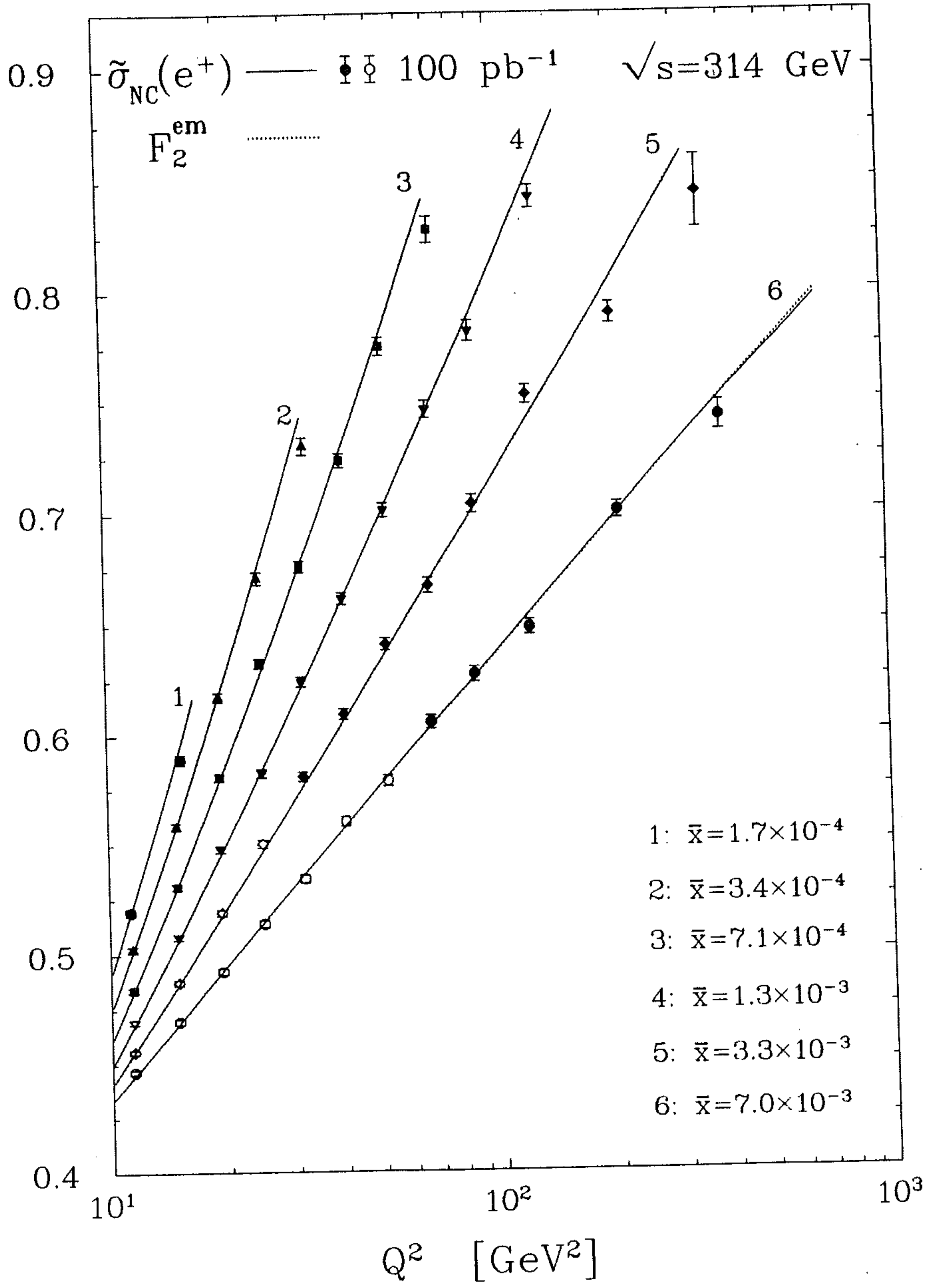

$\tilde{\sigma}_{NC}(e^+)$
100 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
1: $\bar{x}=1.7\times10^{-4}$
2: $\bar{x}=3.4\times10^{-4}$
3: $\bar{x}=7.1\times10^{-4}$
4: $\bar{x}=1.3\times10^{-3}$
5: $\bar{x}=3.3\times10^{-3}$
6: $\bar{x}=7.0\times10^{-3}$
Q^2 [GeV^2]

Fig. 6

PROBLEM:

$$F_i(x, Q^2) \underset{\substack{x \to 0 \\ AP}}{\sim} \exp \sqrt{\ln \frac{1}{x}}$$

⟶ HO TWIST 2

⟶ NEW DYNAMICS TO GET

$$\sigma \sim \pi R^2(s)$$

⟶ SUMMATION OF LARGE EFFECTS AT SMALL x

2. EXTRAPOLATION OF AP TO SMALL x & $O(\alpha_s, \alpha_s^2)$ RESULTS

- CONSIDER ONLY GLUONS (DOMINATING, $x \ll 1$)

$$\mathbb{G}(x, Q^2) := x\,G(x, Q^2)$$

$$\frac{d\mathbb{G}(x, Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dx'}{x'} \Big[\underset{\uparrow\atop \text{LO}}{6} - \underset{\uparrow \atop \text{NTLO}}{\frac{61}{9} N_f \frac{\alpha_s}{2\pi}} \Big] \frac{x^2}{x'^2}\, \mathbb{G}(x', Q^2)$$

!

DF : $\quad y = \frac{8N_c}{\beta_0} \ln \frac{1}{x} \quad , \quad \xi = \ln \ln\left(\frac{Q^2}{\Lambda^2}\right)$

$$\frac{\partial^2 \mathbb{G}(y, \xi)}{\partial y\, \partial \xi} = \frac{1}{2}\, \mathbb{G}(y, \xi) \qquad \text{LO}$$

$$\frac{\partial^2 \mathbb{G}(y, \hat{\xi})}{\partial y\, \partial \hat{\xi}} = \frac{1}{2}\, \mathbb{G}(y, \hat{\xi}) \qquad \text{NTLO}$$

$$\hat{\xi} = \xi + f(\xi) \; ; \; f'(\xi) = -\left[\frac{\beta_1}{\beta_0} \xi e^{-\xi} + \frac{61}{63} \frac{2N_f}{\beta_0} e^{-\xi} \left(1 - \frac{\beta_1}{\beta_0} \xi e^{-\xi}\right)^2\right]$$

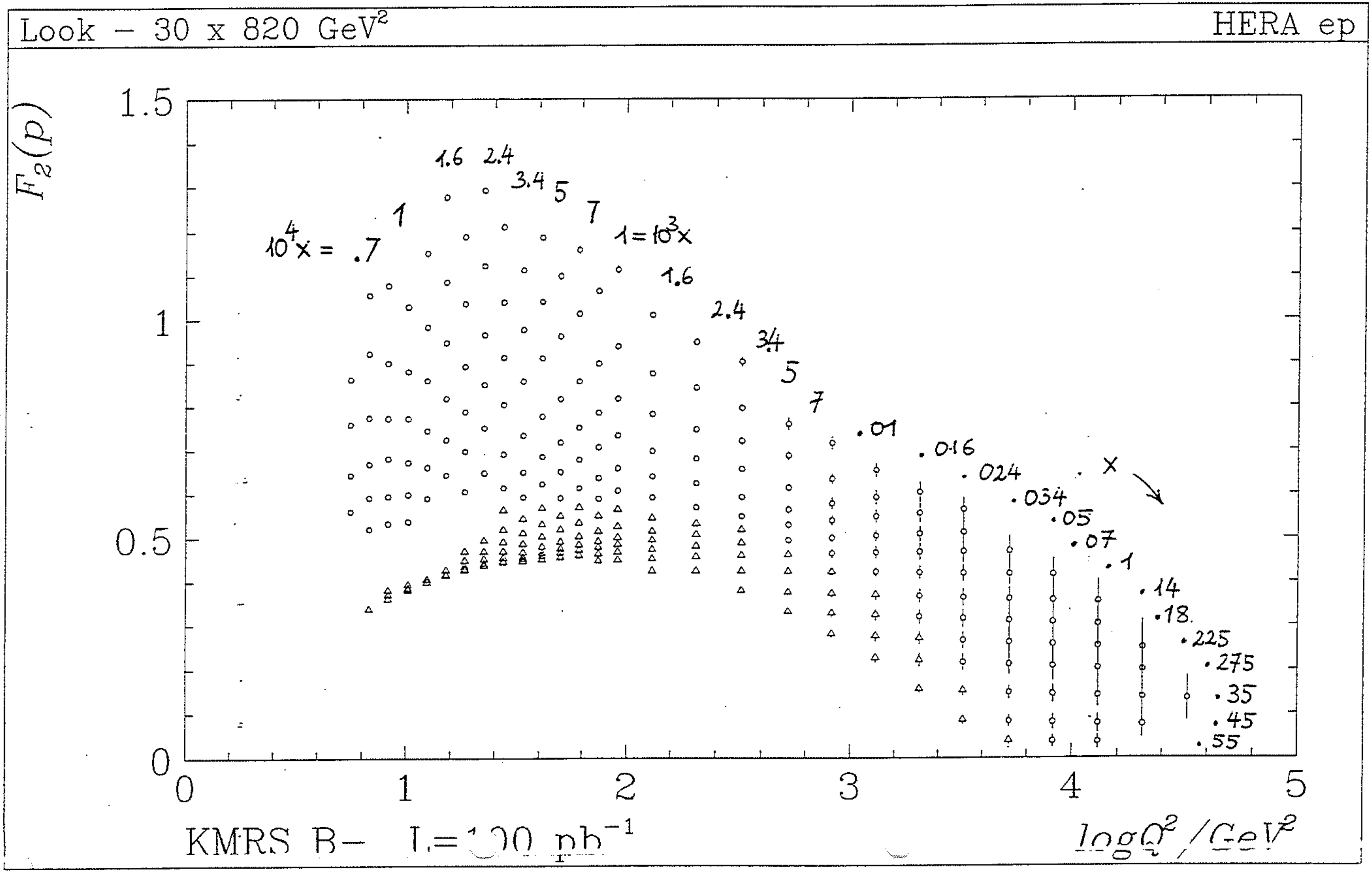
SOLUTIONS :

$$\mathbb{G}(y, \overset{(\wedge)}{\xi}) = \sum_{\nu=0}^{\infty} \left\{ A_\nu \left(\frac{2\overset{(\wedge)}{\xi}}{y}\right)^{\nu/2} + B_\nu \left(\frac{y}{2\overset{(\wedge)}{\xi}}\right)^{\nu/2} \right\}$$

$$\cdot\, I_\nu\left(\sqrt{2\xi y}\right)$$

$\mathbb{G}(y, \xi)_{y \to \infty}$ GROWS FASTER THAN A POWER OF $\ln(1/x)$.

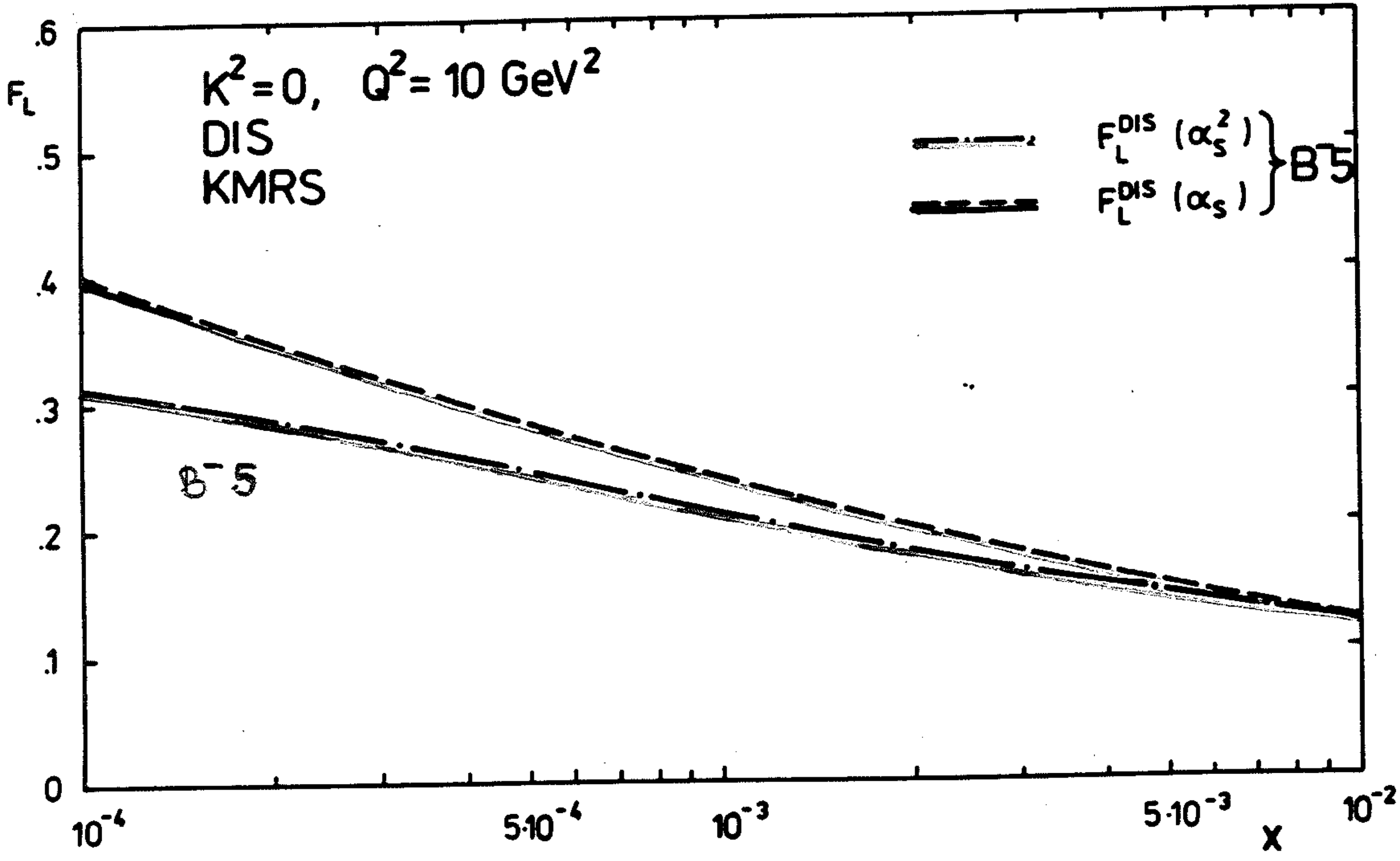
JB, M. KLEIN



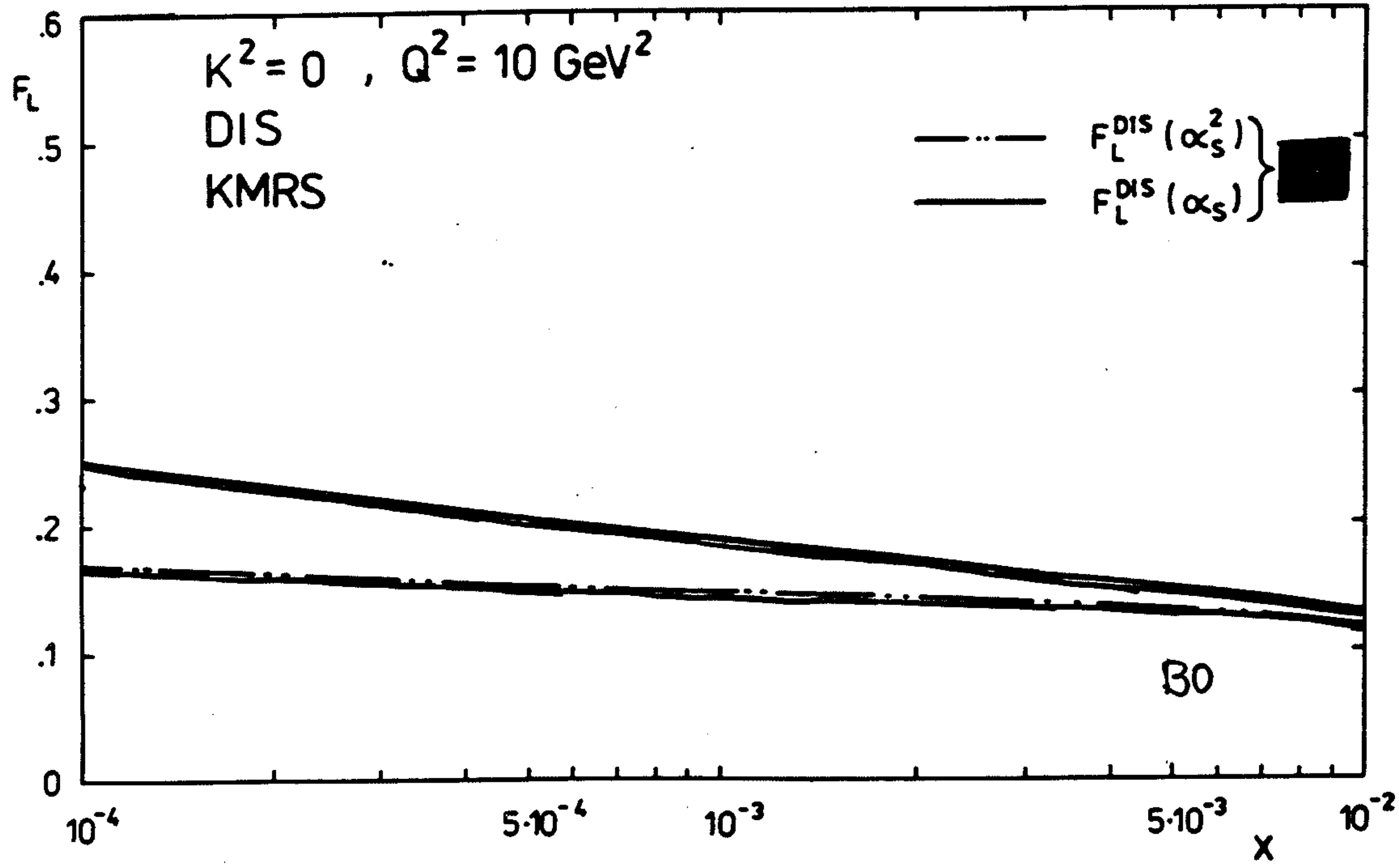
Look – 30 x 820 GeV²
HERA ep
F₂(p)
1.5
1
0.5
0
0
1
2
3
4
5
10⁴x = .7
1
1.6
2.4
3.4
5
7
1 = 10³x
1.6
2.4
3.4
5
7
.01
.016
.024
.034
.05
.07
.1
.14
.18
.225
.275
.35
.45
.55
x
KMRS B– L = 100 pb⁻¹
log Q² / GeV²

IMPORTANCE OF HIGHER ORDER CORRECTIONS

ZIJLSTRA, VAN NEERVEN



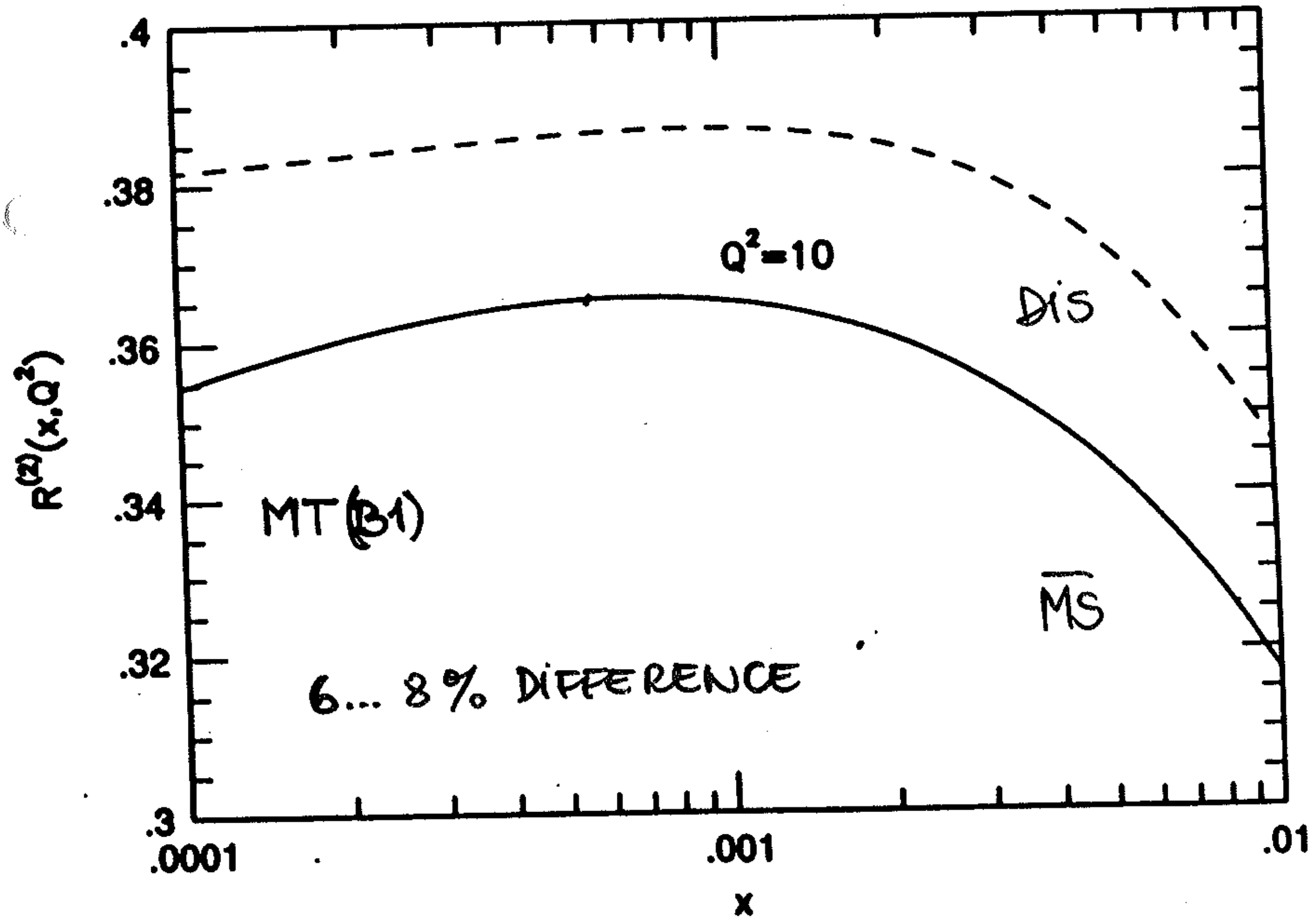
ZIJLSTRA, VAN NEERVEN

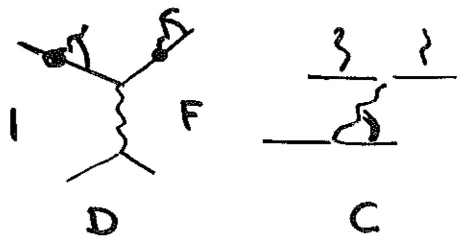


$$R^{(2)}(x,Q^2) = \frac{F_L^{(2)}(x,Q^2)}{\left(1+\frac{4M_p^2x^2}{Q^2}\right)F_2^{(1)}(x,Q^2) - F_L^{(2)}(x,Q^2)}$$

$O(\alpha_s^2)$

ZIJLSTRA, VAN NEERVEN





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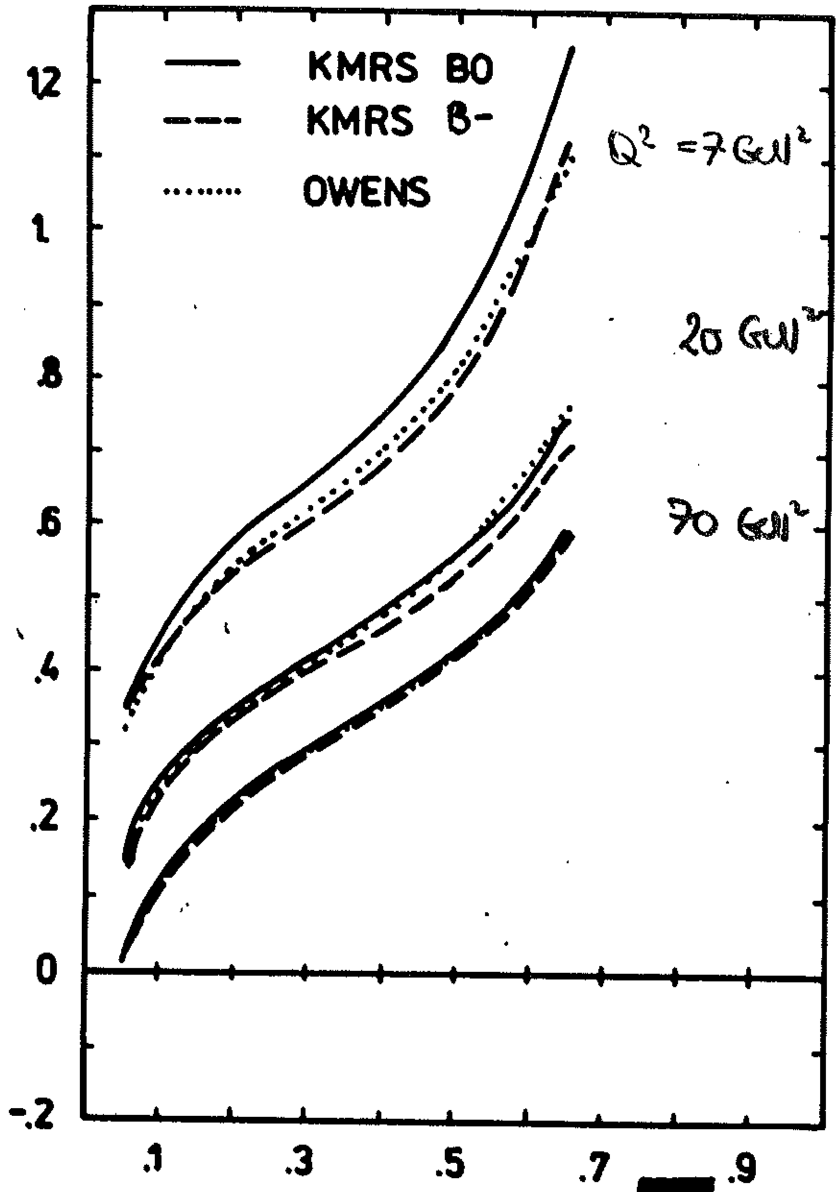


[redacted]

JB

cf. also
RC WG
DESY '91

$$\frac{d^2\sigma^{e^-p}}{dy\,dQ^2} = \frac{d^2\sigma_0^{e^-p}}{dy\,dQ^2}\left(1+\delta^{e^-p}(y,Q^2)\right)$$

$\mathcal{O}(\alpha)$ QED.

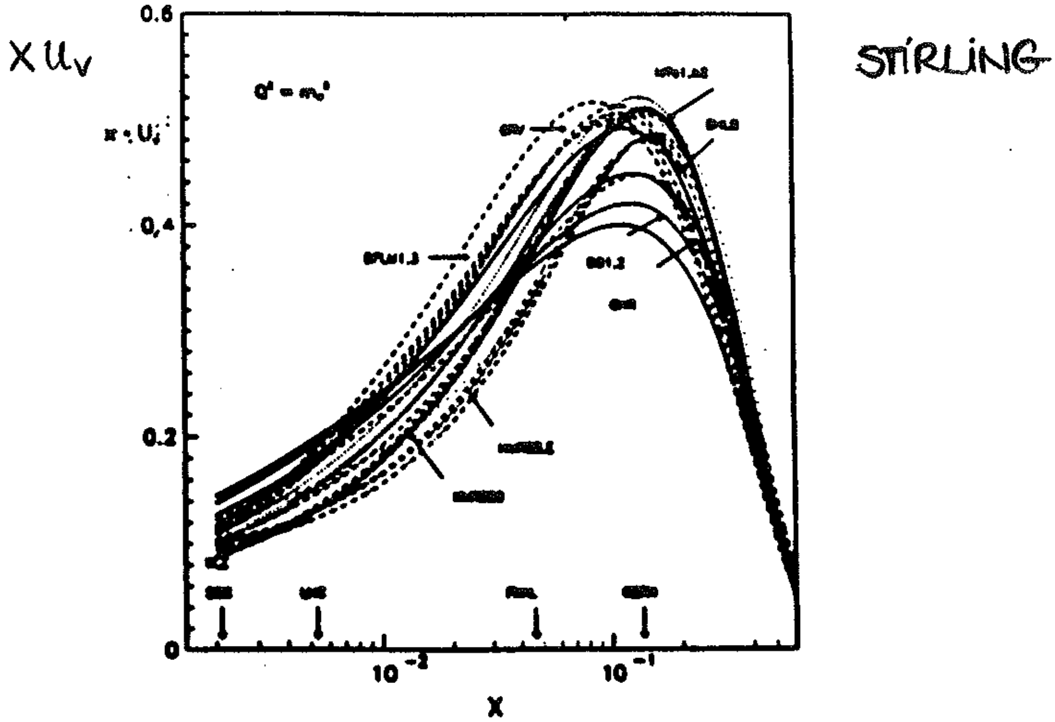


Figure 1: The valence u-quark distribution at $Q^2 = M_W^2$ [4].

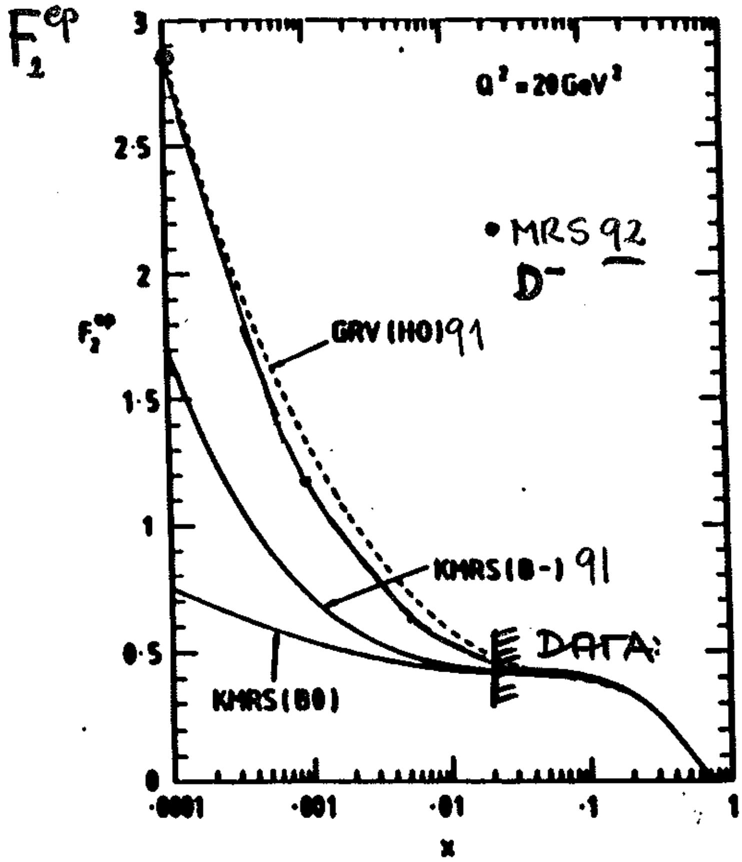


Figure 3: F_2^{ep} structure function predictions.

3. THE FKL - EQUATION

FADIN, KURAEV, LIPATOV; BALITZKII

$$f(n,k^2) = \frac{1}{n-1} f_0(n,k^2) + \frac{3}{\pi}(\mathcal{L} \otimes f)(n,k^2)$$

i) $\alpha_s = \text{const.}$ $\quad \mathcal{L} = L_1$

$$L_1 \otimes f_n = \frac{\alpha_s}{n-1} \int_{k_0^2}^{\infty} \frac{d\bar{k}^2}{\bar{k}^2} \left\{ \frac{k^2}{|\bar{k}^2 - k^2|} \left[f(n,\bar{k}^2) - f(n,k^2) \right] + \frac{f(n,k^2)\, k^2}{\sqrt{k^4 + 4\bar{k}^2}} \right\}$$

$k_0^2 = 0$

$\longrightarrow$ EIGENFUNCTIONS :

$$e(n,\omega) = \frac{e(n,\omega^0)}{n - \left(1 + \frac{3\alpha_s}{\pi} K(\omega)\right)}$$

n_0 - POLE of $e(n,\omega)$ FOR $\omega = 0$

$$n_0 = 1 + 2.64\,\alpha_s$$

COLLINS
KWIECINSKI

$$G(x) \longrightarrow \sim \frac{1}{x^{n_0}}$$

ii) α_s running: $\quad \mathcal{L} = L_2$

$$L_2 \otimes f_n = L_1(\alpha_s \to \alpha_s(k^2),\ k_0^2 > 0) \otimes f_n$$

$$1 + \frac{3.6}{\pi} \alpha_s(k_0^2) \leq n_0 \leq 1 + 4\ln 2 \cdot \left(\frac{3}{\pi}\right) \alpha_s(k_0^2)$$

$$1.31 \lesssim n_0 \lesssim 1.72$$

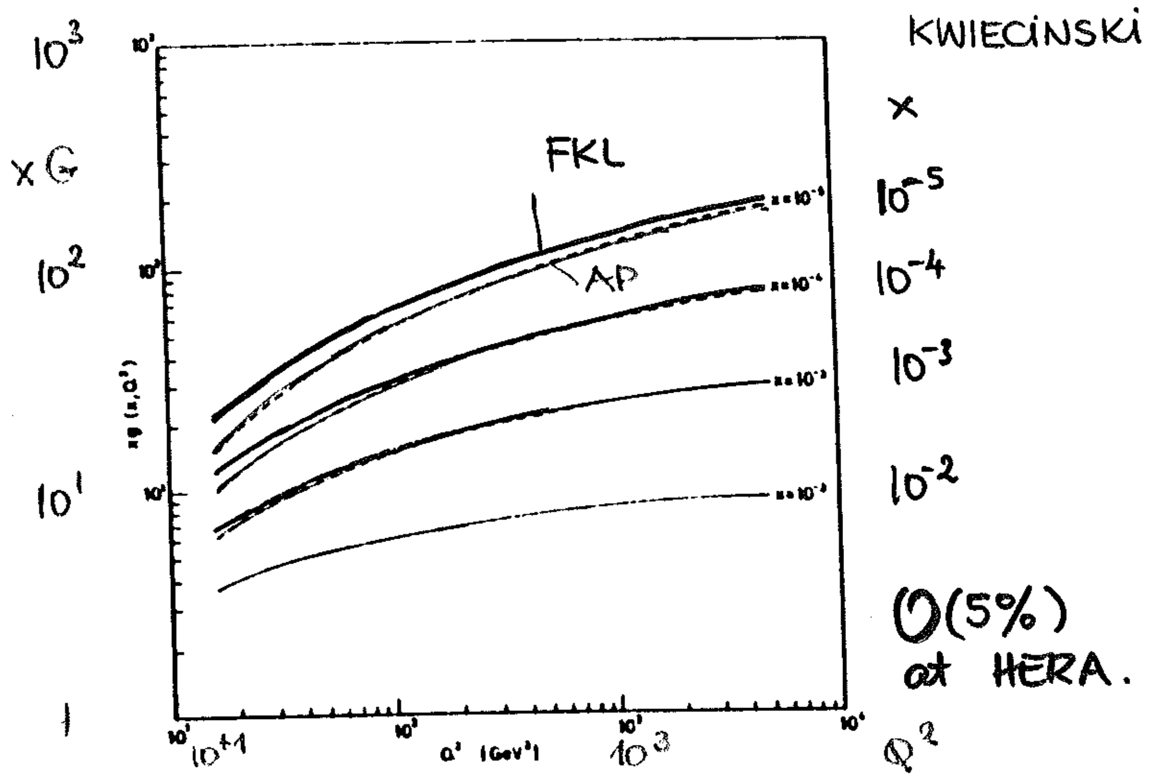


Fig. 3. The Q^2 evolution of gluon distributions beyond the leading ln 1/x approximation after corrections due to the constant term in the product $A_{gg}(\omega)\cdot\Gamma_{gg}(\omega)$ are included. The full line corresponds to the solution of the appropriately modified (3.6) and the dotted line to its leading log Q^2 counterpart

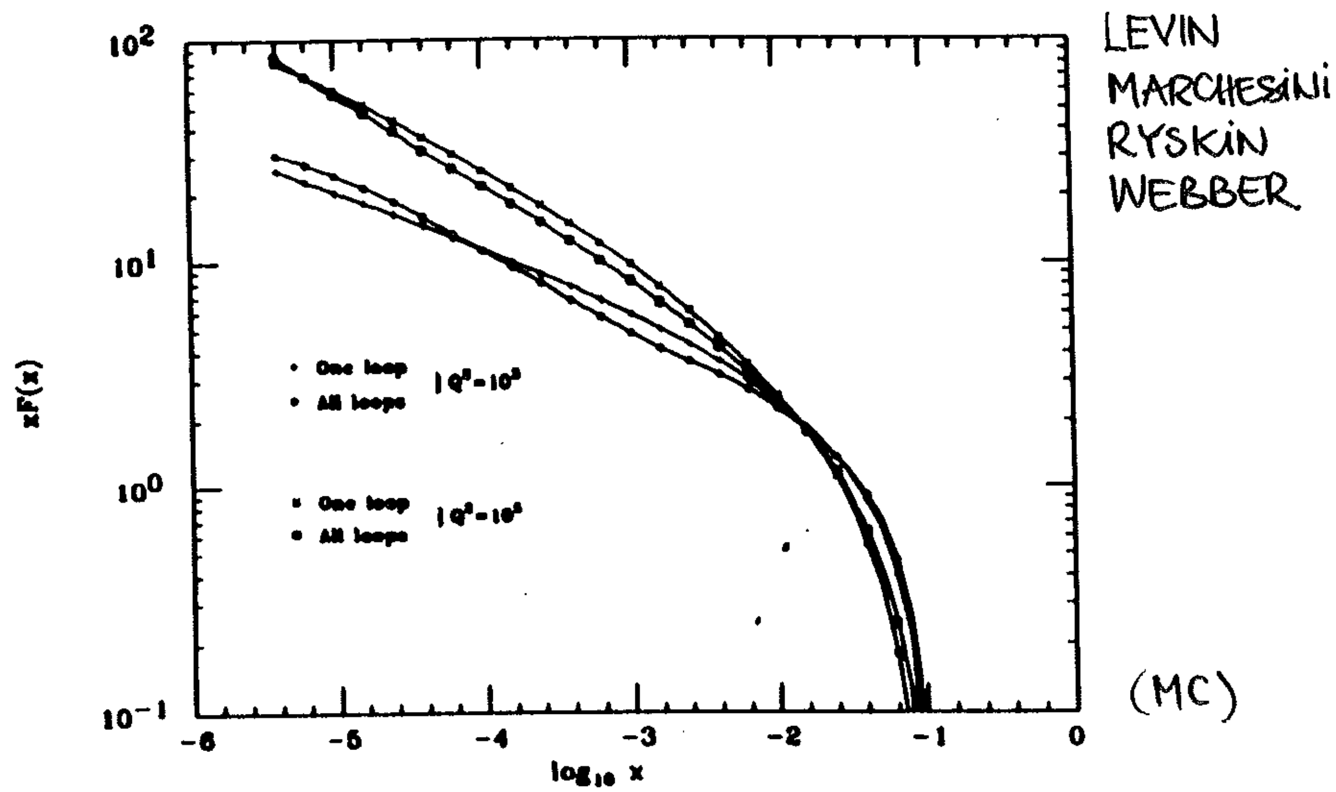
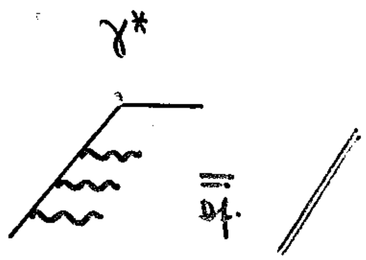


Fig. 6. Monte Carlo structure functions for $Q_0 = 0.4$ GeV with starting condition $xF(x, Q_0^2) = \delta(x - 0.1)$ at $Q_0^2 = 5$ GeV^2.

4. THE GLR EQUATION

a) TWIST-2 : EVOLUTION OF INDIVIDUAL PARTONS

$x \gtrsim 10^{-2}$ $\quad Q^2 > 10\ \mathrm{GeV}^2$



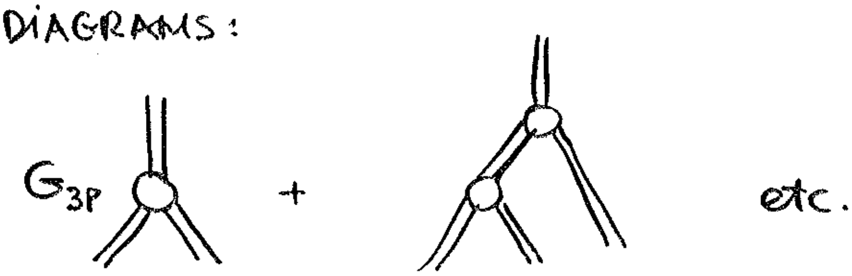
b) SEMI-HARD RANGE : x - GETTING SMALLER

$Q^2 \gtrsim 10 \dots 50\ \mathrm{GeV}^2$

PARTONS RECOMBINE – FINITE 'PROTON' RADIUS

+ HIGH PARTONIC DENSITIES

DIAGRAMS :



- GRIBOV, LEVIN, RYSKIN, 1982: LEADING PART AT EACH NODE
  $G + G \to G$ (Ladders)

- BARTELS 1992 (FURTHER CONTRIBUTIONS)

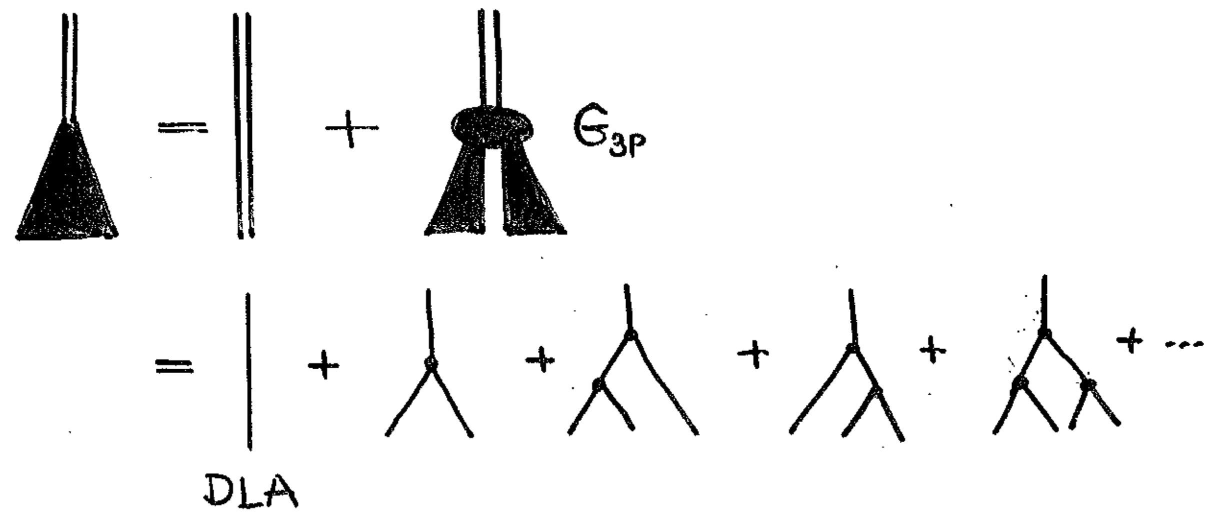
MUELLER, QIU: DLA-EXPRESSION FOR G_{3P}.

$$G_{3P}(\xi) = \frac{3}{4}\pi^2 \frac{1}{\beta_0} e^{e^{\xi_0}} \exp\left[-(e^{\xi}+\xi)\right]$$

$$= \frac{3}{4}\pi^2 \frac{1}{\beta_0} \left(\frac{Q_0^2}{Q^2}\right) \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \quad ; \text{HT}$$

$$= C \cdot \frac{\Lambda^2}{Q^2}$$

SUMMATION OF FAN DIAGRAMS:



GLR:

$$\underbrace{\frac{\partial^2 F(\xi,y)}{\partial\xi\,\partial y} = \frac{1}{2}}_{\text{DLA}} F(\xi,y) - C\exp\left[-(e^{\xi}+\xi)\right] F^2(\xi,y)$$

ASSUMPTION: $G_n(x_1,\dots,x_n) \simeq G(x_1)\dots G(x_n)$

IMPROVE WITH RESPECT TO REALISTIC INITIAL CONDITIONS:

(BARTELS, JB, SCHULER)

$$F(\xi,y) = G(\xi_0,y) + \int_{\xi_0}^{\xi} d\xi' \int_0^{y} dy'\, F(y',\xi')\left[\frac{1}{2} - C\exp\left[-(e^{\xi'}+\xi')\right] \times F(\xi',y')\right]$$

$\uparrow$

INPUT AT Q_0^2.

PROPERTIES OF THE EQUATION

i) SOLUTIONS ARE BOUNDED FROM ABOVE:

$F \leq F_0(\xi, y)$ (DLA) (AND BELOW $F_{Plup} \geq 0$!)

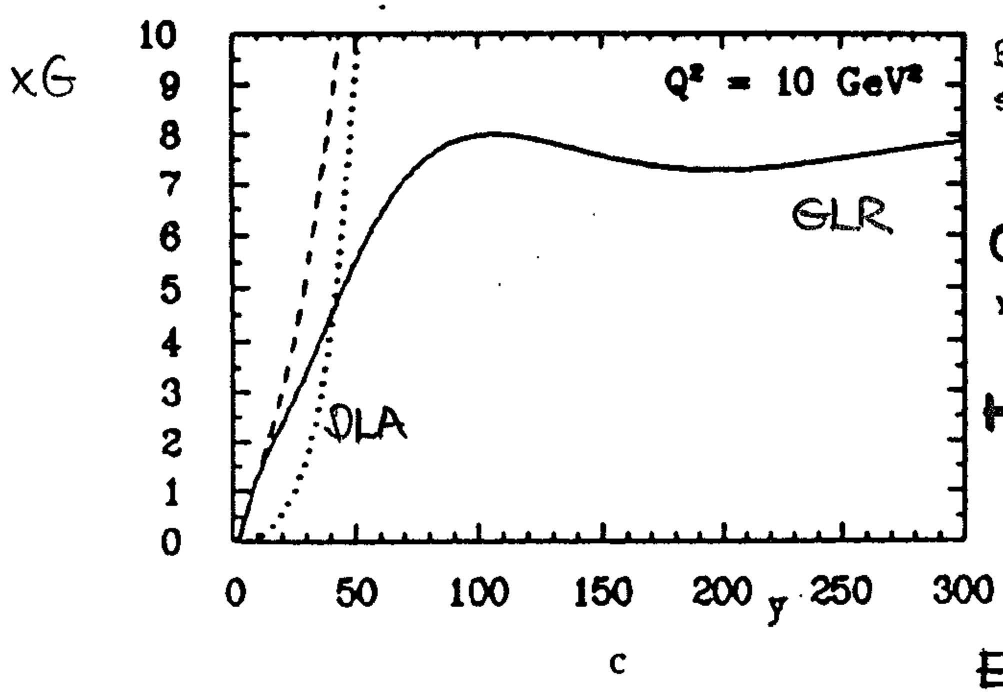
ii) $\lim_{y \to \infty} F(y, \xi) = \frac{1}{2C} \exp\left[e^{\xi} + \xi\right]$

$= \frac{1}{2C} \frac{Q^2}{\Lambda^2} \cdot \ln\left(\frac{Q^2}{\Lambda^2}\right)$ - const ξ or Q^2.

INDEPENDENT OF THE PARTICULAR CHOICE OF $G(y, \xi_0)$!

iii) $\exists!\ F(y, \xi)$

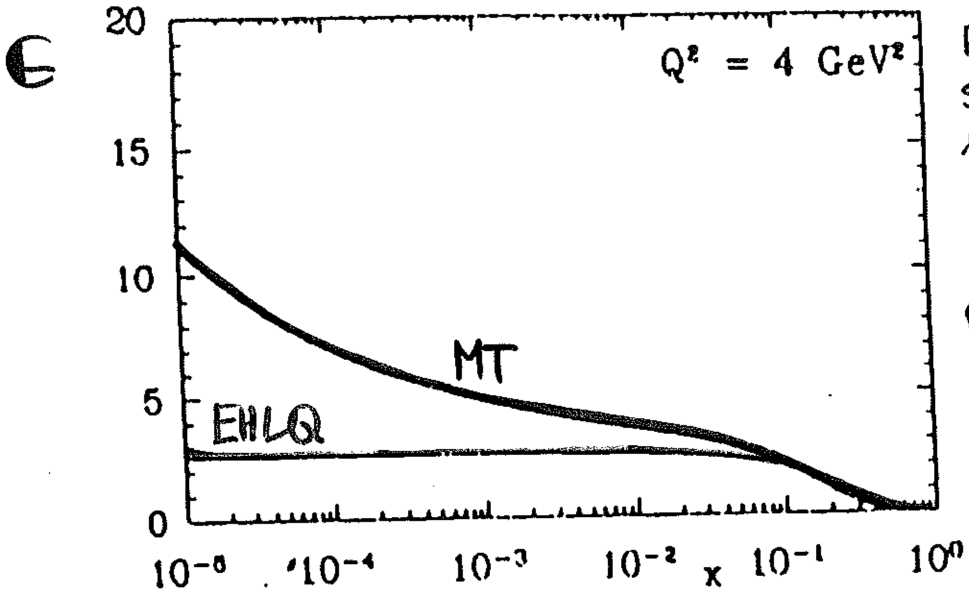
SOLUTION: QUADR. EQU. (LOCALLY) AT A SOFF. FINE GRID IN (ξ, y)
'VOLTERRA-TYPE' EQ. BOTH IN ξ & y.



BARTELS, JB, SCHULER

OSCILLATIONS
'FEED BACK'
$\mapsto$ SATURATION

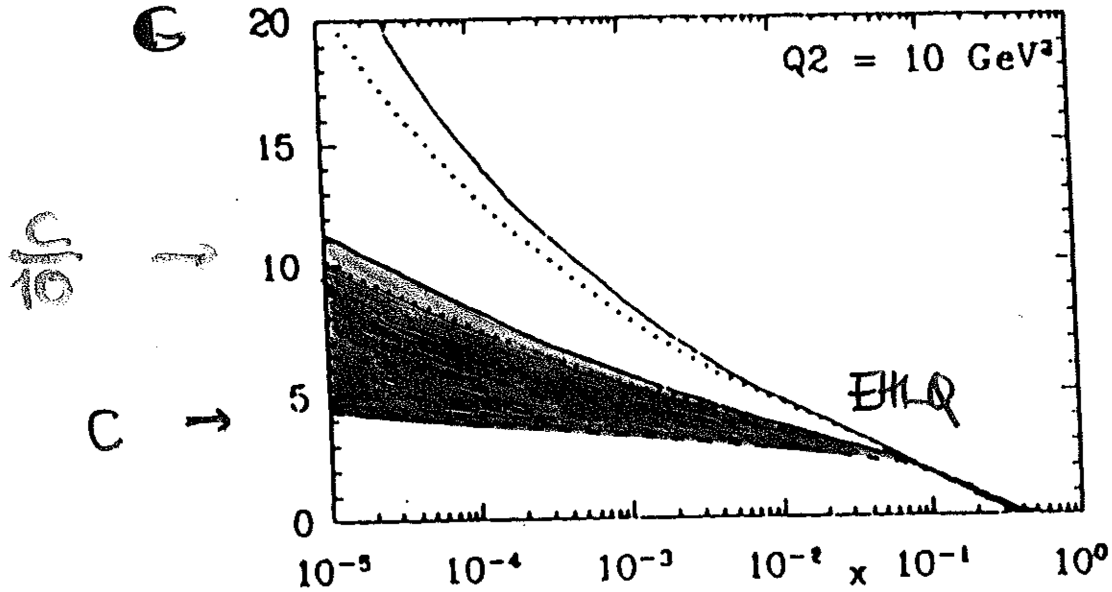
EXTREME y



BARTELS, JB, SCHULER 1990

(cf. COLLINS, KWIECINSKI 1990; ALTMANN, GLÜCK, REYA, 1992 FOR SIMILAR INVESTIG.)

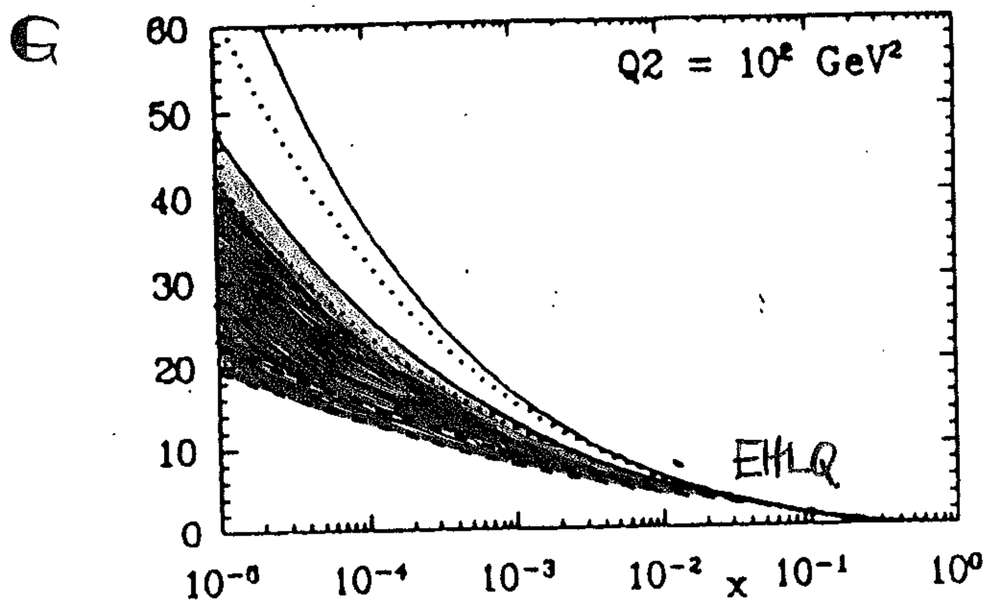
a



$C = C(\hat{Q}_0^2)$

$\hat{Q}_0 = \hat{Q}_0(R_{sc})$

b



c

SEMICLASSICAL SOLUTIONS

BARTELS, JB, SCHULER ; COLLINS, KWIECINSKI

$$F(y,\xi) \underset{Df}{=} \exp\left(S(y,\xi)\right)$$

$$S_{|y} S_{|\xi} = \frac{1}{2} - C \exp\left[S - e^{\xi} - \xi\right]$$

$$S_{|y} S_{|\xi} \gg S_{|y|\xi} \quad (\xi \cdot y \gg 1)$$

$$\dot{y} = S_{|\xi}$$

$$\dot{\xi} = S_{|y}$$

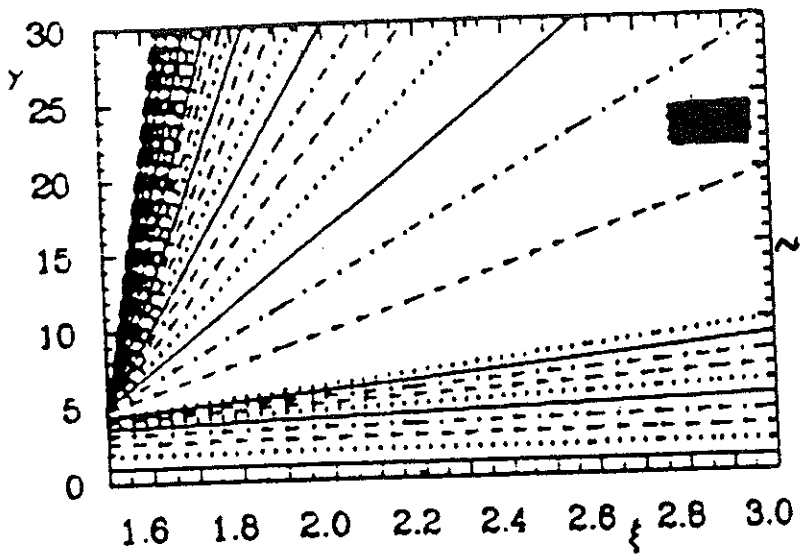
$$\dot{S} = 2 S_{|y} S_{|\xi}$$

$$\dot{S}_{|\xi} = -C\exp\left(S - e^{\xi} - \xi\right) S_{|y}$$

$$\dot{S}_{|\xi} = -C\exp\left(S - e^{\xi} - \xi\right)\left(S_{|\xi} - 1 - e^{\xi}\right)$$

$C = 0$ $\quad \ddot{y} = \ddot{\xi} = 0$: STRAIGHT LINES IN ξ & y.

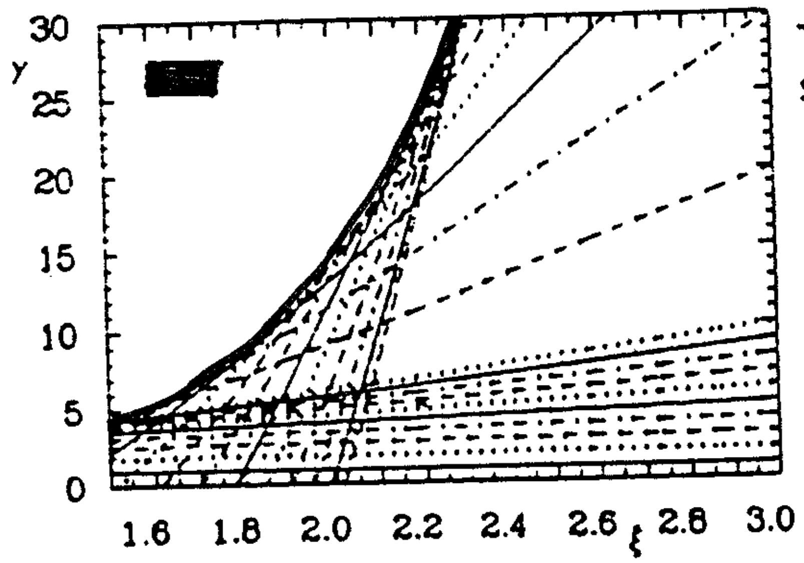
BARTELS, JB, SCHULER



~AP

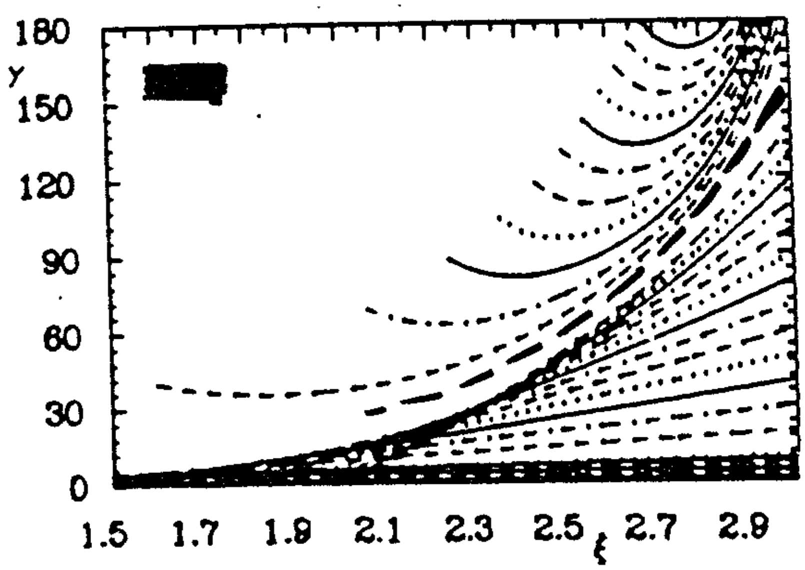
a

TRAJECTORIES STARTING BELOW



b

ABOVE THE 'CRITICAL' LINE



AP + FAN-DIAGRAMS (xG)

MUELLER, QIU

$$\frac{d\,x q_s(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\left[P_{qg}\otimes xG \qquad P_{qq}\otimes x q_s\right]$$

$$- \frac{27\,\alpha_s^2}{160\,R^2 Q^2}\left(x G(x,Q^2)\right)^2$$

$$+ \frac{\alpha_s}{\pi Q^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\,\gamma\left(\frac{x}{x'}\right)\,x G_H(x',Q^2)$$

$$\gamma(y) = -2y + 15y^2 - 30y^3 + 18y^4$$

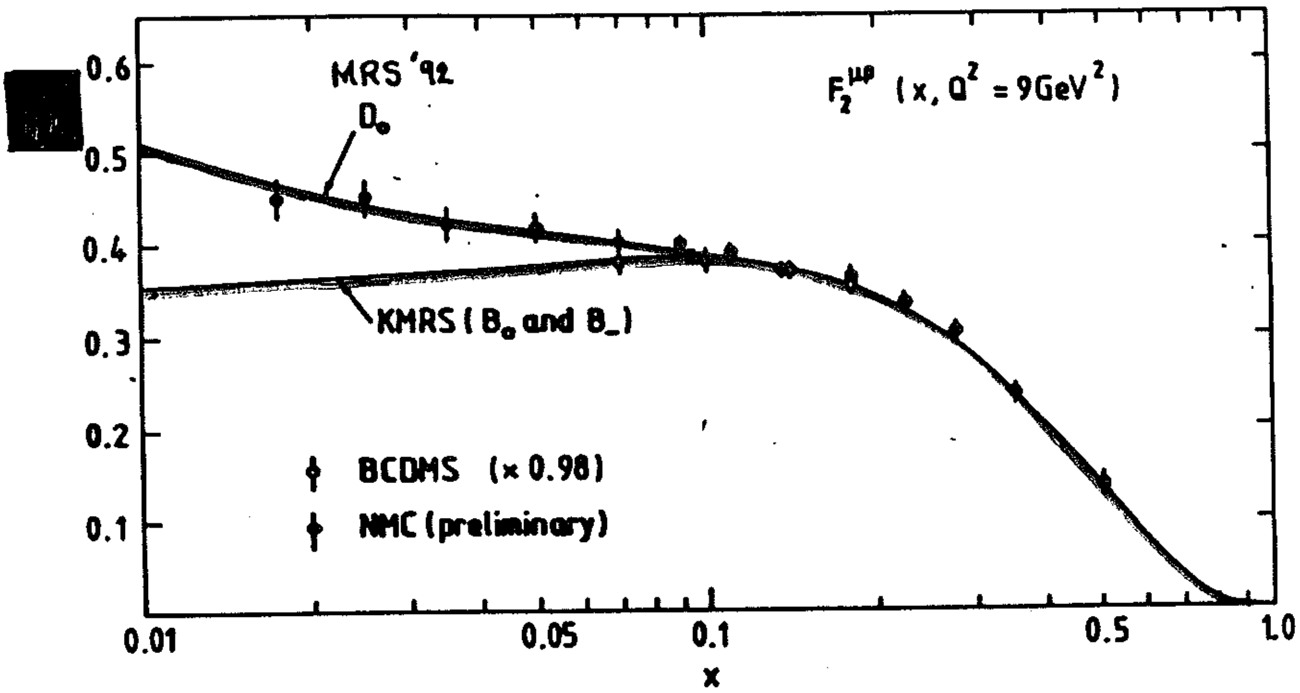
$$\frac{d\,xG_H(x,Q^2)}{d\ln Q^2} = -\frac{81\,\alpha_s^2}{16\,R^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\left[x' G(x',Q^2)\right]^2$$

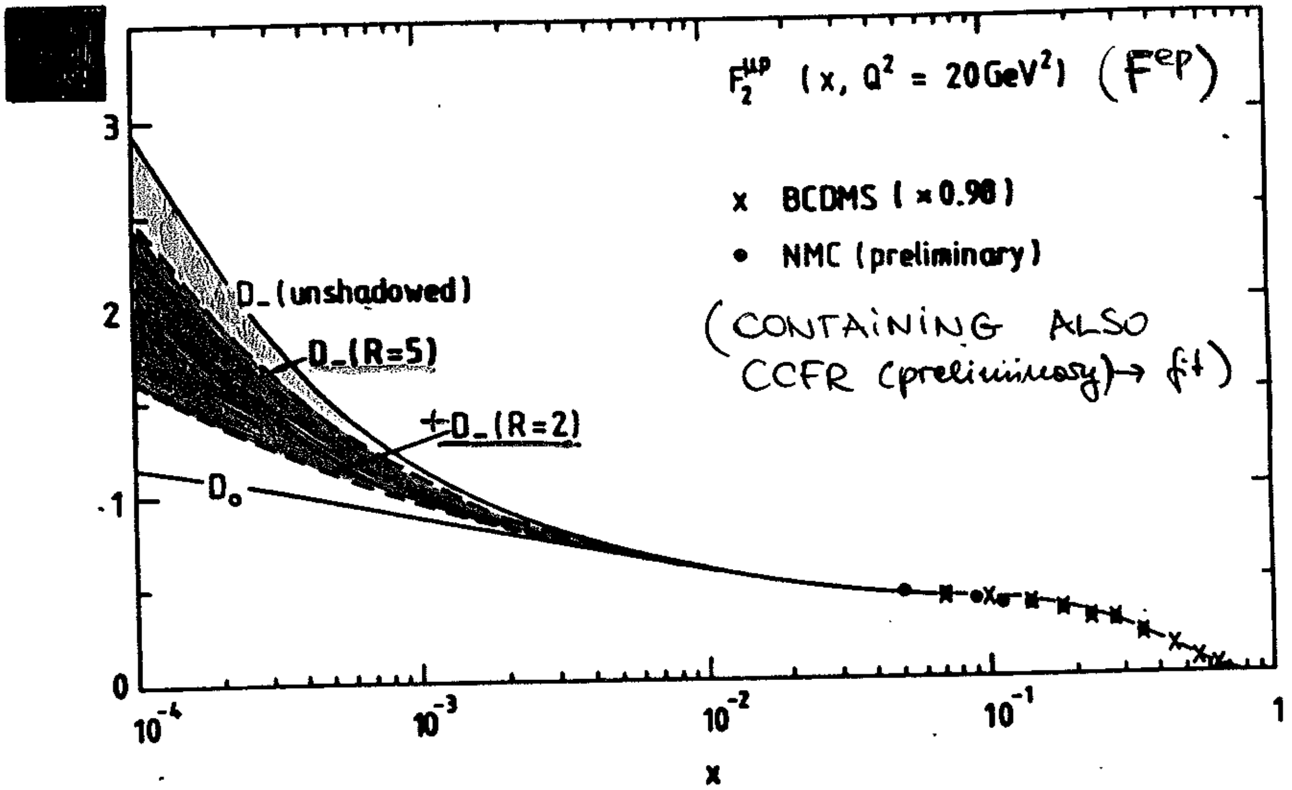
$$\frac{d\,xG(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\left[P_{gg}\otimes xG + P_{gq}\otimes xq\right]$$

$$- \frac{81\,\alpha_s}{16\,R^2 Q^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\left[x' G(x',Q^2)\right]^2$$

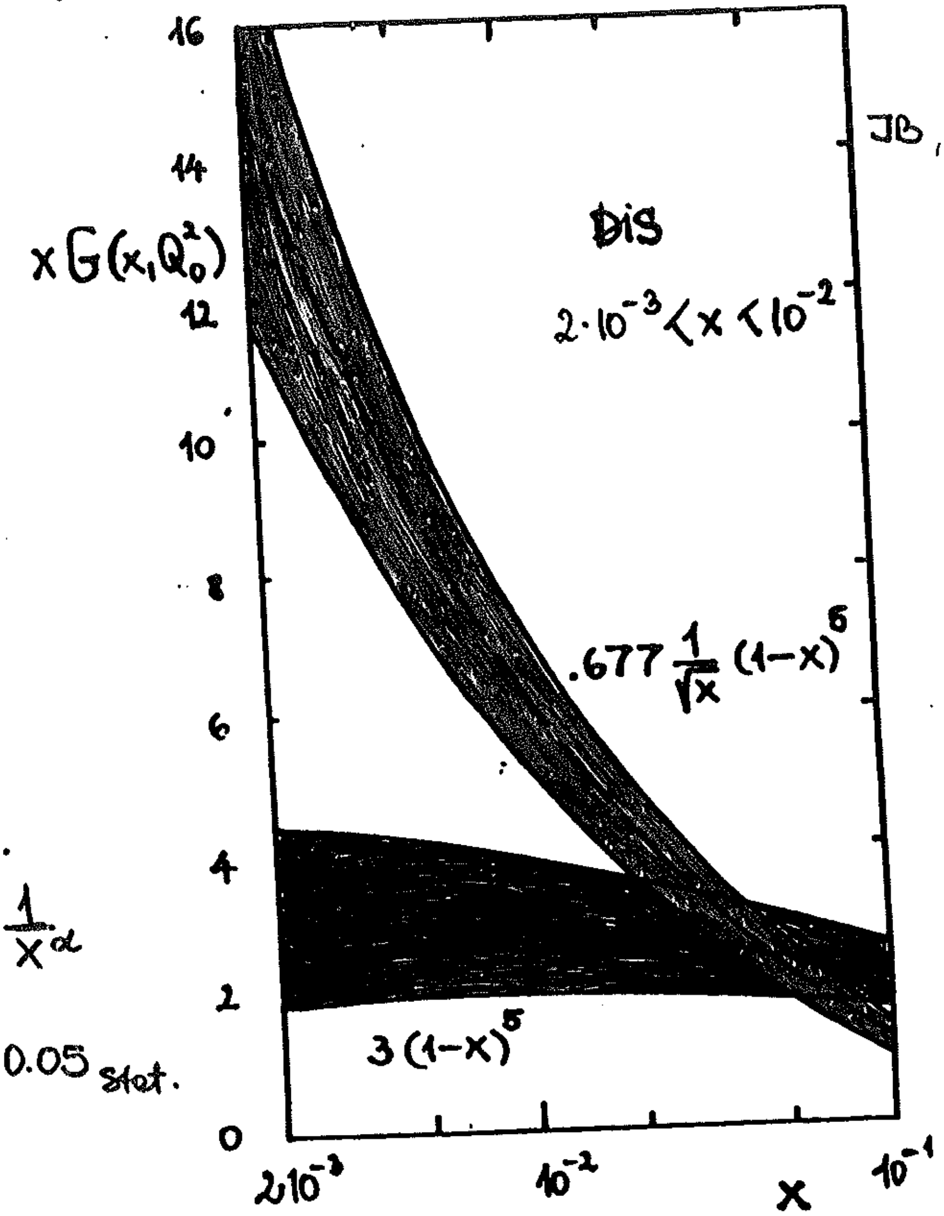
USED IN: KMRS

MARTIN, ROBERTS, STIRLING





JB, M.KLEIN



$xG \propto \frac{1}{x^\alpha}$

$\delta\alpha \simeq \pm 0.05_{stat.}$

Figure 8: Possible determination of $xG(x,Q_0^2)$ in a QCD fit for $x < 0.1$, see text. The upper error band corresponds to the choice $\alpha = -0.5$ and the lower band to $\alpha = 0$, see eq. 15. The inner error denotes the statistical error for $\mathcal{L} = 100\ pb^{-1}$ for both the low and high s option.

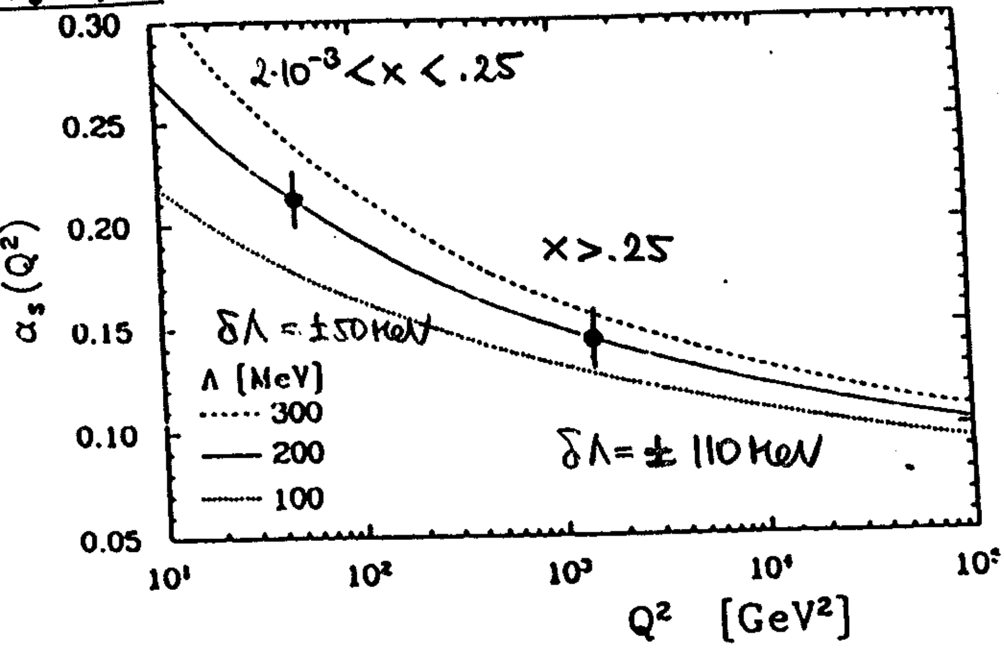
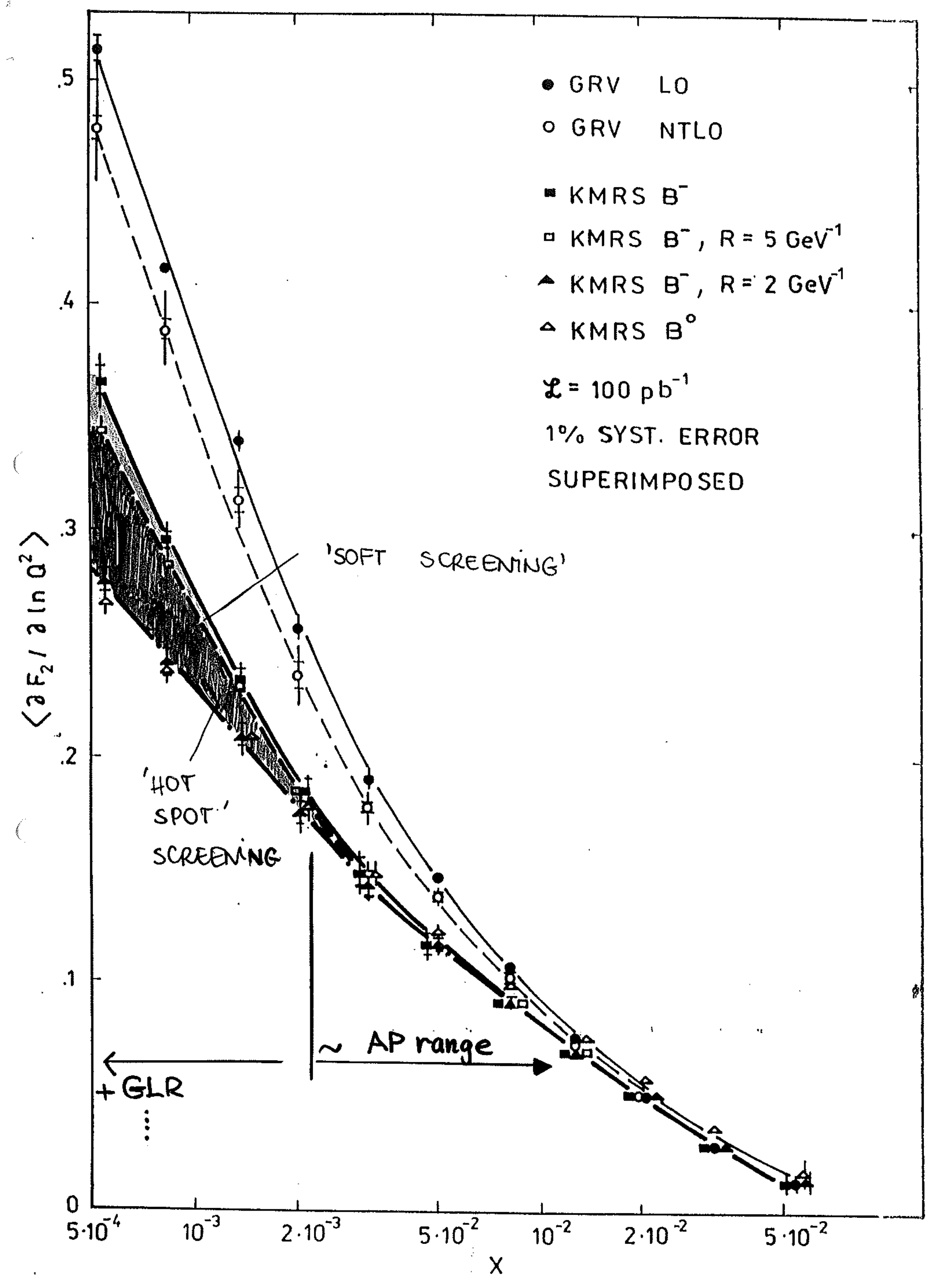
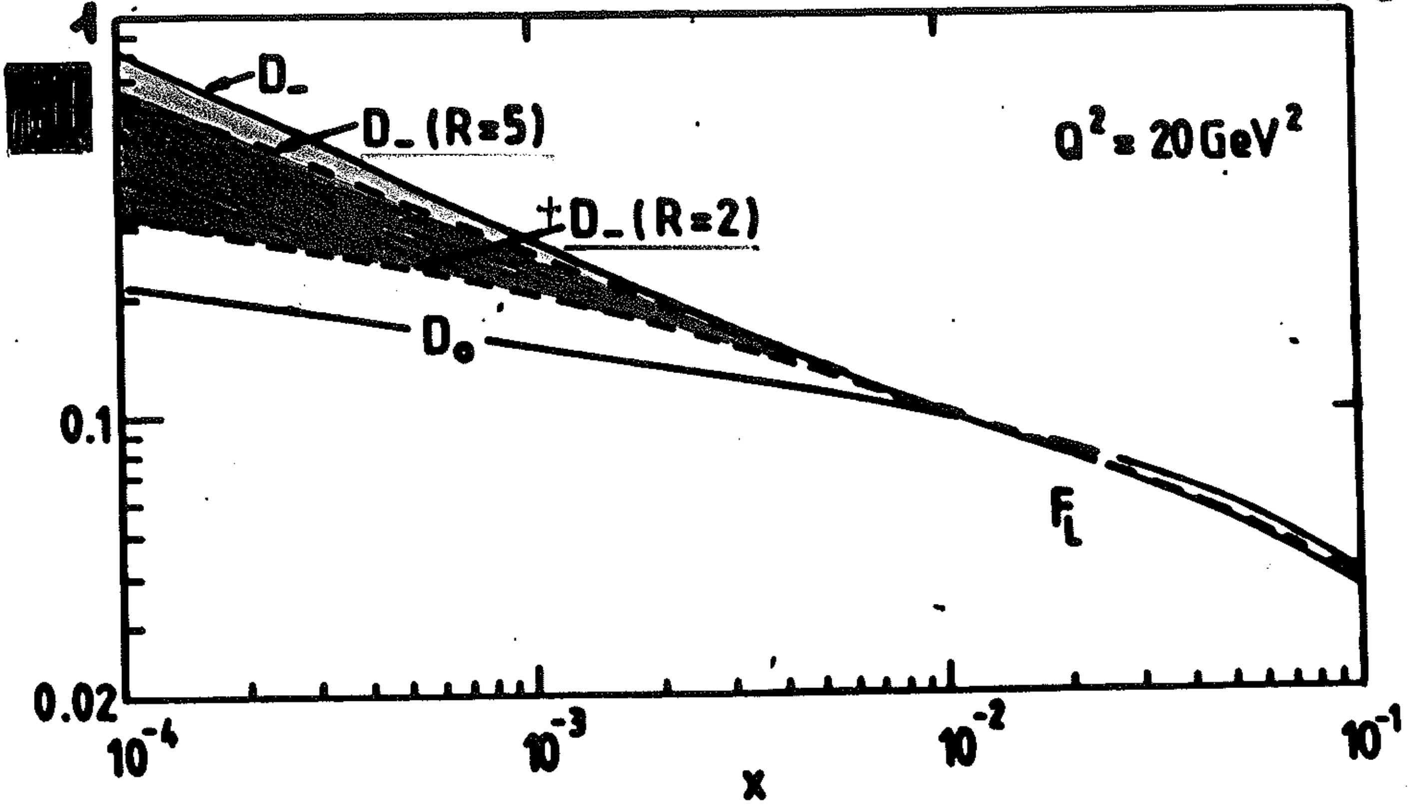


Figure 7: Dependence of α_s on Q^2 from a combined fit using two samples of $\sqrt{s} = 314$ GeV and $\sqrt{s} = 110$ GeV with $\mathcal{L} = 100\ pb^{-1}$ each. The upper point corresponds to a nonsinglet fit for $\theta_e > 5°$ and $x > 0.25$. The lower point at $Q^2 \sim 50$ GeV^2 corresponds to a fit in the



GRV LO
GRV NTLO
KMRS B$^-$
KMRS B$^-$, R = 5 GeV^{-1}
KMRS B$^-$, R = 2 GeV^{-1}
KMRS B^0
$\mathcal{L}$ = 100 pb^{-1}
1% SYST. ERROR
SUPERIMPOSED
'SOFT SCREENING'
'HOT SPOT' SCREENING
~ AP range
+ GLR
$\langle \partial F_2 / \partial \ln Q^2 \rangle$
x

MARTIN, ROBERTS, STIRLING



$O(\alpha_s)$:

$$F_L(x,Q^2) = \frac{\alpha_s}{2\pi}\left\{ \frac{8}{3}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y,Q^2) + 2\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) y\, \underline{G(y,Q^2)} \right\}$$

ROBERTS:

$$xG(x,Q^2) \simeq \frac{3}{5} \times 5.85 \left\{ \frac{3\pi}{4\alpha_s} F_L(0.4x,Q^2) - \frac{1}{2} F_2(0.8x,Q^2 \right\}$$

COOPER-SARKAR, DEVENISH, LANCASTER.

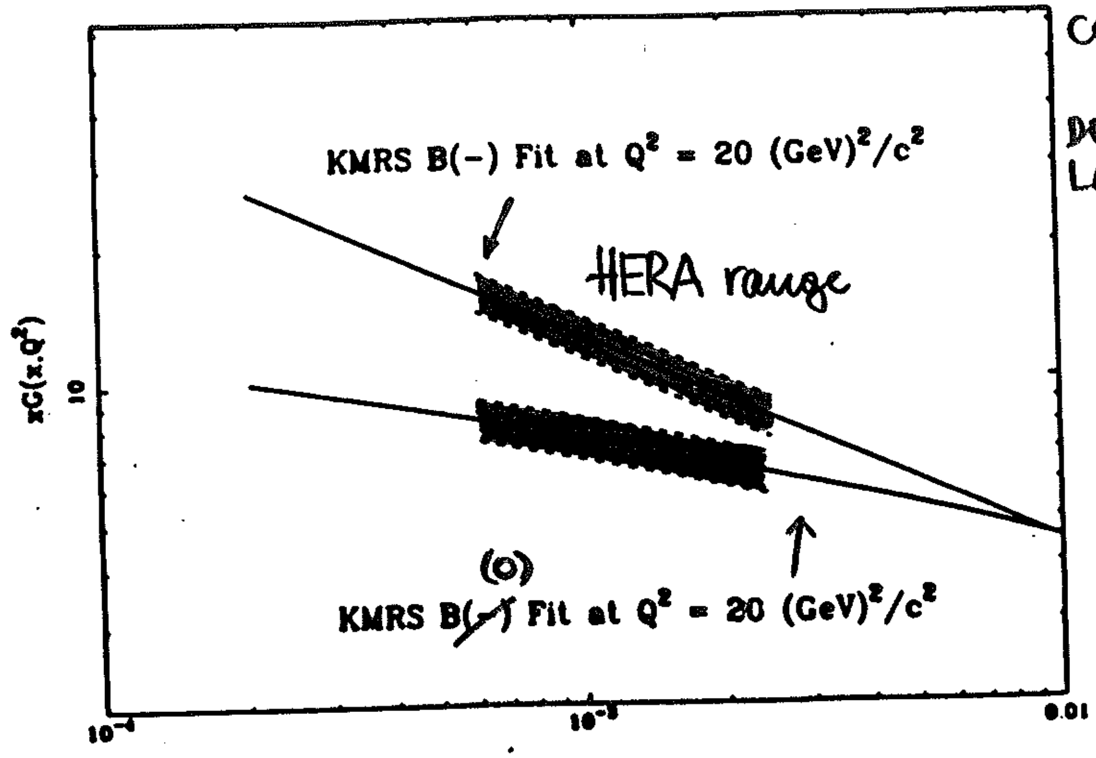


Fig 11 – Gluon at Low-x & measurable domain (with errors)

(FROM F_L^{AP} - O(α_s).)

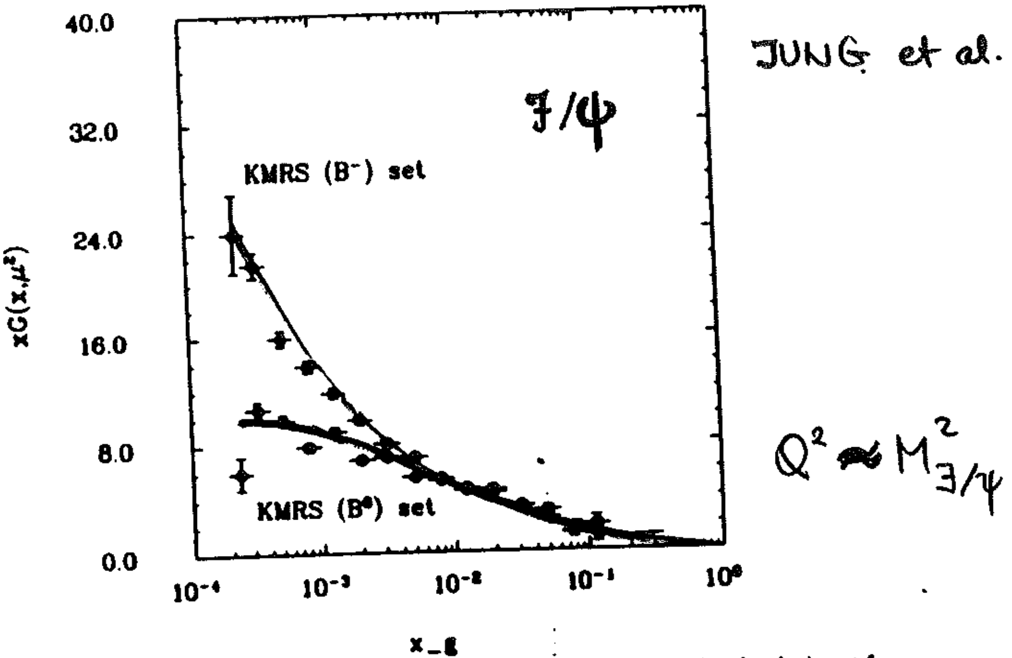


Figure 16: The gluon density reconstructed from inelastic J/ψ production for the input function of KMRS. The statistical error bars correspond to an integrated luminosity of $100\,pb^{-1}$ The curves show the input gluon density.

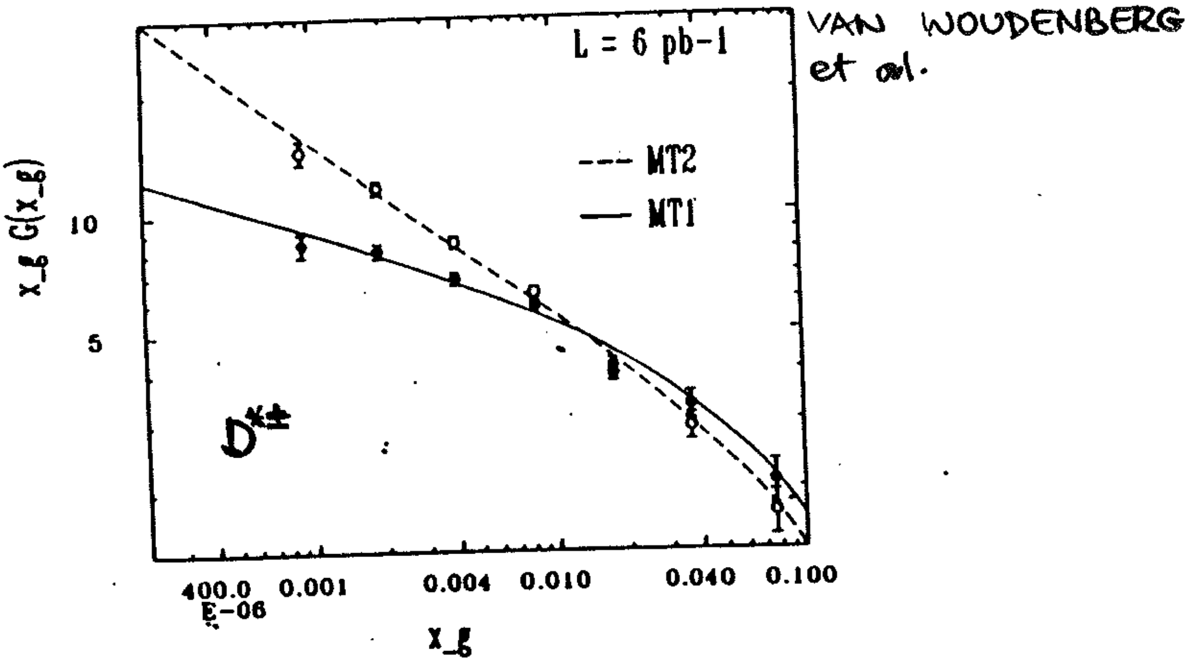
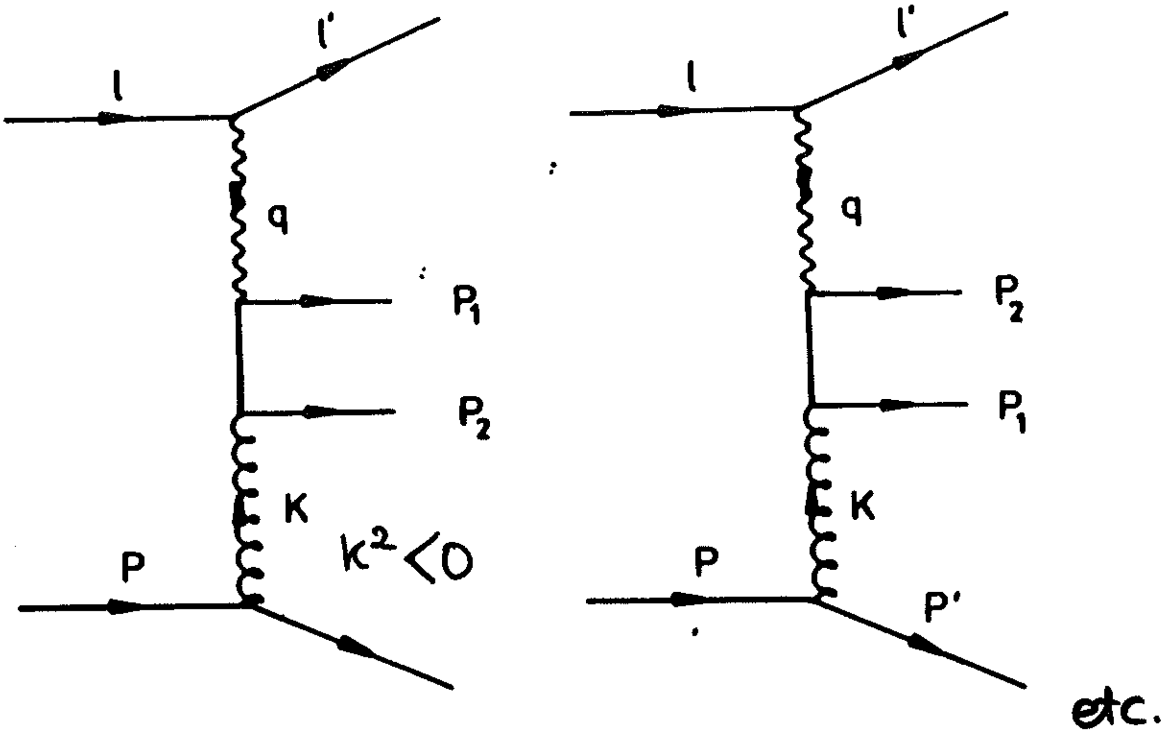


Figure 18

Reconstructed gluon densities from inclusive $D^{*\pm}$ production. The curves show the input gluon functions taken from Morfin and Tung [36]. The error bars include statistical errors for an integrated luminosity of $6\ pb^{-1}$.

EFFECTS DUE TO GLUON VIRTUALITY



FACTORIZATION

$$H(x,\mu^2) = \int d^2k \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) \mathcal{F}(x_1, k, Q_0^2) \sigma_H^{\gamma g}(x_2, \mu^2, k)$$

$$\mathcal{F}(x, k, Q_0^2) = \frac{\partial\, x G(x, k_\perp^2)}{\partial k_\perp^2}$$

— HF: CIAFALONI, CATANI, HAUTMANN
COLLINS, ELLIS, R.K.,
LEVIN, RYSKIN, SHABELSKI, SHUVAEV

SMALL x 'HIGH ENERGY' APPR.: *)

- LEVIN, RYSKIN (GRIBOV, LIPATOV, FROLOV; CHENG, WU)

→ PROBLEMS:

i) CONTACT TO THE USUAL $k^2=0$ PDF's ?!
→ WORK IN A WELL DEFINED FACTORIZATION SCHEME
→ THE DERIVED TERMS ARE <u>NON-SINGULAR</u>!

ii) CONVOLUTIONS IN x REQUIERE THE KNOWLEDGE OF f_{qg} etc. <u>ALSO</u> FOR LARGE x. (→ FULL P_{ij}'s)

iii) PHASE SPACE REQUIREMENTS HAVE TO BE MET.

$x \ll 1$: HOT SPOT : • BARTELS, LOEWE, DE ROECK;
• TANG;
• KWIECINSKI, MARTIN, SUTTON.

$$\frac{d^2\sigma}{dQ^2 dy} = 2\pi\alpha^2 \frac{Ms}{(s-M^2)^2} \frac{1}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$\begin{aligned} L_{\mu\nu} &= 2\left[l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu} l.l'\right] \\ W_{\mu\nu} &= \frac{1}{4\pi} \sum_X \langle P|J^\dagger_\mu(0)|n\rangle\langle n|J_\nu(0)|P\rangle(2\pi)^4\delta^{(4)}(P+q-p_X) \end{aligned}$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) + \frac{1}{M^2}\left[\left(P_\mu - \frac{P.q}{q^2} q_\mu\right)\left(P_\nu - \frac{P.q}{q^2} q_\nu\right)\right] W_2(x,Q^2)$$

$$\begin{aligned} F_2(x,Q^2) &= x T^1_{\mu\nu} W^{\mu\nu} = x\left(-g_{\mu\nu} + \frac{12x^2}{Q^2} P_\mu P_\nu\right) W^{\mu\nu} \\ F_L(x,Q^2) &= x T^2_{\mu\nu} W^{\mu\nu} = \frac{8x^3}{Q^2} P_\mu P_\nu W^{\mu\nu} \end{aligned}$$

$$\begin{aligned} -g^{\mu\nu}\widehat{W}_{\mu\nu} &= 8\pi e_q^2 \alpha_s \left\{ -\frac{\hat{u}}{\hat{t}} - \frac{\hat{t}}{\hat{u}} + \frac{2\hat{s}Q^2}{\hat{u}\hat{t}} - k^2\left[\frac{Q^2}{\hat{t}^2} + \frac{Q^2}{\hat{u}^2} + \frac{2\hat{s}}{\hat{u}\hat{t}}\right]\right\} \\ \eta^2 P^\mu P^\nu \widehat{W}_{\mu\nu} &= 16\pi e_q^2 \alpha_s \eta^2 \left\{ \frac{A_1}{\hat{t}} + \frac{A_2}{\hat{u}} + \frac{\hat{s}A_3}{\hat{u}\hat{t}} + k^2\left[\frac{B_1}{\hat{t}^2} + \frac{B_2}{\hat{u}^2} + \frac{B_3}{\hat{u}\hat{t}}\right]\right\} \end{aligned}$$

$$\begin{aligned} A_1 &= -(k.P)^2 - 2(k.P)(q.P) + 2(k.P)(p_1.P) - (q.P)^2 + 3(q.P)(p_1.P) - 2(p_1.P)^2 \\ A_2 &= -(k.P)^2 - (k.P)(q.P) + 2(k.P)(p_1.P) + (q.P)(p_1.P) - 2(p_1.P)^2 \\ A_3 &= -(k.P)^2 - (k.P)(q.P) + 2(k.P)(p_1.P) + 2(q.P)(p_1.P) - 2(p_1.P)^2 \\ B_1 &= -(q.P)(p_1.P) + (p_1.P)^2 \\ B_2 &= (k.P)^2 + (k.P)(q.P) - 2(k.P)(p_1.P) - (q.P)(p_1.P) + (p_1.P)^2 \\ B_3 &= (k.P)^2 + 2(k.P)(q.P) - 4(k.P)(p_1.P) + (q.P)^2 - 4(q.P)(p_1.P) + 4(p_1.P)^2 \end{aligned}$$

$$F_i^G(x,Q^2) = \int_x^1 \frac{d\eta}{\eta} \int_{\Lambda^2}^{\mathbb{k}^2_{\perp max}} d\mathbb{k}_\perp^2 \; \frac{\partial \mathcal{G}(\eta, \mathbb{k}_\perp^2)}{\partial \mathbb{k}_\perp^2} \times \theta(\mathbb{k}^2_{\perp max} - \Lambda^2)$$

$$\times \left[\sigma^G_{i(0)}\left(\frac{x}{\eta}, Q^2\right) + \sigma^G_{i(1)}\left(\frac{x}{\eta}, Q^2, x\, \mathbb{k}_\perp^2\right) \right]$$

$$\mathbb{k}^2_{\perp max} = \frac{1}{x}\left(\frac{1-\eta}{1-x}\right)(\eta - x)\, Q^2 \quad , \quad \mathbb{K}^2 = \frac{\mathbb{k}_\perp^2}{1-\eta}$$

$$\sup k^2_{\perp max} = \frac{1-x}{4x} Q^2$$

Contact to the $\mathbb{K}^2 = 0$ representations of $F_i^G(x,Q^2)$:

$$\lim_{\mathbb{K}^2 \to 0} \sigma^G_{i(1)}(z, Q^2, x, \mathbb{k}_\perp^2) = 0$$

DIS scheme expr. derived ($\mathbb{K}^2 = 0$) (transition to other schemes as usual).

$\mathbb{K}^2/Q^2 \ll 1$: $\sigma^G_{i(1)} \sim A \cdot \frac{\mathbb{K}^2}{Q^2}(1 + \dots)$

Numerical work in progress:

1st estimates: · little effect for $x \gtrsim 10^{-2}$

· non-negligeble contribution for $x \sim 10^{-4}$

6. CONCLUSIONS

1) FOR $x \to 0$ THE CONTRIBUTIONS OF THE TWIST 2 TERMS YIELD A STRONG GROTH OF THE PDF'S IN LO & NTLO. (NTLO YIELDS A NEGATIVE CONTRIBUTION, BUT DOES NOT STOP THE GROTH.)

2) THE $O(\alpha_s^2)$ TERMS ARE VERY IMPORTANT QUANTITATIVELY : δF_i (HERA) $\sim$ 5%
$O(\alpha_s^2)$: 25% ...

3) THE FKL - TERMS YIELD $\sim$ 5% CONTRIBUTIONS AND SHOULD NOT BE NEGLECTED.

4) CAREFUL TREATMENT OF HF - TERMS IS REQUIRED ($\longrightarrow$ WU KI).

5) THE GLR EQU. LEADS TO SCREENING CORRECTIONS WHICH MIGHT BE REVEALED AT HERA.
IN ITS CONCEPT SATURATION MAY BE OBTAINED FOR $F_i(x, Q^2)$, $x \to 0$.

6) FURTHER WORK IS NEEDED TO CLARIFY WHETHER THE GLR TERMS THE ONLY RELEVANT ONES LEADING TO SHADOWING.

7) 'VIRTUAL' GLUON CONTRIBUTIONS AT SMALL x YIELD A FINITE CONTRIBUTION TO $F_i(x, Q^2)$.

8) HERA WILL PROVIDE EXP. TESTS VIA
F_2, F_L, $\sigma(c\bar{c})$, $\sigma(J/\psi)$ ON xG <u>AND</u> xq_s,
A & R_{sc}.